ROBUSTNESS OF MULTIPLE OBJECTIVE DECISION ANALYSIS PREFERENCE FUNCTIONS

DISSERTATION

William K. Klimack, Colonel, USA

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Abstract
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DISSERTATION

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William K. Klimack, B.S., M.S., M.M.A.S.

Colonel, USA

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William K. Klimack, B.S., M.S., M.M.A.S.
Colonel, USA

Approved:

	Date
______________________________ Kenneth W. Bauer, Jr. (Co-Chairman)	__________
______________________________ Jack M. Kloeber, Jr. (Co-Chairman)	__________
______________________________ Mark E. Oxley (Member)	__________
______________________________ Meir N. Pachter (Dean's Representative)	__________

Accepted:

______________________________ __________

Robert A. Calico, Jr. Date
Dean, Graduate School of Engineering and Management

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William K. Klimack

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List of Notation

Operators:

$\succ$: Is preferred to.

$\prec$: Is less preferred than.

$\sim$: Is indifferent to.

· : Is at least as preferred to.

¶ : Is at most as preferred to.

Diacritical Marks:

$\underset{\cdot}{u}$: Indicates that the utility domain is in v rather than x.

$\hat{u}$: Indicates that the function is an estimate of the true function.

$\bar{u}$: Indicates the mean of multiple functions.

Variables:

α : Current level of wealth.

b: Krzysztofowicz' indicator of local risk aversion in the value to utility transform, or, in RSM, the estimators of the coefficients for the hypothesized model.

β_i : In RSM, the coefficients for the hypothesized model.

$c(x)$: Pratt's measure of local risk aversion in utility.

δ : Acceptable endpoint error associated with the sigmoid function adapted for use to represent a preference function.

δ_{ik} : WRMSE measure of the difference between the mean elicited utility function and the elicited value function for the ith evaluation measure and the kth subject.

ε_{ik} : WRMSE measure of the difference between the two elicited utility functions for the *i*th evaluation measure and the *k*th subject.

E_i : The *elasticity* of a variable *i*.

G: Keller's measure of concordance of the value to utility transform function compared to the linear function.

$h(z)$: Chebyshev polynomial weighting function for WRMSE.

i : Index for discrete outcomes in an uncertain situation (in Chapter II). Subject Evaluation measure index.

j : Evaluation measure index.

J : Number of evaluation measures.

$j(x)$: General weighting function for WRMSE

k : Increment index used in determining goodness of fit.

K: Number of increments used in determining goodness of fit.

l : Domain index.

L : Lottery defined $L = (p_1 x_1, p_2 x_2, \cdots, p_n x_n)$, where the outcomes are represented by x_i and the associated probabilities of their occurrence are represented by p_i.

L_i: The *ith* lottery.

$L^{(i)}$: The *ith* compound lottery, defined $L = \left(p_1 L^{(1)}, p_2 L^{(2)}, \cdots, p_n L^{(n)}\right)$.

$L_i^{\vdots}$: Defined as the lottery $(q_i x_1, (1-q_i) x_n)$ for which a subject is indifferent between the lottery and x_i for evaluation measure *i*.

$\lambda_j^{(m)}$: The difference between successive iterations of the hybrid model for the *j*th alternative, defined $\lambda_j^{(m)} = \left| E\left[\hat{U}_j^{(m)}\right] - E\left[\hat{U}_j^{(m-1)}\right] \right|$.

$\lambda_*^{(m)}$: The difference between successive iterations of the hybrid model for the best alternative at the *m*th iteration, defined $\lambda_*^{(m)} = \left| E\left[\hat{U}_*^{(m)}\right] - E\left[\hat{U}_*^{(m-1)}\right] \right|$.

m: The iteration counter for the hybrid utility algorithm.

$m(x)$: Krzysztofowicz' measure of local risk aversion in value.

n: The number of discrete outcomes in an uncertain situation (in Chapter II). The number of evaluation measures.

$n(x)$: 1. Krzysztofowicz' measure of local risk aversion in the value to utility transform.
2. The risk neutral function $n(x) = x$.

$o(x)$: The objective function, the overall value or utility function.

p, p': The probability of some event.

p_i, q_i: The probability of event i.

Π: An aggregation of proportional data used in calculating a test statistic.

r: The range of $R(f)$, $r = R^*(f) - R^0(f)$.

$R(f)$: The integral of the difference of some function $f(x)$ and $n(x)$ over the domain multiplied by two. For examination of risk attitudes, $f(x) = u(v(x))$, $n(x) = v(x)$, and $R(f) = 2\int_0^1 \left[f(x) - n(x)\right] dx$.

$R^0(f)$: $R^*(f) = \min\left\{R(f_1), R(f_2), \ldots, R(f_j)\right\}$ for the *j* evaluation measures.

$R^*(f)$: $R^*(f) = \max\left\{R(f_1), R(f_2), \ldots, R(f_j)\right\}$ for the *j* evaluation measures.

$RMSE_{ij}^{\text{ex}}$: Root mean square error of the exponential utility model.

$RMSE_{ij}^{\text{lin}}$: Root mean square error of the exponential utility model.

$RMSE_{ij}^{\log}$: Root mean square error of the exponential utility model.

$RMSE_{ij}^{\text{pow}}$: Root mean square error of the exponential utility model.

$RMSE_{ij}^{\text{sig}}$: Root mean square error of the exponential utility model.

ρ_m : Multiattribute exponential utility function constant.

$S_o\left(X_i\right)$: The swing of the objective function.

u : The single dimensional utility function in x.

$\underset{\cdot}{u}$: The single dimensional utility function in v.

$\bar{u}(x)$: The arithmetic mean of the utility functions elicited by the certainty equivalent method and the probability equivalent method.

$u^{CE}(x)$: The utility function elicited by the certainty equivalent method.

$u^{PE}(x)$: The utility function elicited by the probability equivalent method.

$\hat{u}(x)$: An estimate of the utility function obtained by fitting a mathematical model.

$\hat{u}\left(v\left(x\right)\right)$: Transform from single dimensional value domain into the single dimensional utility domain.

$\hat{u}^{\text{ex}}(v(x))$: Exponential transform from value domain into the utility domain.

$\hat{u}^{\text{lin}}\left(v\left(x\right)\right)$: Linear transform from value domain into the utility domain.

$\hat{u}^{\text{log}}(v(x))$: Logarithmic transform from value domain into the utility domain.

$\hat{u}^{\text{pow}}(v(x))$: Power transform from value domain into the utility domain.

$\hat{u}^{\text{sig}}\left(v\left(x\right)\right)$: Sigmoid transform from value domain into the utility domain.

$U\left(\mathbf{X}\right)$: The multi-dimensional (or objective) utility function in x.

$\underset{\cdot}{U}\left(\mathbf{X}\right)$: The multi-dimensional (or objective) utility function in v.

$\hat{U}_*^{(m)}$: The optimum alternative in the hybrid model at the mth iteration, defined as

$\hat{U}_*^{(m)} = \max\left\{\hat{U}_1^{(m)}, \hat{U}_2^{(m)}, \cdots, \hat{U}_J^{(m)}\right\}$.

$v(x)$: The single dimensional value function.

$V(f)$: The integral of the difference squared of some function $f(x)$ and $n(x)$ over the domain multiplied by three. For examination of risk attitudes, $f(x) = \underset{\cdot}{u}(v(x))$, $n(x) = v(x)$, and $V(f) = 3\int_0^1 \left[f(x) - n(x)\right]^2 dx$

$V(\mathbf{X})$: The multi-dimensional (or objective) value function.

x_i: A realization of evaluation measure i, or, in RSM, an encoded independent variable.

$\mathbf{X}$: A vector of realizations of evaluation measure i.

ξ_i: In RSM, an unencoded independent variable.

x_0: The least preferred level of evaluation measure x.

x_*: The most preferred level of evaluation measure x.

x_{ij}: A realization of evaluation measure i for alternative j.

x_{ijk}: A realization of evaluation measure i for alternative j and subject k.

X_i: The set of n possible realizations of evaluation measure i under alternative j where

$X_i = \{x_1, x_2, \ldots, x_n\}$ such that $x_1 \cdot\ x_2 \cdot\ \ldots \cdot\ x_n$.

X_{ij}: The set of n possible realizations of evaluation measure i under alternative j where

$X_{ij} = \{x_{1j}, x_{2j}, \ldots, x_{nj}\}$ such that $x_{1j} \cdot\ x_{2j} \cdot\ \ldots \cdot\ x_{nj}$.

z: Evaluation measure transformed into $[-1,1]$ for use with the weighting function $h(z)$.

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Abstract

This research investigated value and utility functions in multiobjective decision analysis to examine the relationship between them in a military decision making context. New data is presented and data from an earlier study is also analyzed for this relationship. Further, the impact of these differences on the decision model was examined in order to improve implementation efficiency. Specifically, the robustness of the decision model was examined with respect to the preference functions to reduce the time burden imposed on the decision maker.

Data for decision making in a military context supports the distinction between value and utility functions. Relationships between value and utility functions and risk attitudes were found to be complex. Elicitation error was significantly smaller than the difference between value and utility functions. Risk attitudes were generally neither constant across the domain of the evaluation measure nor consistent between evaluation measures. An improved measure of differences between preference functions, the weighted root means square, is introduced and a goodness of fit criterion established. An improved measure of risk attitudes employing utility functions is developed.

Response Surface Methodology was applied to improve the efficiency of decision analysis utility model applications through establishing the robustness of decision models to the preference functions. Which value-utility differences are significant was determined. An algorithm was developed and employs this information to provide a

hybrid value-utility model that offers increased elicitation efficiency for the decision maker.

ROBUSTNESS OF MULTIPLE OBJECTIVE DECISION ANALYSIS PREFERENCE FUNCTIONS

"When all available knowledge has been applied, the problem is reduced to one of preference." (Matheson and Howard, 1968: 21)

I. Introduction

Decision Analysis and Operations Research

This research extends decision analysis theory and knowledge. All Operations Research (OR) is concerned with making decisions. The introductory remarks in Introduction to Operations Research (Hillier and Lieberman, 2001: 3) opines "to be successful, OR must … provide positive, understandable conclusions to the decision maker(s)." Decision analysis (DA) provides a framework for examining complex decisions made under conditions of uncertainty, particularly for single, non-repeatable situations. The DA literature continues to expand, testifying to its success. Besides extensions to theory, DA has found application in many fields.

Decision analysis provides a bridge between the objectivist and subjectivist formulations of probability, linking the decision maker's intrinsic attitudes to the external environment with mathematical tools. In a sense, DA has returned to the roots of

probability theory, to the work of Pascal and Bernoulli. In providing this confluence, DA provides a solid analytical foundation for problems of increasing complexity.

Decision makers likely will face increasingly complex situations because of increasingly complex technologies, more rapid communications, faster business practices, and increasing competition. Frequently decisions involve objectives that are in conflict. Acceptable alternatives are challenging to develop. Decisions often involve intangible aspects and are of interest to multiple groups with differing views and may involve multiple decision makers. Often decisions involve expertise from several unrelated disciplines. Uncertainty also has a significant impact on decisions. Important decisions typically involve high stakes, are complex in nature, are multi-disciplinary, and need to be defensible (Keeney, 1982: 803 – 806). All these aspects of decisions challenge the decision maker, and the operations researcher. Certainly without quantitative reasoning, our society will be less well disposed.

The Purpose of Multiple Objective Decision Analysis

Keeney (1982: 806) defines decision analysis as "a formalization of common sense for situations which are too complex for informal use of common sense." He also offers a more technical definition: "a philosophy, articulated by a set of logical axioms, and a methodology and collection of systematic procedures, based on those axioms, for responsibly analyzing the complexities inherent in decision problems." The term *decision analysis* is sometimes used more broadly to refer to the analysis of any decision (see, for example, [Wallace, 2000]). In this document, the term decision analysis (DA)

refers to the OR methodologies derived from this axiomatic basis, those that produce a preference function.

Multiple Objective Decision Analysis (MODA) is employed to permit the decision maker to clarify in her own mind which alternative she should select. The intent of Multiple Objective Decision Analysis is to be prescriptive, to provide what is the most prudent course available in a bewildering decision context, as established from examination of underlying preference components. Multiple objective decision analysis is not descriptive in that it does not attempt to replicate how an individual makes a decision. It is also not normative in that it accounts for deviation from preferences that would be expected of one who was strictly a formalist in application of mathematical principles. (Keeney and Raiffa, 1993: xv). (A less fastidious taxonomy of decision making considers the categories normative and prescriptive to be identical. For example, see Skinner [1999: 13].)

Operations research is credited as being the first organized scientific discipline devoted to decision-making. Emerging as a profession during the Second World War, operations research analysts played important roles in developing effective tactics for air and sea forces. Postbellum, those skilled in OR techniques found applications in the private sector and operations research continued to develop. The problems that were well suited for application of OR tools were repetitive in nature. Whether searching for enemy aircraft or scheduling production, OR techniques were found to be useful. Operations research flourished at the lower and middle levels of management. But executive decisions tend to be unique, not repetitive, involve large portions of an organization's resources, characterized by high degrees of uncertainty, and typically

involve the decision maker's personal risk attitudes. Operations research techniques were found lacking for application in the boardroom. (Matheson and Howard, 1968: 21 – 22)

The term "decision analysis" is attributed to Howard (1963: 55). That DA arose not until two decades after the genesis of OR is ascribed to three influences. First, the development of electronic computational capability fostered the ability to develop computationally intensive mathematical models. The ubiquity of computers, while assisting greatly in ciphering, had a negative aspect in organizations. Managers were observed to say that computers made them feel as if they had less control, not more, over their organizations. Decision analysis provided an analytic focus on managerial issues, and provided for employing the computer as an appropriate tool. Finally, private corporations evolved more complex managerial structures and techniques. Boards supplanted individual managers. More complex financial arrangements increased auditing activities. These trends required decisions to be more thoroughly reasoned and defensible. (Matheson and Howard, 1968: 22 – 23)

Decision analysis provides a number of advantages. Besides extending analysis beyond repetitive decision situations, it codifies the process of making the actual decision. Prior to the development of DA, many man-months of effort were devoted to analysis of particular problems. This analysis may have been development and employment of a simulation, the application of statistical tools to historical or experimental data, or similar efforts. The analytic results were reduced to graphs, tables, or other conveyances and presented to the decision maker. The decision maker then was forced to absorb the information, process it in some internal, usually unspecified manner, and then announce his decision. Decision analysis permits the tools of operations

research to be brought to bear on the decision itself to assist the decision maker. Also, the decision analytic process itself has inherent advantages. The DA process forces the decision maker to think hard about her preferences. While this requires an investment of time, generally participants indicate that the exercise was beneficial in making them devote the effort to gain insight into the problem. (For example, see Felli, Kochevar, and Gritton, 2000: 60 – 61). Corner and Kirkwood (1991: 206) observe, "Decision analysis provides tools for quantitatively analyzing decisions with uncertainty and/or multiple conflicting objectives. These tools are especially useful when there is limited directly relevant data so that expert judgment plays a significant role." Kirkwood (1992b: 37) notes that DA's ability to perform tradeoffs among multiple objectives makes it valuable for government decision making. Within decision analysis, multiattribute utility theory is considered the leading methodology. Keeney and Raiffa (1993: xi) state, "Decision analysis is widely recognized as a sound prescriptive theory. When a decision involves multiple objectives – and this is almost always the case with important problems – multiattribute utility theory forms the basic foundation for applying decision analysis."

Decision analysis is widely applicable to military decision making. Buede and Bresnick (1992) show that decision analysis is useful in the materiel acquisition process. Vendor engineers viewed value models distributed with the request for proposal for the US Marine Corps (USMC) Mobile Protected Weapon System to industry as helpful. The USMC found that all ten conceptual responses to the request for proposal were "outstanding" (115). Later work for the USMC involved adapting this value model for off-the-shelf procurement of the Light Armored Vehicle. A utility model was employed to determine the best air defense system mix for a US Army heavy division. The authors

used a utility model to assess whether control of the Patriot Air Defense System should be transferred from the US Army to the US Air Force. A separate study group conducted the effort in parallel with a simulation approach. The two efforts were in concordance, the DA results were more easily interpreted and more defensible, and the simulation was an order of magnitude more expensive and slower (121). The authors include a list of 26 other materiel acquisition projects that have successfully employed decision analysis. Other notable military decision analytic efforts are SPACECAST 2020 (Burk and Parnell, 1997) and AIR FORCE 2025 (Jackson, 1996).

Clemen (2001: 6 – 8) describes the decision analysis methodology as a five-step process. The initial step is to identify the decision situation and articulate objectives. The second step is to identify alternatives. Next the problem is decomposed and modeled. Three aspects are modeled: the problem structure, the uncertainty, and the preferences. The structure of the problem is typically modeled using decision trees and influence diagrams. Concepts of probability theory are employed to capture aleatory aspects of the decision problem. Preferences are represented with value or utility functions. This step, according to Keeney (1982: 813), is unique to the discipline of decision analysis. Next, the modeling effort is used to analyze the decision and the best alternative is chosen. A sensitivity analysis is typically performed as the last step to determine the robustness of a solution. This process is iterative, and repeated until an adequate solution is determined. Figure 1 shows the process graphically. (Alternate, but similar, constructs of the decision analysis methodology are available in Skinner [1999: 15] and Keeney [1982: 807 – 820].)

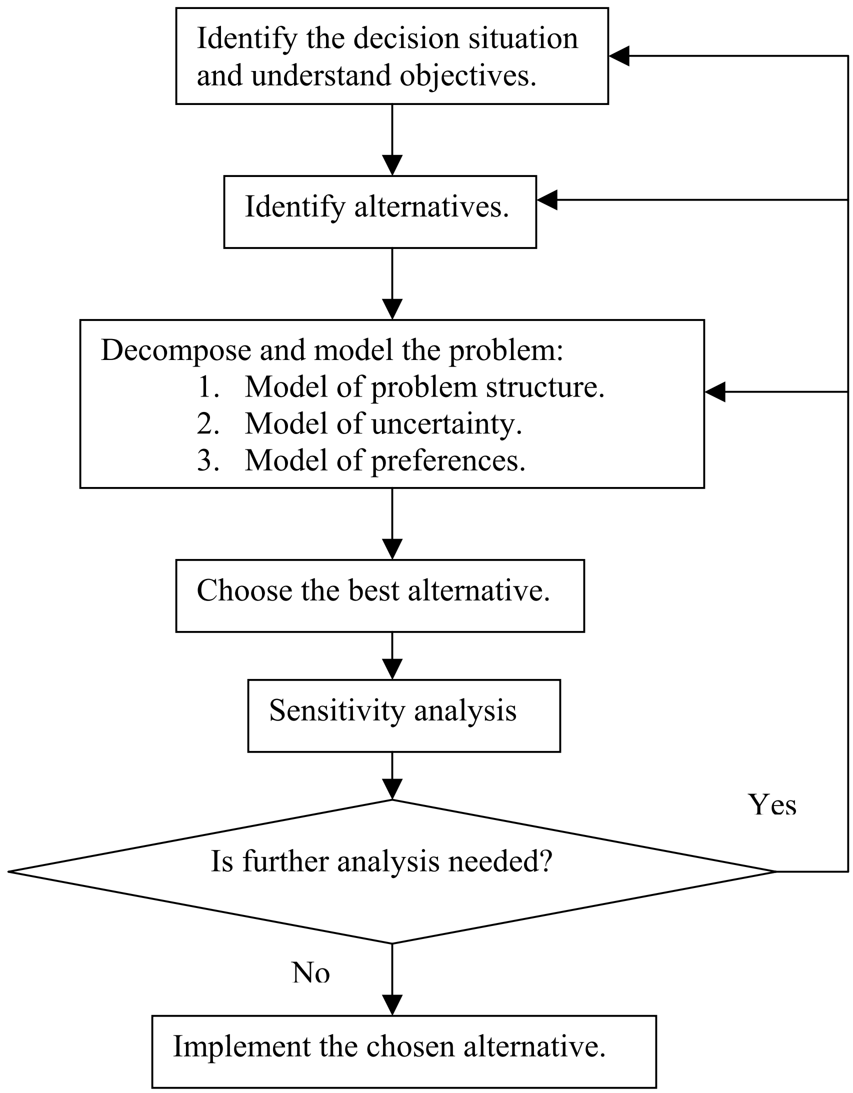


Figure 1. The Decision Analysis Process (after Clemen, 2001: 6).

Problem Description

Research Questions. The research primarily examines two related aspects of decision analysis. Do value and utility functions differ, and, if they differ, when are the differences important and how should this be considered when applying decision

analysis, especially to achieve more efficient use of the decision maker's time? Regarding the first point, researchers disagree about the relationship, if any, between these two preference functions. Empirical data consist of single dimensional decision problems typically presented to subjects who are not subject matter experts. The question exists as to whether professionals exhibit differences in value and utility functions for realistic multidimensional decision problems. If differences are manifest, is a functional relationship discernable? Additionally, the metrics for discerning differences between preference functions is not mature. Only a general metric, root mean square error, has been widely accepted.

Choice of either form of preference function should, except in certain specific cases, potentially have an impact on identification of the optimal alternative. The second research topic addresses the robustness of decision analysis solutions to the form of the preference function. When one accepts that there is a distinction between value and utility functions, when is it necessary to acknowledge that distinction in application? A more pragmatic question is germane, how may a value function-based model be adapted to adequately represent the behavior of the corresponding utility function model?

Preference Functions. In decision analysis, the decision maker's preferences under certainty are represented as a value function and under uncertainty or risk as a utility function. Value functions reflect strictly preferences considered in a deterministic sense, while utility functions reflect those preferences as well as risk attitudes. (In this document, all references to value functions are to "measurable" value functions, which provide cardinality. Non-measurable value functions provide only ordinality and are less useful for multiobjective decision analysis models.) However this interpretation is not

universal. Opinions in the literature vary widely. Some authors have used the terms interchangeably, e.g., Anandalingam (1987: 339). Some argue that they are identical. For example, see Von Winterfeldt and Edwards (1986: 213). Keeney opines that when measurable value functions and utility functions for a subject differ, this may indicate that the elicitation is inaccurate or incomplete (1992a: 187). Others believe that no functional relationship exists, as noted by Bouyssou and Vansnick (1988: 110). Finally, another group believes that a functional relationship is present, e.g., Pennings and Smidts (2000: 1343).

Value functions are elicited through asking questions about preferences under conditions of certainty. Typically pairs of levels of an attribute are compared and adjusted until the decision maker believes that quantitative statements about his preferences have been captured. The decision maker is asked to compare the differences in value between each pair of attribute levels. Often this becomes a process of varying one interval until the value increase is identical to that of the other pair. Utility functions, in order to capture the uncertain aspects, are elicited employing lottery-type questions. The decision maker, when composing an answer, must consider her attitudes about the uncertainty involved as well as preferences for various levels of the attribute in question. As well as this theoretical difference, empirical data tends to support the argument that the functions are distinct. See Pennings and Smidts (2000: 1343) for a summary.

If value and utility functions are functionally related, work by Pratt (1964: 122) provides a hypothesis of that relationship under varying risk attitudes. However empirical data has failed to adequately validate these relationships. See Keller (1985b: 481 – 482) for a discussion. The situations examined by Keller are restricted to the

univariate case. If value and utility are separate constructs, the value function may be elicited from subject matter experts, rather than the decision maker. Then the utility function, in the form of $u(v(x))$ may be elicited from the decision maker. Such a process offers the potential for reducing the contact time required with the decision maker. As decision makers often face great time demands, such a reduction makes employment of decision analysis more practicable. The first phase of the research examines the relationship of value and utility functions.

The initial research question is whether $u(x) = f(v(x))$. In order to address this question, the metric for determining the difference between preference functions will be considered. Are current metrics acceptable? A third question becomes what criterion should be considered representing a significant difference between the preference functions. If $u(x) = f(v(x))$, then what is the form of the function f? Does the multiattribute method of Kirkwood provide a reasonable approximation of single dimensional utility functions?

Regardless of the relationship between value and utility functions, other conjectures and observations may be examined. Do individuals maintain consist risk attitudes? Does elicitation error overshadow the elicited differences between value and utility functions? Does simplifying $u(x)$ to a linear function in x provide reasonable approximations?

The final area of investigation into preference functions involves the decision attitudes of military decision makers. Are there significant learning effects present

during elicitation sessions? Do decision attitudes vary with experience, rank, education, or military occupational specialty?

Preference Function Robustness. The final step of decision analytical effort is a sensitivity analysis. The intent is to see how sensitive the optimal solution is to variability inherent in the problem. A sensitive solution may indicate that additional analysis is desirable. Alternatively, it may indicate that the best alternative is not the one that provides the highest mathematical expectation for the objective function. Instead an alternative with somewhat lesser desirability under nominal conditions may retain its value relative to other alternatives better under varying conditions. Such an alternative is considered "robust."

Typically, in decision analysis this is done in univariate fashion, varying a single input variable and observing the impact on the overall multivariate value or utility function. While this produces valuable insights, it does not permit analysis of interactions. The capability to do pair-wise examination of variables has been added to software packages in the last few years. Until recently, the state of the art was to consider only three-way sensitivity, taking three variables at a time. See Clemen (2001: 183 – 184) or Borison (2000: 530 – 531).

Other techniques have been employed, including Dependent Sensitivity Analysis, Rank Order Stability Analysis, and Response Surface Methodology (RSM). These techniques are more powerful than simple sensitivity analysis and they are discussed in Chapter II. To the author's knowledge, no sensitivity analysis been employed to analyze preference functions.

The research question in this area is it possible to conduct sensitivity analyses on the preference function itself? If this is tractable, then can a sensitivity analysis be employed to increase the operational efficiency of decision analysis employment? A hybrid model, composed of a mixture of single dimensional value and utility functions, may be a useful construct that reduces the time required of a decision maker. Specifically, is it possible to create a hybrid model, using the appropriate value model as a base, that approximates the behavior of the utility model? If so, when is the hybrid model sufficiently accurate and what is an acceptable criterion?

Benefits of the Research

As stated above, the unique aspect of decision analysis is the employment of the preference function. Yet the nature of this function is in dispute. This unsettled central issue leaves decision analysis open to criticism. Further examination of this issue is clearly important for the continued acceptance of decision analysis. This research extends previous work in this area into multivariate cases.

In complex problems the number of attributes to be examined is large. For example, the United States Air Force 2025 Study, which was commissioned to develop a recommendation for allocation of that service's funds and organizational efforts towards development of key system concepts and their enabling technologies, had 134 attributes (Jackson, 1996; and Parnell, Conley, Jackson, Lehmkuhl, and Andrew, 1998). (An effort for the National Reconnaissance Office had 69 evaluation measures [Parnell, 2001].) The attributes ranged from operational performance issues to technical feasibility of untested

technology. Clearly such large sets of attributes, which draw on expertise from multiple disciplines, is beyond the ability of a single individual to make informed preference statements. Instead, the judgment of subject matter experts must be coalesced and presented to the decision maker. Under conditions of uncertainty, common for challenging decision situations, utility functions must be elicited personally from the decision maker. Such an approach is problematic, as for large, important decisions; the decision maker faces many competing demands upon her time. The Chief of Staff of the Air Force, the senior uniformed officer of that service, for example, initiated the 2025 study. A methodology that permits the value model to be adapted to represent the decision maker's utility model with the least decision maker interaction would be extremely valuable and critical for consideration of decision analysis in such cases.

The research extends application of response surface methodology within decision analysis. Current analysis of uncertain aspects of decision analysis problems lags behind developments applied in other research areas. Specifically, response surface methodology will be employed to examine the sensitivity of the decision problem to the preference functions themselves. This is a novel approach in sensitivity analysis.

Finally, the research included analysis of military professionals' preference and risk attitudes for a realistic tactical problem. While certain industries and public sector groups have been subject to intensive decision analytical study, no similar work has been performed on military subjects.

Sequence of Presentation

The balance of the dissertation is divided into five chapters. Chapter II will review the literature regarding value and utility functions in decision analysis and response surface methodology. Chapter III outlines the objectives of the research, delimits the research scope, and describes the methodology. Chapter IV describes research regarding the relationship of value and utility functions. Chapter V presents results from investigation of the robustness of the preference function. Chapter VI briefly summarizes the dissertation. A glossary of technical terms is at Appendix A. Details of results are available at Appendices B and C. Appendix D contains a technical report providing the decision analysis of the automatic target recognition classification systems problem. This work is extended into a hybrid value-utility model in Chapter V. A bibliography is also included.

II. Literature Review

General

As discussed in Chapter I, decision analysis is of recent development in operations research. However, the concepts involved date to the Enlightenment, when mathematical tools were brought to bear on decision-making. That work provided a foundation upon which modern economics rests, and within the last few decades has proven to be the inchoate form of decision analysis.

History of Utility and Value Theory

Pascal. Hacking links the birth of scientific analysis of decision making to Pascal, and his well-known application of probability in what is now referred to as Pascal's Wager (Hacking, 1975: 11). Previous work with probability was limited to solving puzzles about games of chance. Here Pascal wrestles with the concept that God either is, or is not. If God exists, one should lead a pious life leading to salvation. If not, one may enjoy one's vices without regard to consequences after death. However the matter is unresolved until after death, and the choice of how to live must be made now. With the objective probability of God's existence unknown, Pascal focuses on the idea of the differing value of the potential outcomes, extending the ability to analyze uncertain situations beyond those posed about gaming tables. He is responsible for recognizing

that the expected value of an uncertain (discrete) situation with *n* outcomes is equal to $\sum_{1}^{n} x_i p_i$, where x_i represents the potential outcomes and p_i their corresponding probabilities. Pascal vacillated on how to live his life, sojourning in a monastery, and then abandoning his refuge to the exhilaration of the casinos. His final choice, as salvation was preferable to damnation regardless of the probabilities involved, was the decision to return to the monastery for good. (Bernstein, 1996: 69 – 71).

Bernoulli. The development of the concept of utility theory is credited to Daniel Bernoulli. In a 1738 paper published by the Imperial Academy of Sciences in St. Petersburg, Bernoulli states, "the value of an item must not be based on its price, but rather on the utility that it yields." (Bernoulli, 1738/1954: 24). He notes that under conditions of uncertainty, the expected value of an outcome is determined by multiplying the probabilities and the respective "gains." But he finds this inadequate for describing how individuals make decisions. He proposed that an individual would make a decision in order to maximize expected utility, rather than maximizing the expected amount of money (ducats, for Bernoulli's paper) or some other objective measure of worth. The calculation is made by determining the mathematical expectation employing utility in place of the units of currency, employed by previous researchers, yielding $\sum_{1}^{n} u(x_i) p_i$, where *u* is the utility function. He also asserted that for a given decision situation, different individuals would assign differing utilities to various possible outcomes.

Bernoulli extended his analysis by suggesting that an increase in utility resulting from a gain in wealth would be inversely proportional to the amount of previous possessions. This was another key insight. He provided a graph similar to Figure 2. This

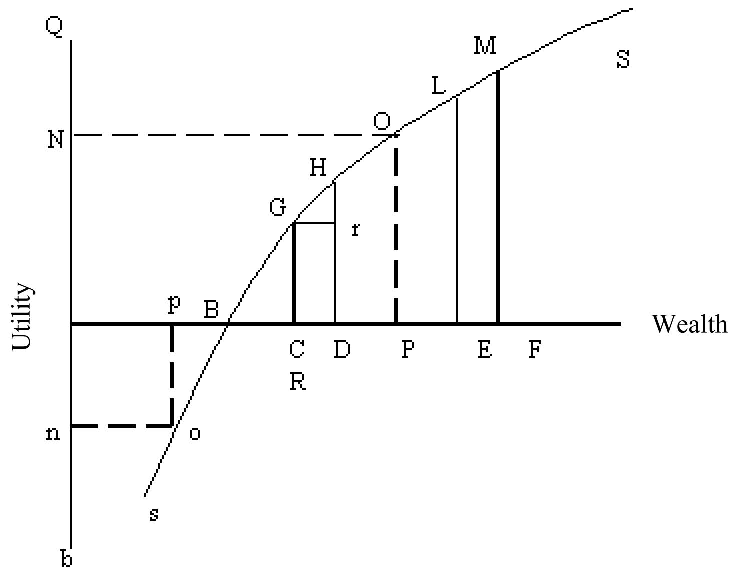


Figure 2. Bernoulli's Utility Curve. After Bernoulli (1738/1954).

inverse relationship causes the concavity shown by the utility function. He also demonstrated that two individuals with identical utility functions, who engage in a wager equally likely to cause either to pay the other a fixed amount, were both facing a loss of expected utility. The concavity means that the increase in utility of winning is less than the reduction in utility of losing the same amount.

Bernoulli used the concept of utility to solve a question originally posed by his cousin, Nicolas Bernoulli in 1713. (With Daniel Bernoulli's solution, this problem has become known as the St. Petersburg Paradox.) Nicolas had developed a thought experiment where two individuals agree to a game between them with no house take. The first player will flip a coin repeatedly until the result is tails. Then he will pay the second player 2^n ducats (or dollars) where n is the number of coin flips. Before the game begins, a third player wishes to buy the right to play from the second. Nicolas notes that the expected value of the game in a monetary sense is infinite, as the payoff increases

infinitely and the probability of achieving another heads never reaches zero. But most individuals would sell the game for some amount far less than infinity, say between 5 and 20 ducats (or dollars). Daniel Bernoulli's utility provides the answer. The concavity of the utility function keeps the perceived worth of the gamble far from infinity. (Bernstein, 1996: 99 – 115).

Using a geometrical argument, Daniel Bernoulli proposed that the increase in utility for an individual with wealth B in Figure 2 to wealth P is given by $PO = b\log \alpha + a/\alpha$ where α is his current wealth (equal to AB), a is the gain (equal to BP), and b is unique to the individual. Bernoulli then uses this relationship inappropriately (by misinterpreting a property of infinity and conveniently dropping the coefficient b, as explained by Sommer, [see Bernoulli, 1738/1954]) to show that for a man with nothing of value at all, the expectation of the game would be two ducats. Owning ten ducats would change his expectation to about three ducats. He argues that very large initial wealth is required before someone would offer more than 20 ducats. (Bernstein [1996: 106] observes that a ducat would be worth about 40 dollars in today's currency.) Menger showed that the player's expectation is actually equal to $b\log\left[(\alpha+1)^{1/2}(\alpha+2)^{1/4}\cdots(\alpha+2^{n-1})\cdots\right]-b\log\alpha$ (Bernoulli, 1738/1954: 32).

According to Bernstein, Bernoulli first applied measurement to something that cannot be counted. Bernstein also credits him with "an equally profound influence on psychology and philosophy, for Bernoulli set the standard for defining human rationality" (1996: 106). Further, Bernoulli also introduced the idea of human capital, that one's ability to produce is intrinsically valuable.

The concept of utility was recognized as valuable. Adam Smith's famous passage from The Wealth of Nations uses the concept. Smith opined "The word value has two different meanings, and sometime expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called 'value in use'; the other, 'value in exchange.'" (Smith, A., 1775/1976: 44) Here we see, continued from Bernoulli, imprecision in the employment of the terms value and utility as value in use is defined as the utility of an object. The problem persists to this day.

Bentham. Jeremy Bentham widely promulgated the concept of utility in England in 1789. Bentham applied the concepts of utility to all aspects of society, including assessing the appropriateness of penal judgments. Utility theory, although dormant for a period following Bentham, provided the foundation for later development of economic theory, particularly the Law of Supply and Demand, and remains an important economic concept today. (Stigler, 1965: 89 – 92).

Von Neumann-Morgenstern. The developments of Bentham, while of interest to economists, contain little of direct interest to the development of decision analysis. John von Neumann provided the next significant development in utility theory, with respect to what would become decision analysis. He developed game theory. In a simple game two opponents each make a binary decision. The winner is determined by the results of the pair of binary decisions. For example, in a game where the players secretly decide to show (not flip) heads or tails on a single coin, player A wins if both show the same side. Player B wins if one shows heads, the other tails. Von Neumann determined the best strategy for the players (which is a random display of heads and tails). The point

of interest is that the laws of probability do not dictate the optimum strategy; the players themselves do so. (Bernstein, 1996: 232 – 235).

Later, in collaboration with Oskar Morgenstern, von Neumann published the book Theory of Games and Economic Behavior. (Von Neumann and Morgenstern, 1953) They proposed an axiomatic basis for utility theory. The axioms provide the necessary and sufficient conditions for the existence of a utility function. Others have rewritten their axioms for clarity. For their description, the presentation generally follows Biswas (1997: 17 – 18). Let E denote the outcome set, so $x \in E$ is an outcome. Let $X = (x_1, x_2, \cdots, x_n)$ be the vector of n possible discrete realizations from the outcome set, each with a respective probability of occurrence, p_i. Represent a lottery of these n outcomes with $L = (p_1 x_1, p_2 x_2, \cdots, p_n x_n)$. Preference relationships will be written for $x_i, x_j \in X$ as $x_i \succ x_j$ when x_i is preferred to x_j, and $x_i \sim x_j$ when a subject is indifferent between x_i and x_j. Similarly, to represent x_i is at least as preferred as x_j, we write $x_i \succsim x_j$. Without loss of generality, we will assume that $x_1 \succsim x_2 \succsim \cdots \succsim x_n$.

Axiom 1 (Ordering). Preferences may be (totally) ordered between any two outcomes. This ordering is a total ordering. So for any (x_i, x_j) either $x_i \succsim x_j$ or $x_j \succsim x_i$. Further, $x_i \succsim x_j$ and $x_j \succsim x_k$ implies $x_i \succsim x_j \succsim x_k$.

Axiom 2 (Reduction of Compound Lotteries). An individual is indifferent between a simple lottery L, (a lottery with outcomes that are fixed) and a compound lottery (a lottery with one or more outcomes that are also lotteries) with the same outcomes when the probabilities of the simple lottery are determined in accordance

with standard manipulation of probabilities. If a lottery $L^{(m)}$ is defined as $L^{(m)} \equiv \left(p_1^m x_1, p_2^m x_2, \cdots, p_n^m x_n\right)$, where the superscripts indicate membership in a compound lottery *m*, it follows that $\left(q_1 L^{(1)}, q_2 L^{(2)}, \cdots, q_o L^{(o)}\right) \sim \left(p_1 x_1, p_2 x_2, \cdots, p_n x_n\right)$ where $p_i = q_i p_i^{(1)} + \cdots + q_o p_i^{(o)}$ and $\sim$ represents indifference.

For clarity, this axiom will be detailed for the case of two lotteries each with possible outcomes x_1 and x_2. These lotteries are $L^{(1)} = \left(p_1^{(1)} x_1, p_2^{(1)} x_2\right)$ and $L^{(2)} = \left(p_1^{(2)} x_1, p_2^{(2)} x_2\right)$. They are shown graphically in Figure 3. Then $\left(q_1 L^{(1)}, q_2 L^{(2)}, \cdots, q_o L^{(o)}\right) \sim \left(p_1 x_1, p_2 x_2, \cdots, p_n x_n\right)$ becomes $\left(q_1 L^{(1)}, q_2 L^{(2)}\right) \sim \left(p_1 x_1, p_2 x_2\right)$ where $p_i = q_i p_i^{(1)} + \cdots + q_o p_i^{(o)}$ or $p_1 = q_1 p_1^{(1)} + q_2 p_1^{(2)}$ and $p_2 = q_i p_2^{(1)} + q_2 p_2^{(2)}$. So an individual would be indifferent between the choice of the compound lottery or the corresponding outcomes with the associated reduced probabilities, $\left(q_1 L^{(1)}, q_2 L^{(2)}\right) \sim \left(p_1 x_1, p_2 x_2\right)$.

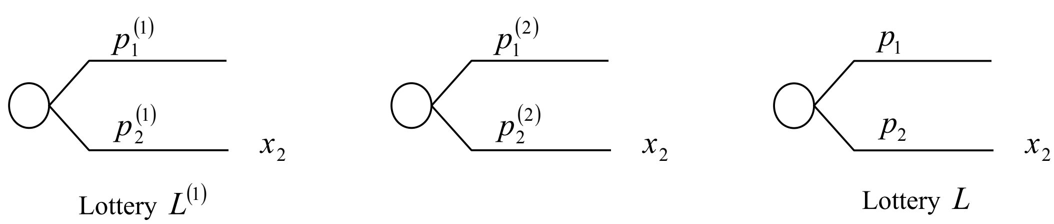


Figure 3. Lotteries $L^{(1)}$ $L^{(2)}$, and L.

Axiom 3 (Continuity). An individual will be indifferent between some x_i and a lottery with outcomes x_1 and x_n. That is, there exists some probability q_i such that $x_i \sim (q_i x_1, 0x_2, 0x_2, \cdots, (1-q_i)x_n)$. Define this lottery $L^{\tilde{x_i}} = (q_i x_1, (1-q_i)x_n)$.

Axiom 4 (Substitutability). In any lottery, $L^{\tilde{x_i}}$ may be substituted for x_i.

Axiom 5 (Transitive Preference over Lotteries). Preference ordering is transitive over lotteries. That is $L^{(i)} \succsim L^{(j)}$ and $L^{(j)} \succsim L^{(k)}$ implies $L^{(i)} \succsim L^{(j)} \succsim L^{(k)}$.

Axiom 6 (Monotonicity). An individual prefers a lottery $L = (px_1, (1-p)x_2)$ to a distinct lottery $L' = (p'x_1, (1-p')x_2)$ if and only if $p > p'$. Similarly, an individual is indifferent to a lottery $L = (px_1, (1-p)x_2)$ when compared to another lottery $L' = (p'x_1, (1-p')x_2)$ if and only if $p = p'$.

As summarized by McCord and de Neufville (1983b: 279), the axioms imply the existence of a cardinal utility function (sometimes referred to as a von Neumann-Morgenstern [vNM] function) that orders alternatives independent of probability distributions. Further, this function may be used to rank uncertain alternatives by taking the mathematical expectation of the vNM utility. That is, select the alternative with the highest expected utility as determined by,

$$U = \sum_{i=1}^{I} w_i E\left[u_i(x_i)\right] \tag{1}$$

for each alternative, where $i, (i = 1, 2, \cdots, I)$, is the index of evaluation measures, w_i are the relative weights, x_i is the level of the *i*th evaluation measure, and $u_i(x_i)$ are the

corresponding single dimensional utility levels. Hence this theory is referred to as expected utility theory.

Criticisms of Expected Utility

Criticisms of Expected Utility (EU) Theory largely are rooted in the observation that it is possible to construct situations where individuals, who subscribe to the axioms, later make choices that violate the axiomatic consequences. The most famous example is that of Allais, and is now well known as the Allais Paradox (Allais, 1953: 503 – 505). Presented two sequential lottery choices, most subjects make alternative selections in each that are mutually incompatible without axiomatic violations. Consider four lotteries. The notation indicates the payoff then, parenthetically, the associated probability.

$$L_1 : \left[\$30,000\ \ (0.33);\ \$25,000\ \ (0.66);\ \$0\ \ (0.01)\right] \tag{2}$$

$$L_2 : \left[\$25,000\ \ (1)\right] \tag{3}$$

$$L_3 : \left[\$30,000\ \ (0.33);\ \$0\ \ (0.67)\right] \tag{4}$$

$$L_4 : \left[\$25,000\ \ (0.34);\ \$0\ \ (0.66)\right] \tag{5}$$

Initially the subject must choose between L_1 and L_2. Most subjects choose L_2. Then the subjects who chose L_2 generally choose L_3 when presented with a choice between L_3 and L_4. This combination of choices violates expected utility. Choosing L_2 evinces

$$0.33u(30,000) + 0.66u(25,000) + 0.01u(0) < u(25,000) \tag{6}$$

which may be rewritten

$$0.33u(30,000) + 0.01u(0) < 0.34u(25,000) \tag{7}$$

When alternative L_3 is preferred in the second decision situation, this leads to

$$0.33u(30,000) + 0.67u(0) > 0.34u(25,000) + 0.66u(0) \tag{8}$$

which simplifies to

$$0.33u(30,000) + 0.01u(0) > 0.34u(25,000) \tag{9}$$

Clearly results (7) and (9) are incompatible. Either the subject has not made choices in a rational manner, or expected utility theory fails to adequately represent rational decisions. Simon (1972) proposed the theory of bounded rationality. Perhaps the ability to calculate the utility differences in the lotteries, which differ only slightly in probabilities, but more noticeably in payoffs, is beyond the ability of a decision maker.

Tversky and Kahnemann (1986) proposed that the presentation, or *framing*, of the decision situation was important and responsible for the Allais paradox. They recast the decisions situations as compound lotteries. When presented in this manner, the majority of subjects made choices in consonance with expected utility. The explanation offered is that the subject's view of what is the status quo is important. In Figure 4 the status quo is seen as a $25,000 gain, and the status quo in Figure 5 is a gain of zero. Decisions are made with this as a base state. Now the results are that someone who chose L_2 usually chose L_4, consistent with utility theory. (Biswas, 1997: 4 – 8)

Ellsberg (1961: 653 – 655) presented another famous example. Given an opaque container in which are 30 marbles, ten of red and the balance being either blue or yellow,

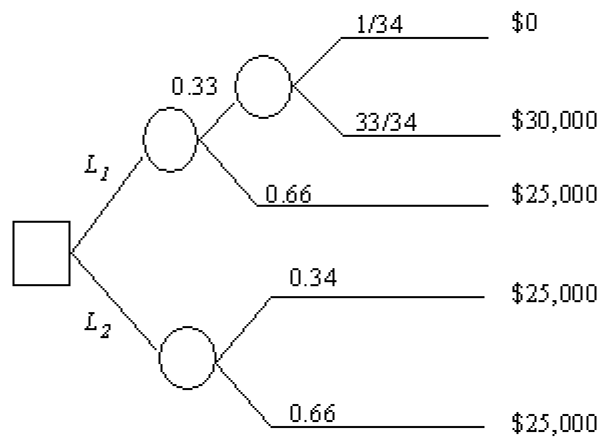


Figure 4. Allais Paradox Initial Lottery, as a Compound Lottery

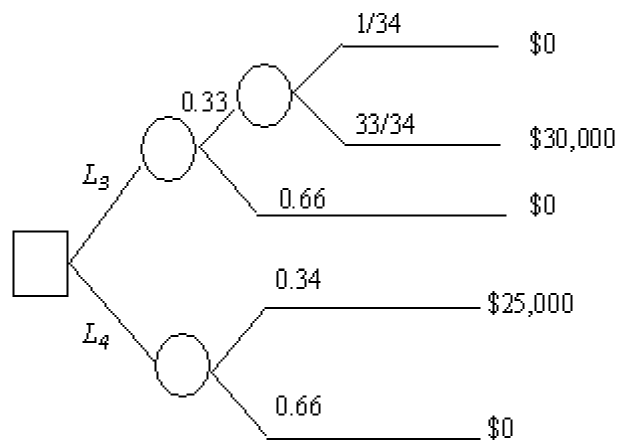


Figure 5. Allais Paradox Choice Two, as a Compound Lottery.

and then offered two sequential bets with replacement: (1) Call red or blue, if correct win $5,000; and (2) Call red or blue, if wrong, collect $5,000; Ellsberg found that most subjects choose red for both bets. This violates expected utility theory because red as the choice in bet (1) evinces

$$p_r \cdot u(5000) + (1 - p_r) \cdot u(0) > p_b \cdot u(5000) + (1 - p_b) \cdot u(0) \tag{10}$$

where the subscripts r and b indicate the respective marble color. The expected utility relationship of bet (2) is captured by

$$p_r \cdot u(0) + (1 - p_r) \cdot u(5000) > p_b \cdot u(0) + (1 - p_b) \cdot u(5000) \quad (11)$$

Multiplying both sides of (11) by -1 produces

$$-p_r \cdot u(0) - (1 - p_r) \cdot u(5000) < -p_b \cdot u(0) - (1 - p_b) \cdot u(5000) \quad (12)$$

which may be rewritten

$$-p_r \cdot u(0) + p_r \cdot u(5000) < -p_b \cdot u(0) + p_b \cdot u(5000) \quad (13)$$

Adding $u(0)$ to each side produces

$$(1 - p_r) \cdot u(0) + p_r \cdot u(5000) < (1 - p_b) \cdot u(0) + p_b \cdot u(5000) \quad (14)$$

Equation (14) may be rearranged to form

$$p_r \cdot u(5000) + (1 - p_r) \cdot u(0) < p_b \cdot u(5000) + (1\text{-p}_\text{b}) \cdot u(0) \quad (15)$$

Equations (10) and (15) contradict each other. This violates expected utility theory. Ellsberg advanced the argument that decision makers are likely to place more confidence in objective rather than subjective probabilities. The red probabilities are objective while those for blue are subjective, leading a decision maker so inclined to select red.

Outside of scientific studies (See [Keller, 1992: 4 – 5] for additional references), human behavior has called expected utility theory into question. In practice, the tenets of expected utility theory have been often observed to be violated, in the opinion of critics, when an individual demonstrates both risk seeking and risk averse behavior. Documentation exists in the work of Appleby and Starmer (1987), Kahneman and Tversky (1979), and Schoemaker (1980). Prakash, Chang, Hamid, and Smyser (1996: 239 – 240) provide a summary of common observations. Some individuals purchase insurance, a risk averse behavior, yet gamble, which is risk seeking. Investors will hedge

in some investments and speculate in others. Investors buy into "long-shot bets." Managers proceed with mergers that are expected to provide lower profit margins.

One explanation is that an individual overestimates the probability of success. An alternate explanation is that the participation in the gamble itself provides utility (Yaari, 1965: 288). A latter developed explanation is that risk aversion is affected by the decision context. Recall that Bernoulli stated that the risk aversion, the concavity of the utility curve, was resultant from decreasing marginal utility as a function of total wealth. Viscusi and Evans (1990: 353) cite research where one's health determines the shape of the utility function. The contextual explanation has also been extended to include the individual's self-assessment of one's own competency.

Obtaining the preference function in expected utility is demanding, and some authors suggest not worth the effort. "It's well documented that decision analysts often run into trouble getting the decision maker's utilities, which turn out to be critical to the analysis," according to Levitt (Samuelson, 2000: 25). Edwards and Barron (1994: 311) argue that elicitation of functions is onerous enough that they recommend treating utility as a linear function in x. Reneau and Blanthorne (2001:148) observe that it is well established in the literature that presentation sequence and presence of extraneous information influences effectiveness of elicitation sessions without causing the decision maker to be aware of the impact.

Humphreys (1982: 147 – 163) provides a summary of several criticisms of Multiattribute Preference Theory. He states that often risk is viewed as a single entity, generated as a by-product, which may be assessed. Then activities generating risk above a threshold are avoided or mitigated. He states that this has been named by others the

“phlogiston theory of risk” (147). Conrad (1982) states that DA is not useful for managing risk in society as political aspects are not included in the analysis.

Most work in the expected utility area has centered on relaxing axioms in an attempt to improve the descriptive ability (Kimbrough, 1994: 618). This has led to the genesis of descriptive theories of decision making, which are beyond the scope of this discussion. Descriptive theories address how an unassisted decision maker makes a choice. They do not attempt to indicate which choice is, in fact, the best alternative. The reader may consult Fishburn (1989).

Keller (1992: 5) visualizes three types of responses to the observed behavioral EU violations. First is the acceptance that EU is normative and compartmentalizes decision problems into those where EU may be used prescriptively and when it is not appropriate. The second are attempts to improve presentation and elicitation tools to assist the decision maker in providing coherent responses. Finally, more generalized utility models have been developed that have descriptive aspects and which, perhaps may be used prescriptively under appropriate constraints.

However, despite the criticisms, Kimbrough (1994: 618) remarked, “For prescribing behavior, expected utility is still by far the most accepted theory.” This position is shared by Edwards (1992: xi), who points out experts in the field at a 1992 conference “unanimously endorsed traditional SEU [subjective expected utility] as the appropriate normative model, and unanimously agreed that people don’t act as the model requires.” Clearly, expected utility is the preeminent model for non-behavioral aspects of decision making.

Measurable value functions

Measurable value functions were developed as a means of comparing an individual's preferences among choices. In brief, given a set of outcomes $\{x_1, x_2, \cdots, x_n\}$ arranged in order of increasing preference, comparisons can be made between pairs of elements. For example, someone may be indifferent between the respective increase in preference between certain element pairs, $(x_i, x_j) \sim (x_k, x_l)$. This is read as the increase in preference from x_i to x_j is equal to the increase in preference from x_k to x_l. By establishing such relationships and normalizing the preferences over the unit interval (or any arbitrary interval), a measurable value function on an interval scale may be determined. Note that preference statements are made under conditions of certainty, and therefore do not involve the subject's risk attitudes. Fishburn (1967) provides a summary and classification of measurable value function methods. For a discussion of measurement theory (scale types), see Stevens (1946), Roberts (1979) or Krantz (1971). A brief summary is available in Forman and Gass (2001: 470).

Measurable Value Function Axioms. Farquhar and Keller (1989: 205) attribute the "strength of preference" concept to Pareto and Frisch. Stigler (1965) provides a history. Also known as "preference intensity," this is a quaternary relationship comparing preference differences. Krantz, Luce, Suppes, and Tversky (1971) provide a set of axioms that establish when relationships between objects may be defined as an algebraic difference structure. (Other sources of the axioms, or alternative formulations of them, are [Fishburn, 1970], [Suppes and Winet, 1955], and [Debrau, 1960].) When

these axioms hold, a real-valued function exists that provides cardinal preference information about the objects.

Definition (Algebraic Difference Structure). The notation ab represents the increase in preference resultant from receiving object b in lieu of object a. Suppose A is a nonempty set and $\succeq$ is a binary relation on $A \times A$. The pair $\langle A \times A, \succeq \rangle$ is an algebraic difference structure if and only if for all $a, b, c, d, e, f \in A$ and all sequences $a_1, a_2, \cdots, a_i, \cdots \in A$, the following axioms hold true:

1. $\langle A \times A, \succsim \rangle$ is of weak order. A relation, R, on a set, A, is of weak order when it is transitive and strongly complete. Transitivity exists when aRb and bRc implies aRc for all $a, b, c \in A$. A relation is strongly complete on a set when aRb or bRa holds for all $a, b \in A$, including when $a = b$. (Roberts, 1979: 20 and 29)

2. If $ab \succsim cd$, then $dc \succsim ba$.

3. If $ab \succsim de$ and $bc \succsim ef$, then $ac \succsim df$.

4. If $ab \succsim cd \succsim aa$, then there exist $e, f \in A$, such that $ae \sim cd \sim fb$. (The requirement that $ab \succsim cd$ be $\succsim aa$ ensures that ab is increasing. This is required so that e, f be members of the interval $[a, b]$. When $e, f \in \overline{ab}$ then it must be true that $e, f \in A$.)

5. If $a_1, a_2, \cdots, a_i, \cdots$ is a strictly bounded standard sequence $a_{i+1}a_i \sim a_2 a_1$ for every a_i, a_{i+1} in the sequence, not $a_2 a_1 \sim a_1 a_1$; and there exists $e, f \in A$ such that $ef \succ a_i a_1 \succ fe$ for all a_i in the sequence), then it is finite. (Krantz, Luce, Suppes, and Tversky, 1971: 151)

Krantz et al show that if $\langle A \times A, \succsim \rangle$ is an algebraic difference structure, then there exists a real-valued function v on A such that, for all $a, b, c, d \in A$, $ab \succsim cd$ iff $v(a) - v(b) \geq v(c) - v(d)$ and v is unique up to a linear transformation. Methods of elicitation of measurable value functions are described by Fishburn (1967 and 1976); Johnson and Huber (1977); Kneppereth, Gustafson, Leifer, and Johnson (1974); Dyer and Sarin, (1979 and 1982); Farquhar and Keller, (1989); and Camacho (1980).

Relationship of Value and Utility.

Opinion regarding the relationship between value and utility functions is varied. Von Winterfeldt and Edwards, (1986: 213) state that there is no significant distinction. They argue that there are no truly riskless situations, so decision making under certainty is never actually encountered. Secondly, they argue that concavity associated with risk aversion may be explained by marginally decreasing return of value or through a regret expression. They also argue that repetition reduces risk aversion and most choices are truly repetitive over a lifetime. Finally they opine that errors associated with elicitation of preference data exceed the distinctions between value and utility functions. McCord and de Neufville (1983a) provide an empirical study supporting this position.

Other authors disagree. Keeney, in his book Value-Focused Thinking, states that value functions are used under conditions of certainty, and utility functions under conditions of uncertainty. He further indicates that his value functions are ordinal only. Only measurable value functions and utility functions provide cardinality. (1992: 132)

He also opines that when measurable value functions and utility functions for a subject differ, this may indicate that the subject has a hidden, or un-elicited, objective (1992: 187). Bouyssou and Vansnick (1988: 110) observe, “Many authors have argued that they see no reason why lottery comparisons should coincide with preference difference comparisons.” Clemen (1996: 553) argues that for decision analysis the distinctions are not significant for applications. Pennings and Smidts (2000) state that significant differences between utility functions and value functions were found by Krzysztofowicz (1983a and 1983b), Keller (1985b), Smidts (1997), and Weber and Milliman (1997). Their “results confirm the proposition that $u(x)$ and $v(x)$ are different constructs.” (1343)

If value and utility functions are distinct, it is intuitively appealing that they are related. Both measure preference in a given decision context. Utility has the additional consideration of probabilities of various desired and undesired outcomes, as well as possibly other factors. Bouyssou and Vansnick (1988: 109) state, “That some idea of strength of preference interferes with risky choices is hardly disputable.” They continue (110), “It seems many features do influence risky choice apart from strength of preference. Among them, we feel regret, … disappointment, … the existence of a specific utility (or disutility) of gambling, the misperception of probabilities … and the avoidance of ruin are the most important ones.” They state (109) “according to Bell [under certain conditions] that value is related to utility either by an affine or an exponential transformation.” They also offer the observation that just because value and utility are both on an interval scale that does not mean that there exists a function that is a linear transformation from one to the other. (107)

Von Winterfeldt and Edwards (1986: 213) observe that as of that writing, little of the literature addressed the relationship between value functions and utility functions. The proposed relationships applied only under constrained conditions. They summarized the concept of mapping from the decision space into a preference space in a graph, adapted in Figure 6. An attribute is measured on some scale, transformed into value, and then the corresponding value is transformed into utility. Short cuts are also possible, bypassing intermediate steps, as shown in Figure 7, also adapted from Von Winterfeldt and Edwards (1986).

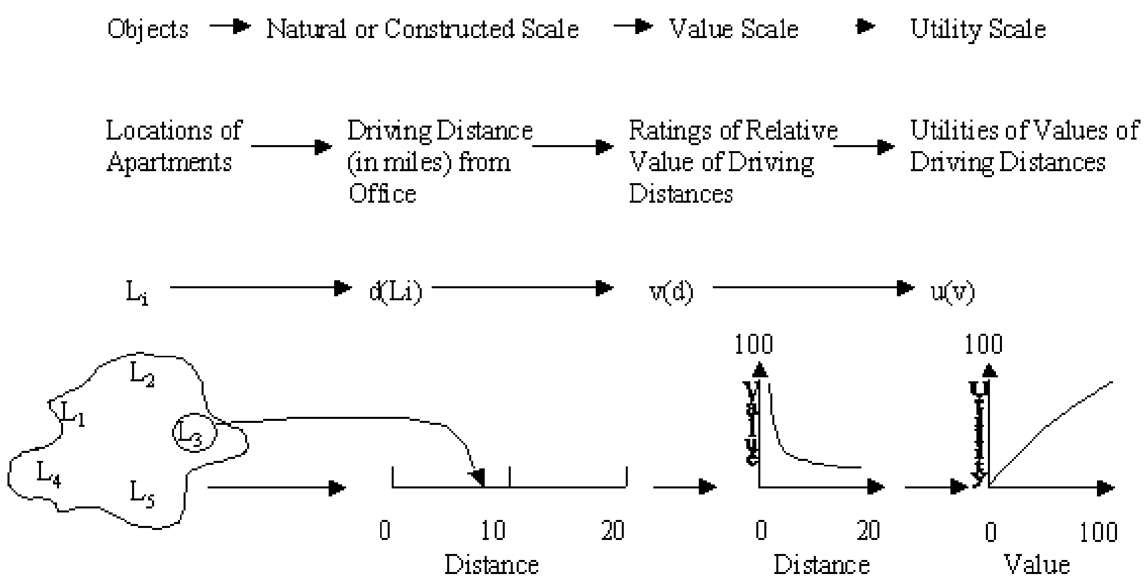


Figure 6. Mapping From Decision Space Into Preference Space (Von Winterfeldt and Edwards, 1986).

Krzysztofowicz (1983b) makes the argument that $u = w(v)$ for some transformation w. He says that according to Pratt (1964) that the value to utility

Full Decomposition

Objects → Natural or Constructed Scale → Value Scale → Utility Scale

<u>No Value Scale (Keeney and Raiffa, 1976)</u>

Objects → Natural or Constructed Scale → Utility Scale

<u>No Natural or Constructed Scales (Bell and Raiffa, 1979)</u>

Objects → Value Scale → Utility Scale

<u>No Natural or Constructed Scale, Value Equals Utility (Edward, 1977)</u>

Objects → Value Scale = Utility Scale

<u>No Natural, Constructed or Value Scales (Raiffa, 1968)</u>

Objects → Utility Scale

Figure 7. Decision Space To Utility Space Mapping Sequences, after von Winterfeldt And Edwards (1986).

transformation w holds under the condition of constant relative risk aversion. Using Pratt's *measure of local risk aversion c*,

$$c(x) = -\frac{d^2u(x)}{dx^2} \Bigg/ \frac{du(x)}{dx} \tag{16}$$

He then proposes an equivalent *value function measure of local risk aversion*, *m*,

$$m(x) = -\frac{d^2v(x)}{dx^2} \Bigg/ \frac{dv(x)}{dx} \tag{17}$$

and for the function *w*, *utility function measure of local risk aversion*, *n*,

$$n(x) = -\frac{d^2w(v(x))}{dv^2(x)} \Bigg/ \frac{dw(v(x))}{dv(x)} \tag{18}$$

He then offers the theorem that a decision maker is relatively risk averse if and only if $n(x) > 0$, relatively risk seeking if and only if $n(x) < 0$, and relatively risk neutral if and

only if $n(x) = 0$. These relationships translate to concavity, convexity, and linearity of the function *w*.

Krzysztofowicz then provides a theorem for the transformation *w*:

$$u(x) = \frac{1 - e^{-bv(x)}}{1 - e^{-b}} \text{ iff } n(x) \equiv b > 0 \tag{19}$$

$$u(x) = \frac{e^{-bv(x)} - 1}{e^{-b} - 1} \text{ iff } n(x) \equiv b < 0 \tag{20}$$

$$u(x) = v(x) \text{ iff } n(x) \equiv b = 0 \tag{21}$$

Krzysztofowicz' (1983b: 110 – 111) analysis of his experimental results, as well as his analysis of data from the literature, supports that:

- Value and utility functions are different measures, both in theory and as supported in use.
- The classic von Neumann-Morgenstern utility function (1953) must be reinterpreted within a relative risk attitude framework.
- Decision makers are relatively risk constant in a single decision context, and that the function *w* holds.
- Relative risk attitude is not constant between decisions.
- Group decision making results in relative risk neutrality.

Keller (1985b) examined the relationship between value and utility functions in 29 subjects for one of four decision situations. She examined the fit of four models of the relationship between value and utility: linear, $v(x) = u(x)$, exponential (using Krzysztofowicz' notation *w*),

$$w(v(x)) = \frac{1 - e^{-c(v(x))}}{1 - e^{-c}}, c \neq 0 \tag{22}$$

logarithmic,

$$w(v(x)) = \frac{\log(v(x) + c) - \log c}{\log\left(\frac{1+c}{c}\right)}, c > 0 \tag{23}$$

and power,

$$w(v(x)) = v(x)^{c} \tag{24}$$

These functions provide concave or convex smooth curves, except for the logarithmic function, which only provides concave curves. She found that the linear model provided the best fit in only one instance, supporting the assertion that value and utility differ. The exponential model was the best fit in nine cases, the power four, logarithmic three, and for twelve subjects no model produced an acceptable fit (root mean square error less than 0.05). Her conclusions, the last two of which she cautions are preliminary, are:

- Generally, $v(x) \neq u(x)$.
- Risk attitudes varied widely.
- Risk attitudes vary for a subject between attributes.
- Constant relative risk aversion appears to not be appropriate for all subjects.

The final point differs with the work of Bell and Raiffa (1982) in that they hypothesized that individuals would exhibit constant risk aversion in all attributes. Keller

proposes that constant relative risk aversion does not always produce an adequate model of decision making. She also observes that her findings are in disagreement of Krzysztofowicz (1983b), who found the exponential family to provide good results.

Methods of Comparing Preference Functions

Several methods are present in the DA literature. Two basic forms exist, those that evaluate preference functions at a specific point, and those that examine preference functions over the entire domain. Pratt's (1964) (and Arrow's) measure of local risk aversion, $r(x) = -u''(x)/u'(x)$, may be used to compare two functions and so is an example of the former. McNamee and Celona (1987: 94) suggests that an exponential utility function, $u(x) = a - be^{-x/R}$, be fit to a subject's preference data and the constant $R > 0$ is defined as the *risk tolerance*. Alternatively, they define a *risk-aversion coefficient* as 1/R. The risk tolerance or risk aversion coefficient may be used to compare preference functions at an arbitrary point (i.e., outcome), if the assumption of the exponential function is acceptable.

McCord, and de Neufville (1983) compared value and utility functions with "relative difference." They selected an ordinate scale position, and then compared the differences between the corresponding abscissa values for two functions divided by the corresponding abscissa value for some selected reference function. Figure 8 shows these values for two functions f and g compared to a reference function h evaluated at

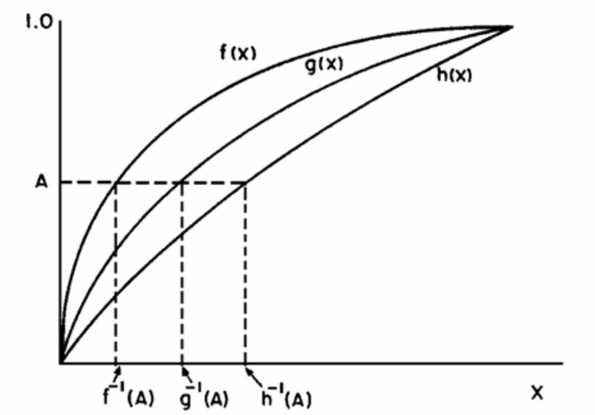


Figure 8. Measure of Relative Difference (McCord and de Neufville, 1983: 292).

$f(x_1) = g(x_2) = h(x_3) = A$. They define the relative difference $D(A)_{f,g}$ as

$$D(A)_{f,g} = \left| \frac{f^{-1}(A) - g^{-1}(A)}{h^{-1}(A)} \right| \tag{25}$$

Fishburn (1979) proposed RMSE, which permits evaluation across the entire domain rather than being limited to a specific point. Keller (1985b) employed RMSE in her comparison of value and utility functions and also introduced a measure, *G*, for comparing functions to the linear model of the relationship of value and utility, $u(x) = v(x)$. *G* is defined as

$$G = 1 - \frac{MSE(BestFittingModel)}{MSE(LinearModel)} \text{ for } MSE(Linear\ Model) \neq 0 \tag{26}$$

When MSE for the linear model is zero, then $u(x) = v(x)$ and she defines *G* to be zero. She says Fishburn and Kochenberger (1979) presented a similar measure.

Another possible discriminator exists. Kimbrough and Weber (1994: 627) defined a measure called the *preference area*. This is the area under the utility curve,

with the domain normalized to the unit interval. A risk neutral DM would have a preference area equal to 0.5. A risk averse DM would have a preference area greater than 0.5, and a risk seeking decision maker would have a preference area less than 0.5. When analyzing data, they equated risk neutrality to the interval [0.45, 0.55]. The preference areas of the preference functions could be compared, although this was not the intent of this construct. However this metric would not discriminate between two non-identical functions that had the same area under the curve.

Decision Analysis Methodologies

A number of textbooks are dedicated to the subject of decision analysis and offer methodological approaches. A selection of the more influential, in order of increasing age, is Clemen and Reilly (2001), Kirkwood (1997b), Keeney and Raiffa (1993), Kleindorfer, Kunreather, and Schoemaker (1993), Watson and Buede (1987), and von Winterfeldt and Edwards (1986). Additionally, the work by Keeney (1992a), while not a textbook, has been very influential in decision analysis.

SMARTS. A number of methods employing the underlying tenets of value and utility for decision analysis have been developed. Within the decision analysis community, the SMARTS (Simple Multiattribute Rating Technique using Swings) technique is well known. Originally developed by Edwards (1977) it has undergone minor modifications over time. It is summarized in Edwards and Barron (1994) along with some simplifying modifications not discussed here. The steps of SMARTS, in the author's terminology, are:

1. Purpose and Decision Makers. Identify the purpose of the decision and those who will make it.
2. Value Tree (Hierarchy). Elicit a structured view of the objectives for the decision.
3. Objects of Evaluation. Define the alternatives with their attribute scoring scales and domain limits.
4. Objects-by-Attributes Matrix. Create a matrix comparing alternatives and scores.
5. Dominated Options. Remove from consideration all dominated alternatives.
6. Single-Dimensional Utilities. Elicit, and then replace the scores in the matrix with utility levels.
7. Rank Order Attributes. Order in declining importance.
8. Quantify Attribute Weights. Quantify in terms the most important attribute.
9. Decide. Analyze the decision, employing an overall utility model. The most common is the additive model.
10. Determine the utility of each alternative as given by

$$U_h = \sum_{i=1}^{I} w_i E\left[u_i\left(x_{hi}\right)\right] \tag{27}$$

where h $(h = 1,2,\cdots,H)$ is an index of alternatives, i $(i = 1,2,\cdots,I)$ is an index of attributes, w_i are the weights, x_{hi} is the *i*th evaluation measure under the *h*th alternative, and $u_i\left(x_{hi}\right)$ are the corresponding single dimensional utility levels.

For our purposes, without loss of generality, the preference functions will range in the unit interval and all attribute weights sum to one. That is,

$u(x_i), v(x_i) \in [0,1] = \{r \in \mathbb{R} : 0 \le r \le 1\}$ and $\sum_{k=1}^{K} w_k = 1$ with $0 \le w_k \le 1$ for all k. Further, in this chapter, increasing levels of x are preferred. Define x_0 as the least preferred level of x and x_* as the most preferred. Then we have $u(x_0) = v(x_0) = 0$ and $u(x_*) = v(x_*) = 1$. These conventions are well-known in the decision analysis literature.

Utility Elicitation. There are several approaches for assessing the single dimensional utility functions. The Lock-Step Procedure selects points in the attribute domain at some fixed interval (Kirkwood, 1997: 233). At each point, indifference curves are constructed permitting a utility function to be established. A more efficient, and widely used, approach is the Midvalue Splitting Technique (Kirkwood, 1997b: 233 – 235, 235 – 237). The domain is bisected, and the utility elicited at that point. The two half-domains are similarly bisected, and those points elicited. This procedure may be extended until the desired resolution is achieved.

The elicitation of the utility at these points is frequently done employing one of two techniques, the certainty equivalent method or the probability equivalent method. In both methods, the decision maker is presented a decision involving a choice between uncertain alternative A and a certain alternative B. Alternative A has two possible outcomes. One occurs with probability p and yields outcome x_*, the most preferred level of x. The other occurs with probability $(1-p)$ and produces x_0, the least

preferred level of x. Alternative B provides a sure payoff of x_B, where $x_0 < x_B < x_*$.

This is shown in Figure 9.

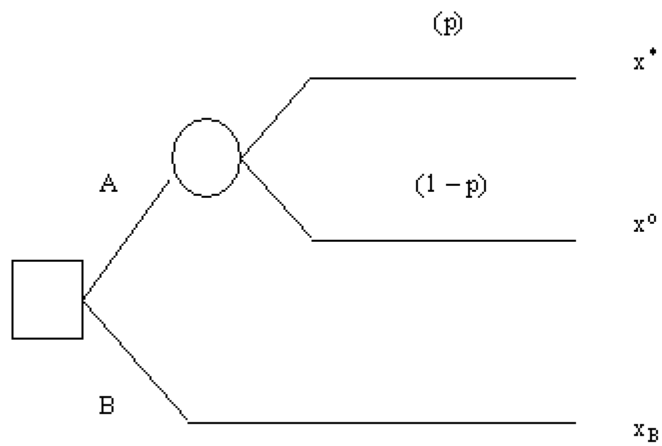


Figure 9. Decision Diagram. Squares represent decisions, circles uncertain events, parentheticals probabilities, and outcomes are expressed as the variable *x*.

In the certainty equivalent method, the probability p is equal to 0.50, so Alternative A is a 50-50 bet between the high and low x values under consideration. The decision maker is presented various values of x_B until she is ambivalent in choosing between the two alternatives. By the expected utility axioms, the utilities of both alternatives must be equal at ambivalence. Further, the utility of x_* is equal to one, and the utility of x_0 is equal to zero, by definition. So

$$u(x_B) = (1-p)u(x_0) + (p)u(x_*) \tag{28}$$

$$u(x_B) = (.5)u(x_0) + (.5)u(x_*) \tag{29}$$

$$u(x_B) = (.5)(0) + (.5)(1) = .5 \tag{30}$$

The utility may next be determined for the midpoints of the subregions $[x_0, x_B]$ and $[x_B, x_*]$.

In the probability equivalent method, x_B is fixed at some value and the probability p is varied until the decision maker is ambivalent between the alternatives. This provides

$$u(x_B) = (1-p)u(x_0) + (p)u(x_*) \tag{31}$$

$$u(x_B) = (1-p)(0) + (p)(1) = p \tag{32}$$

Since p is known, the utility of x_B is known. The utilities of subregions are then determined in similar fashion as in the certainty equivalent method.

One the utility has been elicited at discrete points the utility function is established. A typical assumption, that of constant risk aversion permits the fitting of an exponential curve. In practice, often only a single midvalue point is used, and the curve fitted. (Kirkwood, 1997b: 235)

Value Elicitation. Comparing the increase in preference for pairs of attribute levels permits value function elicitation. Often this is done my determining a midvalue level. If a decision maker is indifferent, for a given price paid, to go from x_0 to x_m as from x_m to x_*, then x_m has value equal to 0.5. This from

$$(x_0, x_m) \sim (x_m, x_*) \Rightarrow [v(x_m) - v(x_0)] = [v(x_*) - v(x_m)] \tag{33}$$

$$v(x_m) - 0 = 1 - v(x_m) \tag{34}$$

$$2v(x_m) = 1 \tag{35}$$

$$v(x_m) = \frac{1}{2} \tag{36}$$

Similar comparisons over the resultant subregions permit additional value point estimates.

Weight Elicitation. Essentially, this is typically performed by starting with all attributes at their lowest levels, then "swinging" one attribute at a time from its lowest to highest levels. Take the smallest reported increase in worth, and then express the other increase in terms of the smallest. Normalize the weights so that they sum to one. (Kirkwood, 1997b: 68 – 70) Clemen (1996: 546 – 552) also provides a description of this technique as well as pricing out and lottery weight techniques.

Sensitivity Analysis. Sensitivity analysis has a role in all decision making, not just within decision analysis. "In solving a problem, a sensitivity analysis should be made in order to determine the variables to which the outcome is most sensitive" (Operations Analysis Study Group, 1977: 12). A typical decision analysis effort involves first constructing a deterministic model of the decision opportunity. Then the model is examined to see in where uncertainty causes important effects. Those variables where uncertainty has a significant impact are then modeled as stochastic while the remaining variables are handled in a deterministic manner. The determination of which variables should become stochastic members of the model is called *sensitivity analysis* and is and is ascribed to Howard and sometimes referred to as the "Stanford approach" (Reilly, T., 2000: 551). "The motivation behind sensitivity analysis is to reduce the assessment burden" (Reilly, T., 2000: 556). Eschenbach (1992: 41) defines sensitivity analysis as "examining the impact of reasonable changes in base-case assumptions." Skinner (1999:

358) links the employment of sensitivity analysis specifically to uncertain aspects of a decision problem, rather than assumptions. Eschenbach provides that the concern is for the sensitivity of the response variable to differing levels of independent variables. He lists several considerations for sensitivity analyses: (1) establish reasonable bounds for independent variables, (2) establish the "unit impact" of these changes, (3) establish the maximum impact of the independent variables, and (4) establish the change required in each independent variable to effect alternative choices.

Stanford Approach to Sensitivity Analysis. In the Stanford approach, sensitivity analysis is usually conducted by singly perturbing each variable of a deterministic DA model. Reilly (Reilly, T., 2000: 553 – 554) provides a summary of this approach. This is often referred to as "one-way sensitivity analysis" (Clemen and Reilly, 2001: 179 – 183). Variables are varied singly, and the effect on the overall preference model is observed. When the objective function score changes considerably, and especially when the preferred alternative changes within the range of change of the variable, the decision problem is sensitive to this variance. Two-way sensitivity analysis is also done, where two variables are varied simultaneously.

Reilly (Reilly, T., 2000: 552 – 554) summarizes the Stanford approach commonly as modeling the variables that create the greatest swing, $S_o\left(X_i\right)$, where the swing is defined

$$S_o\left(X_i\right)=\max\left\{o\left(x_{il}\right),o\left(x_{ib}\right),o\left(x_{ih}\right)\right\}-\min\left\{o\left(x_{il}\right),o\left(x_{ib}\right),o\left(x_{ih}\right)\right\}$$

where $\mathbf{X}=\left(X_1,X_2,\ldots,X_n\right)^T$ is the column vector of all input variables (evaluation measures and relative weights); $x_{il},x_{ib},$ and x_{ih} are the elicited possible low, most

expected (base), and possible high values of x_i; and $o(x)$ is the preference (objective) function, with all $X_k \neq X_i$ fixed at x_{kb}. A measure, the percent variance of X_i, $PVE_o(X_i)$, may be used to select the number of variables to represent as stochastic variables. The measure is defined by

$$PVE_o(X_i) = \frac{\left[S_o(X_i)\right]^2}{\sum_{k=1}^{n} S_o(X_k)^2} \tag{37}$$

Reilly states that it is common to see use of $PVE_o(X_i)$ lead to statements similar to "these four variables capture 95 percent of the variance in the value function. (Reilly, T., 2000: 554). He points out that such an interpretation is correct only if the variables are independent and jointly multivariate normal.

Rank Order Stability Analysis. Jolsh and Armstrong (2000: 537 – 538) indicate that sensitivity analysis may be accomplished through a technique referred to as Rank Order Stability Analysis (ROSA) and attributed to Einarson, Arikian, and Doyle (1995). This approach is basically that of a standard sensitivity analysis. Instead of varying a parameter through a range of values of interest and observing the response of the objective function and noting the change in optimum alternative, ROSA merely notes the range for the variable that retains the optimum alternative. This is restating the standard approach results but presents less information. However, this is a desirable technique when the decision maker wishes to control variables to maintain the optimum solution rather than indicate how the decision responds to changes.

Jolsh and Armstrong also define the *elasticity* of a variable, E_i , as the relative change in the response variable to a relative change in a predictor variable,

$$E_i = \frac{\Delta y / y}{\Delta x_i / x_i} \tag{38}$$

where y is the response variable and x_i an independent variable. The elasticity provides information analogous to the response surface methodology (RSM) coefficient for x_i . However RSM provides statistical tests of the significance of the term and also is conducted in a multivariate framework rather than the ROSA univariate approach. A multivariate approach permits the consideration of variable interactions.

Dependent Sensitivity Analysis. Reilly (Reilly, T., 2000) offers a methodology, dependent sensitivity analysis (DSA), for sensitivity analysis where the singular value decomposition of a correlation matrix is used to combine the independent variables into approximately uncorrelated synthetic variables. The advantages of his method are that independent variable dependencies are not disallowed and the implementation is straightforward. The disadvantages are the requirement for additionally eliciting measures of dependence relationships – Spearman's rank correlation, the requirement to assess or assume the standard deviation of the predictor variables, the loss of specificity when considering independent variables grouped into the synthetic variables, and the additional comprehension burden for the decision maker of the concept of the synthetic variables themselves.

Reilly's DSA uses Spearman's rank correlation to infer dependency among predictor variables. Asking the subject directly assesses this statistic, although he advises

that the research has not definitively validated this approach (2000: 555). Using Spearman's rank correlation requires that monotonicity be assumed and that the correlation matrix be positive definite. The DSA approach constructs synthetic variables that are affine combinations of the predictor variables. Let the matrix of rank correlation estimates $\mathbf{R_S} = (r_{ij})$ where r_{ij} is the rank correlation for X_i and X_j, $i, j = 1, 2, \ldots, n$. Then $\mathbf{R_S}$ is decomposed through singular value decomposition

$$\mathbf{R_S} = \mathbf{L\Lambda L}^T \tag{39}$$

where $\mathbf{LL}^T = \mathbf{L}^T\mathbf{L} = \mathbf{I}$, the columns of $\mathbf{L}$ being composed of the orthonormal eigenvectors have $\mathbf{R_S}$, and $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues of $\mathbf{R_S}$. The estimates of the variable standard deviations, s_i for each σ_i associated with the X_i, are placed into a diagonal matrix $\mathbf{S}$. The synthetic variables are determined by

$$\mathbf{Y} = \mathbf{L}^T\mathbf{S}^{-1}(\mathbf{X} - \mathbf{M}) \tag{40}$$

where $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ and $\mathbf{M} = (x_{1b}, x_{2b}, \ldots x_{nb})^T$. The Y_i are approximately uncorrelated. Rearranging _ to provide sensitivity analysis produces $\mathbf{X} = \mathbf{SLY} + \mathbf{M}$. For each X_i in $o(\mathbf{X})$ the substitution becomes $X_i = x_{ib} + s_i(L_{i1}Y_1 + L_{i2}Y_2 + \cdots + L_{in}Y_n)$ for $i = 1, 2, \ldots, n$. Reilly states that this substitution is readily done using a spreadsheet.

Varying a synthetic variable Y_i causes each X_j to be perturbed by the addition of $L_{ji}Y_is_i$. The direction of the perturbation is determined by L_{ji} and the magnitude by the product of L_{ji} and s_i. The influence of the L_{ji} provide that correlated X_j vary together. The perturbation of each Y_i should cover four standard deviations. Because the synthetic

variable variation is approximately equal to the eigenvalues, Reilly recommends using varying Y_i from $-2\sqrt{\lambda_i}$ to $2\sqrt{\lambda_i}$. The $\mathbf{L}$ matrix is also useful is indicating which X_j have significant influence. Influential X_j will have $\mathbf{L}$-coefficients "close to" $|1|$. Interpretation of the results is similar to that of the predictor variables except that the synthetic variables are combinations thereof. For example, a tornado diagram (see below) may be constructed of the synthetic variables just as would be done with the basic predictor variables.

Response Surface Methodology. Reilly (Reilly, T., 2000: 570) comments that "No systematic approach has emerged to uncover influential groups of variables since the sheer number of all possible multiple-way sensitivity analyses make an exhaustive search prohibitive." This is not quite correct, although the information was not available to Reilly when he prepared his initial draft. An area of promise is the work by Bauer, Parnell and Meyers (1999: 162 – 180), where response surface methodology (RSM) was employed as a demonstration of its efficacy for decision analysis sensitivity analysis. Response surface methodology is "a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes" (Myers and Montgomery, 1995: 1).

Response surface methodology provides the ability to model some system that behaves according to $y = g\left(x_1, x_2, \ldots, x_n\right) + \varepsilon$, where *g* is unknown, with $y = f\left(x_1, x_2, \ldots, x_n\right) + \varepsilon$, where *f* is determined with empirical data. Generally *f* is either a first or second-degree polynomial, and forms the response surface. Figure 10 illustrates how a system may be viewed as transforming inputs ξ_i into some response *y* or

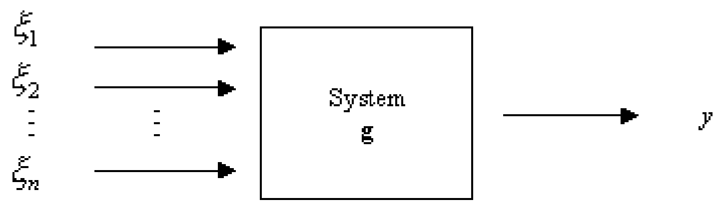


Figure 10. Representation of a System Modeled by RSM.

$g(\xi_1, \xi_2, \ldots, \xi_n) = y$. Response surface methodology replaces *g* with a low degree polynomial *f*. Response surface methodology is frequently used as a substitute for *g* or to gain insight into the important aspects of *g*. Response surface methodology is sometimes referred to as regression analysis or meta-modeling. (Bauer, 2001: 71)

Response surface methodology may be thought of a polynomial surface over a region that approximates the surface of *g*. Each alternative in a decision would generally have a differing surface *g*. The independent variables, ξ_i, have upper and lower bounds L_i and U_i respectively, when considered across all alternatives. Generally in RSM the independent variables are coded $x_i = 2\left(\frac{\xi_i - \xi_{i0}}{U_i - L_i}\right)$ for $i = 1, 2, \ldots, n$ where ξ_{i0} is the midpoint between L_i and U_i. This maps L_i and U_i to –1 and +1, respectively, offering the advantage of working with unitless variables and facilitating experimental design. (Bauer, Parnell, Meyers, 1999: 164)

Response surface methodology produces a metamodel which models the response of the decision analysis model as the input is varied. The response is the multiattribute preference function, and the input the decision parameters. The hypothesized model describes the response

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2, \quad \varepsilon \sim N\left(0, \sigma^2\right) \tag{41}$$

where the β are unknown coefficients. This approach is analogous to a Taylor's series expansion of degree 2 for the true functional relationship. These coefficients provide insight as to contribution of each variable, as well as the strength of variable interactions. Equation (41) may be simplified by omitting the x_i^2 term and the $x_i x_j$ interaction term may also to neglected. As simplified, the expression is easier to solve and so is often used as a screening device early in the RSM process. Employing the complete Equation (41) near the subregion of concern provides a better fit of the approximated surface. The least squares estimated coefficients of the hypothesized model

$$\hat{y} = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i<j} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} x_i^2 \tag{42}$$

is, in vector notation,

$$\mathbf{b} = \left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{X}'\mathbf{y} \tag{43}$$

The fitted surface is provided by

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b} \tag{44}$$

and residual error is

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \tag{45}$$

The coefficient provides a ready indicator of the contribution for the associated variable and statistical tests provide a measure of significance. (Myers and Montgomery, 1995: 16 – 21)

Bauer, Parnell, and Meyers present Figure 11, a graphical representation of iterative application of RSM to a DA problem (1999: 165 – 166). To reduce the analytic burden, variables that are believed to behave similarly may be grouped and a RSM screening conducted. Those groups that have significant effects are then screened as individual factors. Significant variables are then modeled with a first order equation and the fit is tested. If the fit is insufficient, a second order model is employed.

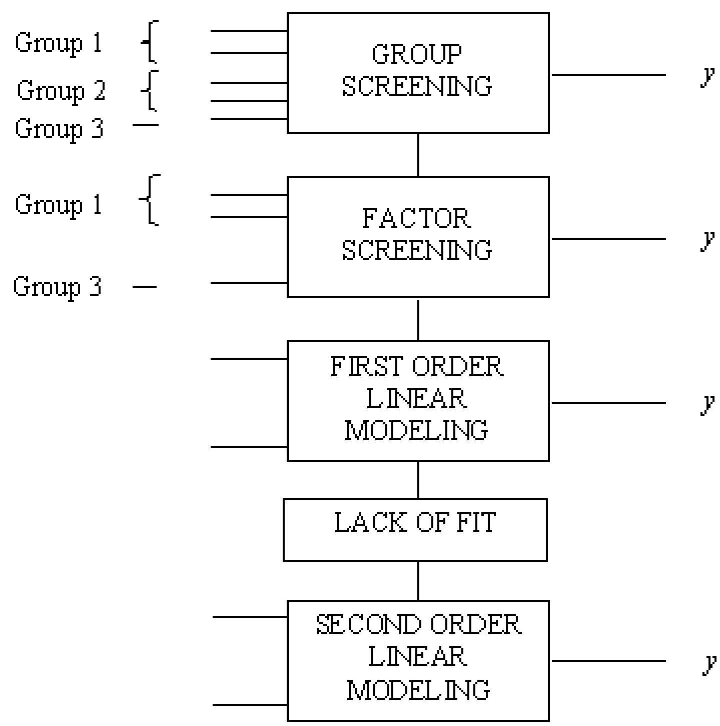


Figure 11. RSM Paradigm (After Bauer, Parnell, and Meyers, [1999: 166]).

Group Screening. As Bauer, Parnell, and Meyers (1999) employ group screening, it deserves additional discussion beyond a review of RSM. Meyers and Montgomery (1995) fail to mention group screening in their RSM text. A screening design in experimentation is a selection of experimental inputs that permit establishment

of which factors are significant while minimizing the number of experimental iterations (usually referred to as runs in modeling) and so reducing associated costs. Large numbers of factors force a prohibitively large number of runs or, when the number of runs is constrained, causes factors to be confounded and effects masked. Madu and Kuei (1992: 96) caution that screening large numbers of variables is to be avoided. Group screening reduces the number of factors considered by consolidating them into groups. These groups are then manipulated simultaneously so the experimental design has been reduced from the total number of factors to the number of groups.

O'Geran and Wynn (1992) provide a history of the group screening methodology. The concept was first employed in 1943 to quickly screen human blood. Samples from donated units were combined and tested. With no positive results in pathological tests the units were quickly cleared for use. When test results indicated pathogens were present, individual units were screened. This concept was applied to the problem of detection of significant factors by Watson (1961). Kleijnen (1975: 488) indicates that Patel and Li generalized this two-step process into n-steps independently in 1962.

In the procedure, stated simply, variables are first aggregated into groups. Various authors have developed recommended differing grouping assumptions and strategies. Then an acceptable experimental design based on the groups is selected. When the runs are complete, all variables contained in the groups found not to be significant are assumed to be non-significant. Significant groups are decomposed and new groups formed, if required, and the process repeated. Group screening has not been found in the DA literature beyond the work by Bauer, Parnell, and Meyers (1999). Their efforts are shown graphically in Figure 11.

While standard decision analysis sensitivity analysis identifies which variables, without interaction, cause the solution to be sensitive, and DSA addresses interactions employing synthetic variables, RSM provides a quantitative estimate of the impact of each variable and interactions.

Communicating Sensitivity Analysis Results. Single-dimensional sensitivity analysis results are usually presented as a *rainbow diagram*. This is a Cartesian plot of the objective function as a function of one of the predictor variables, the others held constant at their expected value levels. The area under the curve is normally color-coded to indicate which alternative is the most optimal and this is responsible for the name. An example rainbow diagram is shown in Figure 12. A *tornado diagram* is used to show the effect of singly varying multiple predictor variables. An example is provided in Figure 13. The change in the dependent variable as each independent variable is varied is indicated by horizontal bars. Bar endpoints are positioned on a scale of the independent variable, so bar length is a measure of the sensitivity of the dependent variable to the corresponding independent variable. Bar color is changed to indicate differing alternatives have achieved optimality. The bars are arrayed in descending order, creating a graphic evocative of a tornado moniker. This graphic device can accommodate a large number of independent variables and quickly shows their relative importance.

Eschenbach (1992: 42 – 43) promulgates the spiderplot as an adjunct graphical device to display sensitivity analysis results. The spiderplot is a Cartesian plot of response the dependent variable to percentage change in independent variables where each independent variable is represented by a curve. The curves intersect at the base point (unperturbed independent variables) creating a spider web-like appearance. The

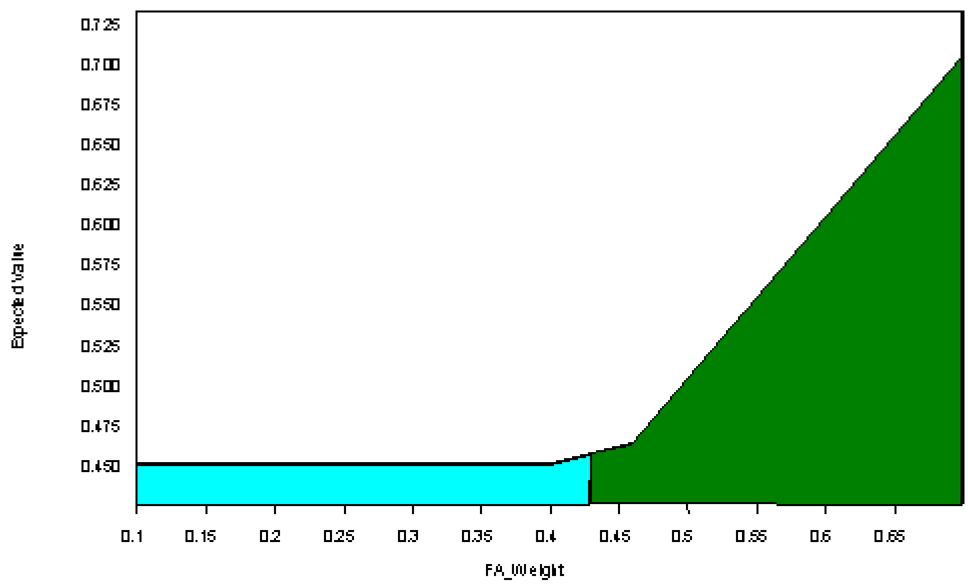


Figure 12. Sample Rainbow Diagram.

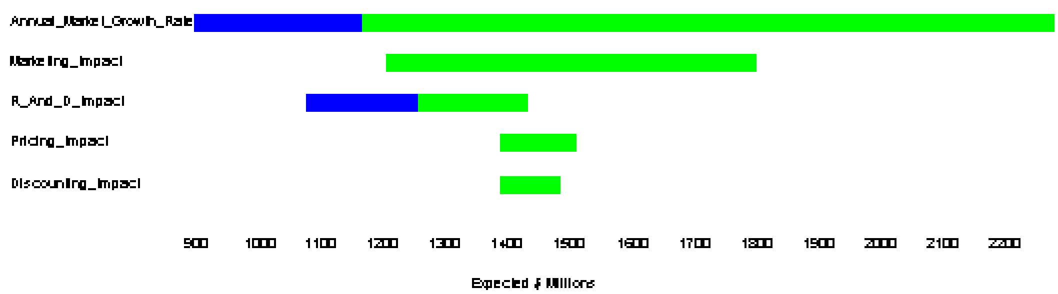


Figure 13. Sample Tornado Diagram.

advantages of a spiderplot as compared to a tornado diagram are that the former shows the respective selected independent variable limits, the unit effect of changes in the independent variables, and the level of maximum impact of the independent variables. The disadvantages are the increased sophistication required to understand the diagram, tendency for the diagram to be incorrectly prepared, and the number of independent variables is restricted to a small number for clarity. Eschenbach does not discuss presenting the effects of interactions. An example of a spiderplot is shown in Figure 14.

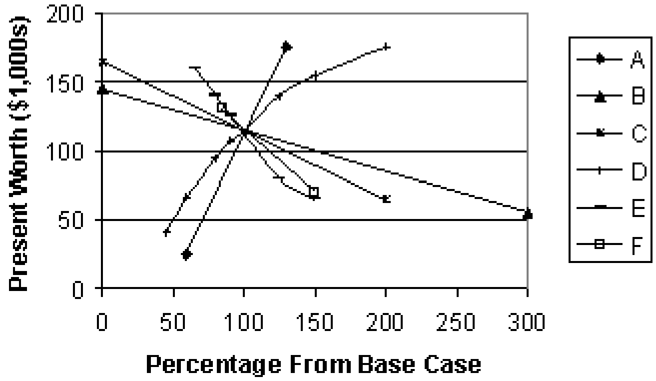


Figure 14. Sample Spider Plot. After Eschenbach (1992: 43).

Clemen and Reilly refer to the problem of presenting results of sensitivity analysis of two or three stochastic variables (2001: 183 – 192). They observe that this is a "difficult problem" (2001: 183). Their solution is the sensitivity graph. For two variables, a Cartesian plot is prepared with each axis representing various levels. The plot area is partitioned into regions by optimal solution, generating a plot analogous to a phase diagram in chemistry. The plot may be extended into three dimensions to present a response surface. This represents the state of the art in DA software. For example, DPL® provides a "two-way rainbow diagram" that examines changes in two variables simultaneously (Borison, 2000: 530 – 539) as illustrated in Figure 15. For three variables, if these may be expressed as a mixture, a ternary (or trilinear) chart may be used (Schmid, 1954: 170 – 171). At least one software package, Data®, provides for "three-way sensitivity analysis" in the form of a "two-way" animated graph (Data 3.0, 1996: 219 – 220). The user interactively varies the third variable and observes the affects on the two-dimensional plot.

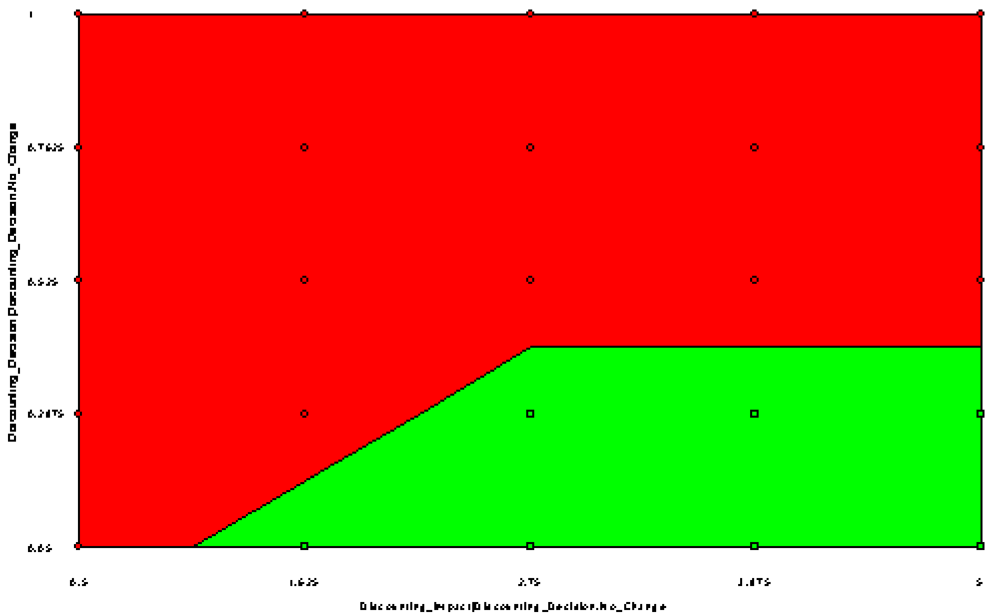


Figure 15. Sample Two-Way Rainbow Diagram.

Sensitivity Analysis of Preference Functions. Sensitivity analysis in the decision analysis literature excludes discussion of preference functions. The literature does not address the issue, although it does provide evidence that the issue is of import. Ghosh and Ray (1997) found that decision maker's choices are both a function of risk attitudes and ambiguity tolerance. Additionally, their results suggest that the decision maker often interprets ambiguity in a decision situation as risk. This indicates that the preference function likely involves some inherent uncertainty.

Besides the uncertainty present within the decision maker, elicitation sessions are vulnerable to human perception frailties. An example of this includes framing, where the same situation presented in differing manners elicits consistently differing preferences. Anchoring is the phenomenon in which an individual presented with some estimate of a parameter of interest will often provide biased elicitation, assessing higher probabilities

against the estimate while significantly underestimating probabilities of severe events. Further, it is well documented that individuals will demonstrate more confidence in objective probabilities than subjective probabilities without discernable justification. See Kirkwood (1997b: 110 – 127; 299 – 320) for discussion of these issues.

Recommended Future Research

Keller examined the relationship between value and utility functions in 29 subjects for one of four decision situations. She recommended that future research examine risk attitudes and relative risk attitudes across multiple attributes within subjects, and similarly within attributes across subjects. She also recommended that future work examine Bell and Raiffa's (1982: 345) conjecture that constant relative risk aversion should be observed. Finally, she recommended repetitive elicitations in order to assess elicitation error. Keller has indicated that she believes these remain "a fruitful area for further study" (2000: 1).

Keller (1989a), Brown (1992), and Howard (1992) have all called for research on *prescriptive* decision analysis methods. While differing in recommended approaches and definition of the term, they all use the term prescriptive to mean procedures that reduce the negative aspects of employment of decision analyses, while retaining the value DA provides. In general, they see reducing the burden placed on the decision maker as fostering acceptance of the employment of DA.

III. Research Overview

The research had two major objectives, both pertaining to the relationship of value and utility preference functions in decision analysis. This chapter will restate the objectives, summarize background material, provide more detail, delineate the scope of the research, and provide a summary of the methodology. Detailed explanation of the methodology accompanies the presentation of the analysis and results in Chapters IV and V. Results are summarized in Chapter VI.

Objectives

The major objectives were decomposed into supporting subobjectives. Additionally the first major objective, examining the relationship of value and utility functions, supports the second. The second objective is predicated on the observation that value and utility functions differ.

Objective 1. Examine Relationship Between Single Dimensional Value And Utility Functions. Examine the single dimensional preference (value and utility) functions of a group of military subjects within the context of a multiattribute decision situation.

Objective 1.1. Determine Functional Relationship of Single Dimensional Value and Utility Functions. Examine whether single dimensional value and utility functions have any consistent functional relationship, including whether they are equivalent. Consider the exponential, logarithmic, power, linear, and sigmoid

functions as candidates for this relationship. In particular, scrutinize the linear case, which corresponds to the situation where the value function equals the utility function.

Objective 1.2. Examine Characteristics And Performance Of Various Common Decision Analysis Elicitation Methodologies. Examine whether single dimensional utility functions elicited using the certainty equivalent and probability equivalent methods provide significantly differing constructs of a subject's preferences. Examine whether the order of elicitation affects information quality. Establish whether the multiattribute utility approach provides an acceptable substitute for single dimensional utility models. When a subject fails to reach ambivalence under the multiattribute utility approach, examine whether bounding of the multiattribute utility function provides a sufficient answer.

Objective 1.3. Improve The Ability to Compare Between Preference Functions. Develop an improved method for discriminating between preference functions. Ascertain whether the elicitation error exceeds the differences between preference functions. Establish acceptance criteria for fits of such models to empirical data.

Objective 1.4. Examine The Subject's Risk Attitudes. Examine the subject's risk attitudes for constant relative risk aversion. Examine the concept of relative consistent risk attitude, where a subject maintains consist risk attitudes across evaluation measures.

Objective 1.5. Improve The Metric For Assessing Risk Aversion. Develop a new measure of risk aversion.

Objective 2. Establish Methods Of Increased Efficiency Of Decision Analysis Elicitation By Considering Robustness Of Employing Single Dimensional Value Functions As Surrogates For Utility Functions. Examine the performance of a hybrid value-utility model as a more efficient, with respect to elicitation, DA model than a standard utility model.

Objective 2.1. Establish When Value-Utility Differences Are Significant. Employ response surface methodology and principles of design of experiments techniques to assess when the form of the preference function itself is significant with respect to the decision analysis model.

Objective 2.2. Develop an Algorithm for Employment of a Hybrid Value-Utility Model. A hybrid DA model adapts an existing value model to adequately represent the corresponding utility model. The algorithm will prioritize the iterative procedure as well as specify stopping criteria.

Background

In decision analysis, a model of the decision problem is manipulated to gain insight. The model must satisfactorily mimic the decision to be studied. The model estimates will deviate from some central value due to external and internal variation. This variation must be low enough to prevent interference with determining the proper recommendation.

External variation is that present in the environment in which the decision must be made. For example, a decision often must be made where each alternative may produce

one of several possible outcomes. The probability distribution of the outcomes is a source of variation external to the model, regardless of how the model is constructed. Internal variation is a function of the model itself. In Expected Utility Theory, a single dimensional utility function is used to map levels of some measure into the unit interval preserving a decision maker's attitudes regarding the levels and associated uncertainty. An example of this would be the function $u(x) = (1/10) \cdot x$ where x is the number of days to complete a project, assuming that the function captured the attitudes and preferences of the decision maker. However preference functions must be elicited from subjects and are subject to elicitation error. This error is an example of internal variation.

A taxonomy of decision analysis model variance is presented in Table 1. External variation is often categorized as: *none*, *known*, or *unknown*. Decision making under these conditions is referred to as being under *certainty*, *risk*, or *uncertainty*, respectively. (Some authors group risk and uncertainty in a single category employing the latter term.) These external sources of variation must typically be estimated in some fashion. For example, a business decision may require that a projection be made for the interest rate for a loan in a future time period. Likely the estimate will not exactly coincide with the rate that occurs. These estimates are listed separately in the taxonomy as they are clearly not the external variation itself, but also are not due to the model proper, as any appropriate model may be applied. Further, these estimates may be generated by the decision maker himself or by separate subject matter experts. Therefore these subjective error sources may not be directly discernable.

The internal error, that due to the model itself, is type specific. A decision problem may be approached with a wide variety of techniques available to the operations

Table 1. Taxonomy of Variation.

- External Sources
  - No Variation (Certainty).
  - Known (Risk). Probabilities are known.
  - Unknown (Uncertainty). Probabilities are unknown – sometimes referred to as states of nature.

- Estimates of External Sources
  - Random Error. Subjective estimates of probabilities contain error, $\hat{p} = p + \varepsilon$
  - Bias Error. Subjective estimates where probabilities are consistently high or low for certain portions of the domain.

- Internal (Model) Error
  - Model Type Error
    - MCDM, LP, …
    - DA
      - Model Basis
        - Formalistic (Normative)
        - Behavioral (Descriptive)
        - Axiomatic (Rational)
      - Decision Maker
        - Objectives/Value Hierarchy Elicitation Error
        - Preference Function Elicitation Error
          - Preferences
          - Risk Attitudes
        - Relative Weights Elicitation Error
  - Alternatives

- Time

research practitioner. These techniques include linear and non-linear programming (optimization), simulation, multi-criteria decision-making (MCDM), and other techniques as well as decision analysis. Each technique will engender its own error sources. For example, an infeasible optimization problem may still be solved, in a fashion, through relaxation of constraints. However MCDM may provide a better solution. When two

methodologies produce the same rank ordering of alternatives, they are considered strategically equivalent.

For this research, only decision analysis expected utility modeling, or multi objective decision analysis (MODA), was considered. Two major categories of variation are observed. The first is the model basis. There are three model categories. Formalistic models are strictly mathematical. Referred to as normative models, they are the standard against which human preferences are measured. Problems with formalistic models were recognized in early probability studies, and, as discussed in Chapter II, led to the St. Petersburg Paradox. They will not be considered further here. Descriptive models are designed to describe how decisions are made, and to predict what those decisions would be. Such models are useful in adversarial situations where knowledge of a competitor's decisions would be useful. Obviously they have use in the economic private sector, sports, and international dealings in war and peace. However descriptive models do not provide a recommendation of what would in fact be the best decision. Rational models are designed to provide such recommendations. They are referred to as rational because they provide the best decision given a model of a decision maker's attitudes about the decision components. (Again, inconsistency is present in the literature with respect to nomenclature. Some use the term normative to represent rational, not formalistic models as described above.)

The other internal error source is that incurred from the process of describing the pertinent aspects of the decision maker himself. These aspects are not directly observable, hence not directly measurable. They must be established through elicitation, and the resultant estimates will have error associated with them. The objectives of the

decision maker must be established, to provide the structure for the balance of the elicitation. Single dimensional preference functions are elicited by asking questions that are designed to determine the decision maker's preference ordering and risk attitudes. The relative importance of the objectives must be assessed. All these estimates are sources of error.

An additional possible source of internal error is the process of generation of alternatives. Failing to recognize an alternative may deprive the decision maker of best solution. However this error source is not observable when comparing models. Further, Keeney argues that Value-Focused Thinking provides the best methodology for avoiding this problem (Keeney, 1992). Finally, variations in decision problems may be introduced when considering the time dimension. It is possible that the decision context and also the decision maker's preferences and attitudes may change over time. The sources of variation are shown graphically in Figure 16.

Decision analysis studies often examine external sources of variation. Research regarding the internal sources is often limited to examining the preference weights, or to contrast one model basis with another. For example, critics of Expected Utility Theory employ descriptive models that better explain choices contraindicated by the decision maker's axiomatic preferences. When the preference function is (rarely) considered as a source of variation, it has not been examined in the sense of its contributions to overall robustness.

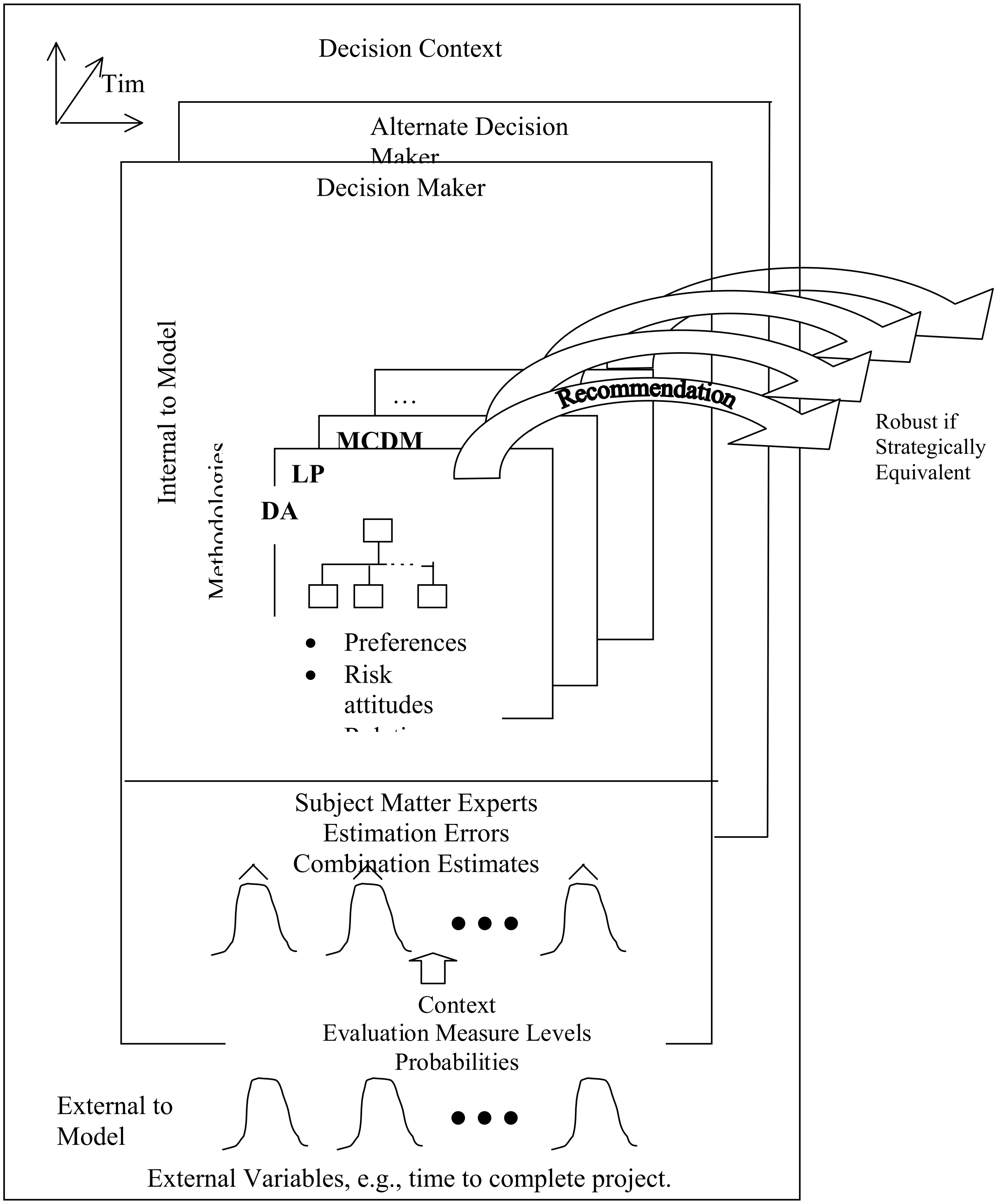


Figure 16. Sources of Decision Model Variation.

Detailed Research Objectives

Objective 1. Examine Relationship Between Single Dimensional Value And Utility Functions. The literature review in Chapter II summarizes the various positions regarding the relationship of value and utility. Keller (1985b) examined the relationship between value and utility functions for single dimensional problems, and recommended that the work be extended into multiattribute problems and include repetitive elicitations in order to assess elicitation error. Typical decision analysis empirical data involves subjects who are not professionals in the field of interest of the problem and monetary costs. Military operational decision making

Objective 1.1. Determine Functional Relationship of Single Dimensional Value and Utility Functions. Decision analysis modeling under uncertainty or risk has been performed both with single dimensional value functions or single dimensional utility functions. Various positions are argued regarding these functions. Some argue that there is no theoretical difference between these functions. Others assert that while differing in theory, the differences are not significant in application. Finally, some argue that they differ significantly in theory and in employment. Various functional relationships have been proposed. Based on the work of Pratt (1964a: 353 – 375), researchers have examined the suitability of a transform from single dimensional value functions to a utility function. These functions are the linear, exponential, logarithmic, and power functions. The only linear function is when value and utility are equal. The remaining functions form curves that are strictly concave or convex, (depending on the

coefficients). This suggests the subject is strictly risk averse or risk seeking, respectively, over the entire domain. Empirical evidence suggests that this is not always correct for example, see McCord and de Neufville, 1983a). The sigmoid function permits both concavity and convexity to occur within the range but has not been examined for adequacy of representing empirical data for DA preference functions. Establishing that the preference functions differ significantly for military decision makers is an essential task regarding the other objectives.

Objective 1.2. Examine Characteristics And Performance Of Various Common Decision Analysis Elicitation Methodologies. McCord and de Neufville (1983a) provide data that suggest that the differences encountered in eliciting theoretically identical single dimensional utility functions by differing methods exceeds that of the differences found between corresponding single dimensional value and utility functions. Kirkwood (1997b) also offers a multiattribute utility approach that converts from the value space to utility space on a multidimensional basis.

Objective 1.3. Improve The Metric For Comparison Between Preference Functions. McCord and de Neufville (1983a: 184) point out that the literature provides few tools for comparing preference functions. They offer a method, "average absolute difference," which compares two functions against a reference function. When comparing the degree to which a preference function approximates another, another measure used has been root mean square error (RMSE). This metric weights each observation equally even though points near the limits of the domain are more constrained. The domain endpoints have commonly defined ordinate values for all

preference functions. It is logical that equal differences are less likely closer to the endpoints, or conversely, that equal differences are more significant near the endpoints.

Objective 1.4. Examine The Subject's Risk Attitudes. Bell and Raiffa (1988d: 395) suggest that constant relative risk aversion is "an appealing idea." Relative risk aversion means that the utility function operating in the value domain, $\underset{\cdot}{u}$, exhibits concavity.

Object Objective 1.5. Improve The Metric For Assessing Risk Aversion. A measure of local risk aversion was developed by Pratt (1964a). However no measure of risk aversion for an entire evaluation measure for a subject has been widely accepted. The basic measure is the utility curve shape with concavity (convexity)(linearity) indicating risk aversion (seeking) (neutrality). However this fails to quantify the degree of the risk attitude when the curve is not strictly convex (concave) (linear). The concept of a preference area was proposed, defined as the area under the utility curve (Kimbrough and Weber, 1994: 627). However the set of all utility curves does not have a bijective relationship with the set of all preference area values so the measure cannot distinguish between different functions.

Objective 2. Establish Methods Of Increased Efficiency Of Decision Analysis Elicitation By Considering Robustness Of Employing Single Dimensional Value Functions As Surrogates For Utility Functions. Sensitivity analysis in DA examines the impact of uncertainty and perturbation of model parameters. However review of the literature has not produced an example of perturbing the preference function. This is a key concept when consider the difference between value and utility functions. By establishing when perturbations in the preference function are significant, it is determined

when changing form one preference function form (value or utility) to the other is significant.

For large, complex decision situations, value functions are often constructed with the aid of subject matter experts. When a utility model is desired, only the value functions that are significant when perturbed need be converted to utility functions. Such a process has the potential to greatly reduce the time burden on the decision maker. Because for large, complex problems the decision maker typically has a critical schedule, this fosters acceptance of the process.

Objective 2.1. Establish When Value-Utility Differences Are Significant. As discussed previously, work regarding value and utility functions focused on how the functions are related, or examined their impact when the entire DA model was constructed as a value model or as a utility model. The idea of development of a hybrid model, with both single dimensional value and utility functions, is not recorded. Such a hybrid, when properly constructed, should function as a surrogate for the utility model.

Response surface methodology techniques have demonstrated efficacy in determining significant contributors from a set of factors for a variety of problem types. Bauer, Parnell, and Meyers (1999) demonstrated that RSM is applicable to DA problems, enhancing the information provided during the sensitivity analysis phase. RSM has not been applied to the set of preference functions.

Objective 2.2. Develop an Algorithm for Employment of a Hybrid Value-Utility Model. No previous work has been done in this area.

Methodology and Scope

Objective 1. Examine Relationship Between Single Dimensional Value And Utility Functions. Twenty subjects, all US Army soldiers, were interviewed. Provided with a tactical situation requiring a decision and four evaluation measures, single dimensional value and utility and multiattribute utility functions were elicited. Relative weights for the evaluation measures, and some demographic data, completed the elicitation session. These data permit construction of value, utility, and multiattribute utility DA models. Additionally, unpublished data from a study by Doyle (1998) was obtained for examination of value and utility function relationships.

Objective 1.1. Determine Functional Relationship of Single Dimensional Value and Utility Functions. Single dimensional value and utility and functions were examined to see which functional relationships performed best as transforms between the two preference functions. Possible transformation functions examined were linear, exponential, power, logarithmic, and sigmoid. The best fitting transformation was identified.

Objective 1.2. Examine Characteristics And Performance Of Various Common Decision Analysis Elicitation Methodologies. Single dimensional utility functions elicited using the certainty equivalent and probability equivalent methods were compared to determine elicitation error. This error was compared to the differences between the mean utility function and the corresponding value function. The two single dimensional utility functions, the value function, and multiattribute function-based

decision analysis models were investigated for each subject. Decision analysis models differences for the differing preference function bases were examined.

Objective 1.3. Improve The Ability to Compare Between Preference Functions. A weighted root mean square error was established to improve the measure of goodness of fit for Objective 1.2.

Objective 1.4. Examine The Subject's Risk Attitudes. The risk attitudes were examined to see if Bell and Raiffa's conjecture regarding constant relative risk aversion held. The risk attitudes were also examined within each subject to see if the risk attitudes were consistent across evaluation measures.

Objective 1.5. Improve The Metric For Assessing Risk Aversion. A measure similar in concept to mean and variance was constructed to improve the ability to compare subjects' risk attitudes.

Objective 2. Establish Methods Of Increased Efficiency Of Decision Analysis Elicitation By Considering Robustness Of Employing Single Dimensional Value Functions As Surrogates For Utility Functions. After establishing an algorithm, it was tested in an automatic target recognition problem. A US Air Force engineer served as the decision maker. Full value and utility models were elicited, allowing employment of the approach.

Objective 2.1. Establish When Value-Utility Differences Are Significant. Standard response surface methodology and design of experiments techniques were employed with respect to the robustness of the value function. An exponential form for $\underset{\cdot}{u}$ was assumed and the exponential coefficient varied throughout a reasonable range. Analysis of results from a screening experimental design provided a

measure of significance, p value, and coefficients for a least squares model. Non-significant utility functions indicated that the respective value functions were adequate surrogates.

Objective 2.2. Develop an Algorithm for Employment of a Hybrid Value-Utility Model. The significant utility functions were prioritized based on the magnitude of the least squares coefficients. The algorithm consisted on iteratively substituting a significant utility function for its corresponding value function. The iterations are terminated when the hybrid model estimate for the utility is sufficiently close to the true utility or when the rank ordering of the alternatives will no longer be affected. The hybrid model then is an adequate model of the true utility model.

Detailed Methodology and Results

The methodology and results for Objectives 1 and 2 are presented in Chapters IV and V, respectively. Detailed results are contained in the appendices, along with the raw data. A summary of the research is provided in Chapter VI.

IV. Examination of Relationship of Value and Utility Preference Functions

General

As discussed in Chapter II, opinions vary widely with regard to the relationship between value and utility functions. Often, research has employed money as a metric when assessing these functions. Research has also suggested that industries or professional groups may share common preference function characteristics. Little decision analysis research is available about preference function relationships for military personnel. Military decision making presents a unique environment as typically operational military decision making does not consider monetary cost. Most decision analysis research involves a dollar cost aspect, whether the decisions are public or private sector. Additionally, military operational decisions rarely quantify risk, an aspect of the utility function but not the value function. There are also military decisions that consider monetary cost, such as acquisition and managerial decisions. Military operational decision making offers an area of decision analysis study that is little examined, yet has unique aspects. This chapter will describe the elicitation of preference attitudes of military professionals, improved analytical techniques, and the analysis of these functions.

Army Tactical Scenario Value-Utility Data

Tactical Preference Function Data. Data was obtained from 20 Army professionals through elicitation sessions. The elicited data permitted construction of alternate models of the subject's preferences in a realistic military scenario.

Description of the Elicitation Sessions. A decision problem was developed to be pertinent to military professionals, regardless of grade or area of expertise. The problem was developed from realistic situations a unit leader might face in an operational environment. The decision problem is an example of administrative forward movement of a recently deployed unit in the communications zone. The communications zone is, by US Army doctrine, the area to the rear of the tactical units conducting the fighting. It is not intended that units in the communications zone encounter enemy units or fires. Such encounters are possible.

The subjects were told that they were in charge of a tactical unit, of size and organization commensurate with their (unspecified) experience and background. Their unit was located in a marshalling area and was required to move to a forward tactical assembly area (TAA). After arriving at the TAA their unit would receive an operational assignment. The move to the TAA, which was nominally 600 miles, would be restricted to a single route during a 24-hour movement window beginning twelve hours from the present. The situation is portrayed in Figure 17.

The subjects were told that they would have to select one route from several choices. Normally the decision maker communicates her concerns for a decision situation. To facilitate comparison between subjects, for this study, the key values were

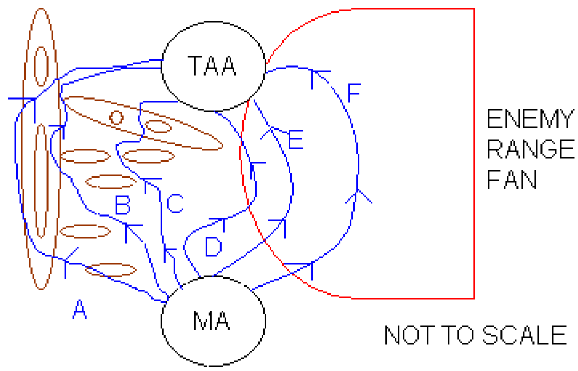


Figure 17. Situational Graphic.

instead presented to the subjects. They were also told that, based on the situation, the pertinent aspects of the situation were force protection and logistical considerations. All other potential considerations were either not of significant impact (e.g., weather) or were identical for all potential routes (e.g., enemy air threat). The force protection aspects of interest were exposure to enemy artillery fire and exposure to enemy ambushes. The scales for these two evaluation measures are the number of route miles within range of enemy artillery and the number of choke points suitable for establishment of ambushes. The logistical impact characteristics are the amount of basic load fuel remaining after completion of the move, in percent, and the required time to negotiate the route, in hours. The subjects were told that these measures were independent from each other. For example, while longer routes required more fuel consumption, some routes had fuel resupply available. The considerations of merit were presented in a value hierarchy that was made available to the subjects during the interview. (Figure 18.) The domains of the pertinent evaluation scales were presented to the subjects. This information is

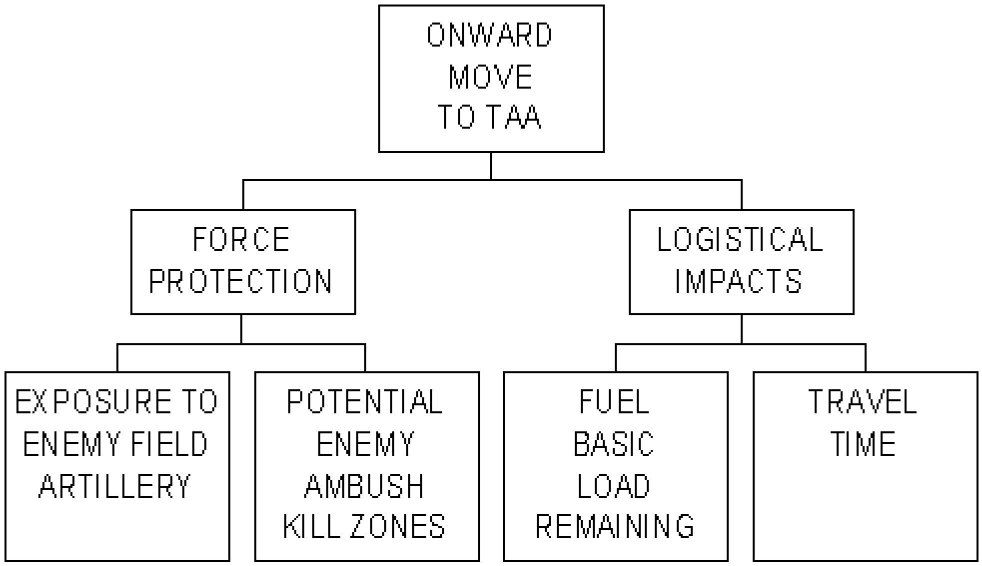


Figure 18. Value Hierarchy.

summarized in Table 2. The data for each route was not presented to the subjects. Additional minor details were presented to make the scenario plausible.

Table 2. Route Characteristics.

	Miles in Enemy FA Range	Potential Enemy Ambush Sites	Percent of Fuel Basic Load Remaining	Total Movement Time
Maximum Value	500	50	100	24
Minimum Value	0	0	0	12

The Subjects. The twenty subjects were all US Army soldiers assigned to, or in support of, the 2nd Battalion, 28th Infantry, a Basic Combat Training unit at Fort Jackson, South Carolina. All were volunteers and received no compensation for participation. Twelve of the subjects were officers; eight were enlisted soldiers. The

officers varied in rank from First Lieutenant to Lieutenant Colonel. The enlisted soldiers varied in rank from Private Second Class to Command Sergeant Major. Two officers had previous enlisted service, one in the US Navy. Eighteen of the subjects were male; two were female. Years of service varied from two to 26 with a mean of 11.4. Three subjects had spent part of their career in the Army Reserve. All were on active duty at the time of the interview. Thirteen of the subjects were from combat arms backgrounds, two from combat support, and five from combat service support.

Elicitation. The elicitation was accomplished through private interviews with the subjects. The interviewer employed a questionnaire to guide the session. Figures 1 and 2 were displayed throughout the interview process for the subject's reference. Subjects were oriented to the interview with the explanation that they would be given a tactical scenario and then asked sets of questions about their preferences and attitudes. These sets of questions would ask for similar information but in different ways. The subjects were told that there were no right or wrong answers in any absolute sense. Instead the intent was to examine the internal consistency of their answers despite altering the question structure. They were told for this reason any assumptions they made should be maintained throughout the interview. Subjects were advised that there was no attempt to trick them. At any time the subject was free to change any information provided earlier. Interviews took between sixty and ninety minutes to conduct, and took place in early 2000.

The single dimensional weights for an additive value function were elicited following the procedures presented in Kirkwood (1997b: 68 – 70). Then the single dimensional value and utility functions were elicited. The utility functions were elicited

utilizing two methods: certainty equivalent and probability equivalent techniques (Clemen, 1996: 474 – 477). These functional elicitations were done in two different sequences. Each subject either provided information for his or her value function, followed by the certainty equivalent utility function, and finally the probability equivalent utility function, or the order of the first two elicitations was reversed. Subjects were assigned to a sequence at random. Then information regarding the subject's multiattribute utility (Kirkwood, 1997b: 155 – 164) was elicited. For subjects for whom there was no "swing" combination of attributes that provided ambivalence, four different certainty equivalents were established using values besides the domain bounds. Finally some demographic data about the subject was recorded. The interviews lasted about an hour per subject. Several subjects were briefly interviewed a second time to elicit additional information to correct an error made by the interviewer. The author believes that the correction did not alter the integrity of the experiment.

Tactical Questionnaire Results. The data analysis began with fitting piecewise linear curves to the points elicited for the value and single dimensional utility functions. This permits visual examination of the elicitation results. An example is provided in Figure 19, which shows the three elicited preference functions for subject FJ1 for the enemy artillery attribute. In this case, all three preference functions appear to agree well. This is in contrast to the results shown in Figure 20 for the same attribute but elicited from a different subject. For subject FJ15 the single dimension utility functions agree well, but differ greatly from the corresponding value function. Figure 21 shows an example of reasonable agreement between the value function and the probability

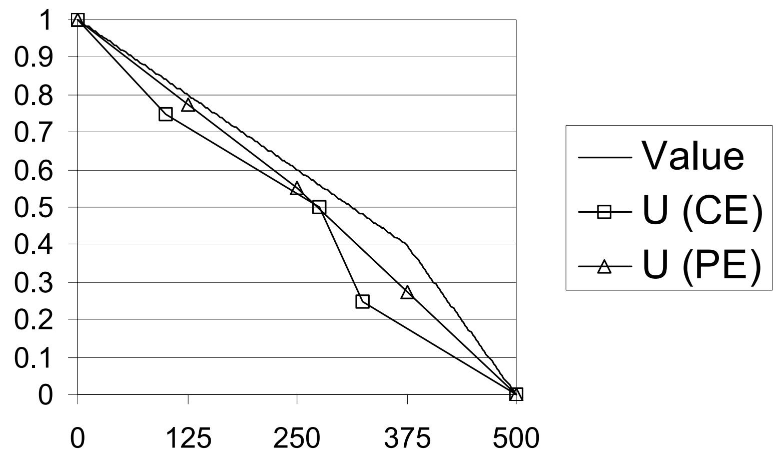


Figure 19. Single Dimensional Preference Functions for Enemy Artillery, Subject FJ1.

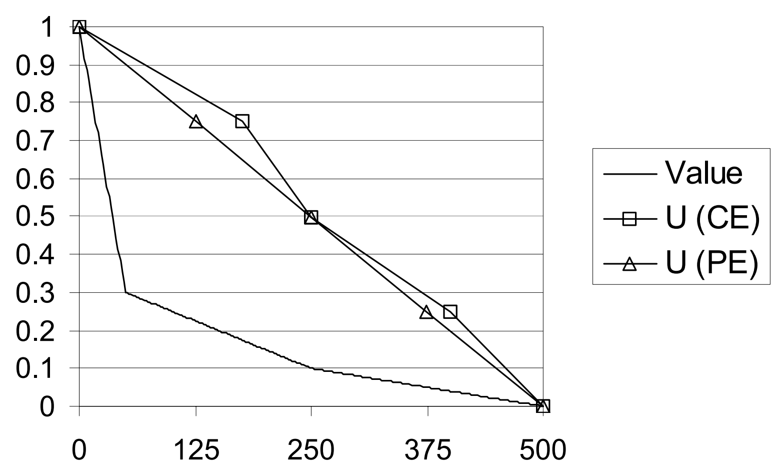


Figure 20. Single Dimensional Preference Functions for Enemy Artillery, Subject FJ15.

equivalent utility, but poor agreement for the certainty equivalent utility function for the ambush site attribute.

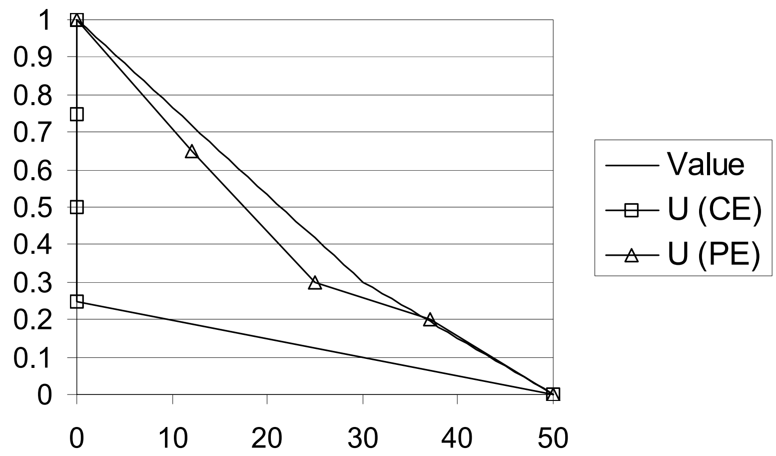


Figure 21. Single Dimensional Preference Functions for Subject FJ18.

Additionally, multiattribute utility functions were elicited. When the subject could not accept any combination of possible outcomes employed in Kirkwood's method (1997b: 163), Kirkwood states the results may be used to bound the exponential form of the multiattribute utility function. Figure 22 shows the results of this procedure for subject FJ18. Another interesting observation is that subject FJ15's responses indicated single dimensional utility functions that were either essentially linear or were moderately concave. The standard interpretation of these morphologies is that they indicate risk neutral and risk averse behavior, respectively. However, the subject's multiattribute utility curve was extremely convex (Figure 23). Results for each subject are contained in Appendix B.

The elicited utility functions were compared to several models of transforms between the value and utility functions. The single dimensional value and utility

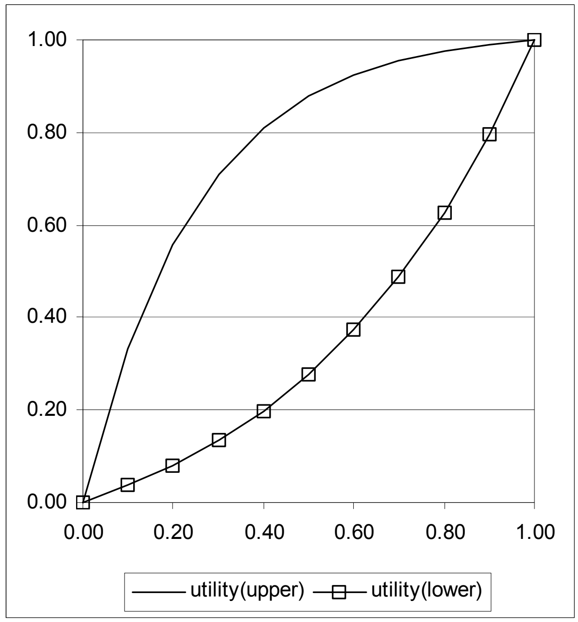


Figure 22. Multi Dimensional Preference Functions for Subject FJ18.

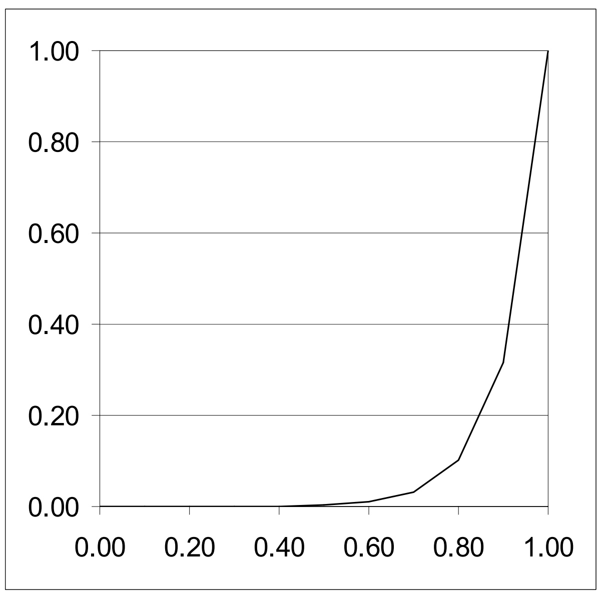


Figure 23. Multi Dimensional Preference Functions for Subject FJ15.

functions, $v(x)$, $u^{CE}(x)$, and $u^{PE}(x)$, respectively, were elicited at three domain points and linear interpolation employed to provide continuous curves for each evaluation

measure x_i. The two single dimensional utility functions, u_{CE} and u_{PE}, were averaged to produce a mean single dimensional utility function, $\bar{u}_{ij}\left(x_k\right)=\left(u_{ij}^{CE}\left(x_k\right)+u_{ij}^{PE}\left(x_k\right)\right)/2$ for the curves for each subject, *i,* and each evaluation measure, *j*, at point x_k.

The examined models of the relationship between the value and utility functions were those of Keller (1985): linear,

$$\hat{u}^{lin}\left(v\left(x\right)\right)=v(x), \tag{46}$$

exponential,

$$\hat{u}^{ex}(v(x))=\frac{1-e^{-c(v(x))}}{1-e^{-c}}, c\neq 0, \tag{47}$$

logarithmic,

$$\hat{u}^{\log}(v(x))=\frac{\log(v(x)+c)-\log c}{\log\left(\frac{1+c}{c}\right)}, c>0 \tag{48}$$

and power,

$$\hat{u}^{pow}(v(x))=v(x)^c, c>0 \tag{49}$$

plus the sigmoid function.

The sigmoid function,

$$f\left(x\right)=1/\left(1+e^{-ax}\right) \tag{50}$$

maps from the reals to the unit interval, so this makes it attractive. The function may be made monotonically increasing or decreasing by selecting $a>0$ or $a<0$, respectively. (When $a=0$, $f\left(x\right)=0.5$.) The difficulty of employing the sigmoid function for decision analysis is that the domain is $\left(-\infty,\infty\right)$ and that it is equal to $f\left(0\right)=0.5$. In

order to use this function, the domain must be translated from the attribute domain to a domain, *z*, where $f(0) = 0.5$ and

$$f\left(z^0, z^*\right) \approx \begin{cases} \{0,1\} & \text{for decreasing levels preferred,} \\ \{1,0\} & \text{for increasing levels preferred} \end{cases} \tag{51}$$

To facilitate this, the midvalue x_m, the point that satisfies the relationship $u\left(x_m\right) = 0.5$ should be selected as the point that will translate to zero. Unless the DM is risk neutral, this will not be the midpoint of the interval, but as the function rapidly approaches the codomain limits this will likely provide little error in the tails and guarantee that the function fits at the midvalue. By inspection the interval [-10, 10] provides reasonable behavior for a wide range of sigmoid coefficients *a*. By this, we mean that the curves rapidly approach values of zero or one for a large range of coefficient values *a*. However when the coefficient is close to zero, the function approaches the constant function $f(x) = 0.5$. Because of this, the assumption for interval should be checked. If the coefficient is in the interval [-0.3, 0.3] the fit will likely be improved by extending the domain.

A method for this is proposed:

1. Given the evaluation measure domain $\left[x^0, x^*\right]$, define the midvalue x_m where $v(x_m) = 0.5$ or $u\left(x_m\right) = 0.5$, as appropriate.
2. Define the sigmoid function domain $\left[z^0, z^*\right]$. Initially select $z^0 = -10$ and $z^* = 10$.

3. Determine the maximum distance D_x from x_m to the endpoints of $\left[x^0, x^*\right]$ employing $D_x = \max\left\{x_m - x^0, x^* - x_m\right\}$.

4. Similarly define the distance D_z on the sigmoid function domain. Initially this is equal to 10.

5. Define the ratio of the two scales $R = D_x / D_z$.

6. Determine the proper offset required to align the zero values on the two scales. If $x_m - x^0 < x^* - x_m$, then $s = \left(x^* - x_m\right) - \left(x_m - x^0\right)$, else $s = 0$.

7. Transform the evaluation measure scale to the sigmoid function scale using $z_i = \left(\left(x_i + s\right)/R\right) - D_z$.

8. Fit the sigmoid function to the set $\left\{\left(z^0, v\left(x^0\right)\right), \left(z^1, v\left(x^1\right)\right), \cdots, \left(z^*, v\left(x^*\right)\right)\right\}$.

9. Check that $v\left(z^0\right),\ 1 - v\left(z^*\right) < \delta$ or $u\left(z^0\right),\ 1 - u\left(z^*\right) < \delta$ where δ is the acceptable value of tolerance of fit at the endpoints. For this research, a sigmoid endpoint fit was deemed acceptable if $\delta \le 0.1$. If the tolerance criterion is not met, extend the interval of the sigmoid function domain by selecting new z^0 and z^* values and return to step three.

The sigmoid function of interest is then given by

$$\hat{\underset{.}{u}}^{sig}\left(v\left(z\right)\right) = 1 \Big/ 1 + e^{-cv(z)} \tag{52}$$

The coefficient, *c*, of each model was selected to obtain the best least squares fit of the function, $\hat{\underset{.}{u}}\left(v\left(x_i\right)\right)$, to $\bar{u}\left(x_i\right)$, for each subject. The five fitted $\hat{\underset{.}{u}}\left(v\left(x_i\right)\right)$ were examined to see how good was the fit to $\bar{u}\left(x\right)$.

Measuring Goodness of Fit

Root Mean Square Error. In order to assess the success of a model in fitting the elicited utility function, a measure of the fit must be employed. Keller (1985b: 479) recommended employing Root Mean Square Error (RMSE) as a measure of the differences between the two functions. In this case the transformed utility function, $\hat{u}(v(x))$, derived from the empirically based value functions must be compared to the mean empirical utility function, $\overline{u}(x)$. Keller (1985b) followed the work of Dyer, Farrell, and Bradley (1973) and Fishburn and Kochenburger (1979) employing RMSE to test this fit. She also proposed an "ad hoc" criterion of accepting a fit when RMSE ≤ 0.05.

The RMSE is determined employing the well-known formula

$$RMSE = \left[\frac{1}{K} \sum_{i=1}^{K} \left[\hat{u}\left(v\left(x_i\right)\right) - \overline{u}\left(x_i\right) \right]^2 \right]^{1/2} \tag{53}$$

For the purpose of examining the error of $\hat{u}\left(v\left(x\right)\right)$, Equation (53) becomes

$$RMSE_{ij}^{\text{model}} = \left[K^{-1} \sum_{k=1}^{K} \left[\hat{u}_{ij}^{\text{model}}\left(v_{ij}\left(x_{jk}\right)\right) - \overline{u}_{ij}\left(x_{jk}\right) \right]^2 \right]^{1/2} \tag{54}$$

where the index type model $\in \{\text{ex,lin,log,pow,sig}\}$ representing the exponential, linear,

logarithmic, power, and sigmoid models described in equations (46) through (52),

$i \in \{1,2,\ldots,20\}$ is the index for the subject,

$j \in \{1,2,3,4\}$ is the index for the evaluation measure, and

$k \in \{1,2,\ldots,K\}$ is the index for the increment.

However, while RMSE is an attractive measure of fit, it possesses the displeasing characteristic of equally weighting all observations. Utility functions by definition equal zero at the least preferred levels of the independent variable and equal one at the most preferred. Because of this, the difference between $\hat{u}(v(x))$ and $\overline{u}(x)$ will be more constrained as x approaches the domain endpoints. This suggests that a weighted RMSE (WRMSE) that gives more weight to the differences the further from the center of the independent variable midpoint would be a more accurate measure. Such a weighted least squares fit is described by Maybeck (1979: 120).

Weighted Root Mean Square Error. Employment of a weighted root mean square error (WRMSE) approach required selection of an appropriate weighting function. Once accomplished, comparison of functions with WRMSE proceeds as with RMSE.

Weighting Function Desired Characteristics. Designate a weighting function for the WRMSE as $j(x)$. Define the domain as $[x_0, x_*]$ and the midpoint $x_m = \frac{1}{2}(x_* - x_0) + x_0$. Let n represent the number of observed data points. The desirable characteristics for $j(x)$ are: (1) $j:[x_0, x_*] \to \mathbb{R}^{+}$ (Vinogradov, 1993:464); (2) that $j(x)$ be symmetric about x_m; (3) that $j(x)$ be strictly monotonically decreasing on $(x_0, x_m]$ (and this, with property (2), provides that $j(x)$ will be strictly monotonically increasing on $[x_m, x_*)$, further these properties provide that $j(x)$ will be minimized at x_m); (4) that $j(x) \to \infty$ as $x \to x_0, x_*$; and (5) $j(x_m) = 1$. Note that when $n = 1$, $WRMSE = RMSE$ as the error is provided by the single datum point.

Chebyshev Polynomial Weighting Function. Orthogonal polynomials provide a method of fitting curves without employing multiple regression techniques given the condition of evenly spaced data (Considine, 1989: 2105). Orthogonal polynomials P_n of degree *n* with respect to the weight *h* satisfy the condition

$$\int_a^b P_n(x)P_m(x)h(x) = 0;\ n \neq m \tag{55}$$

where the weighting function $h(x) \geq 0,\, x \in [a,b]$ (Hazewinkel, 1991: 30 – 33). One of the classic orthogonal polynomials is the Chebyshev (or Tchébichef, and also sometimes Romanized as Tschebysheff) polynomial of the first type where $h(x) = \left(1 - x^2\right)^{-1/2}$ and is defined on $(-1,1)$ (Beyer, 1987: 374 – 378). This function is shown in Figure 24.

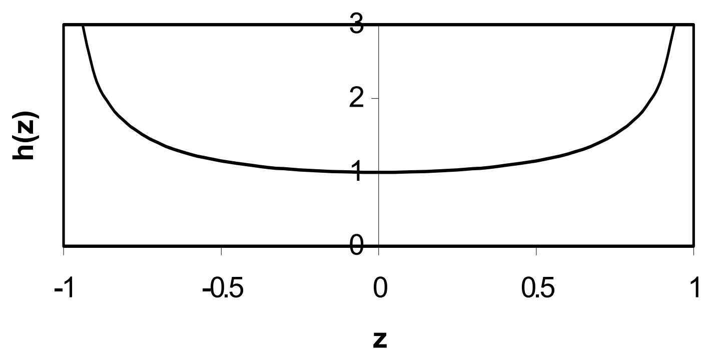


Figure 24. Chebyshev Polynomial Type 1 Weighting Function, *h(x)*.

The function $h(x)$ meets the characteristics discussed above when the domain is transformed from $x \in [x_0, x_*]$ to $z \in (-1,1)$, employing the relationship

$z_i = 2\left(\frac{x_i - x^0}{x^* - x^0}\right) - 1$. When the domain is in $v(x)$ vice in x (i.e., when the function under consideration is $\hat{u}(v(x))$ rather than $u(x)$), the domain is restricted to the unit interval. In this case the transform from $v(x)$ to z is simply $z = 2v(x) - 1$. The points $z = -1, 1$, where $h(x) = \infty$, are neglected as there is no error contribution. This function is illustrated in Figure 25. Such an arrangement modifies the RMSE to

$$WRMSE_{\text{ij}}^{\text{model}} = \left[\frac{1}{\sum_{k=1}^{K} h(z_{jk})} \sum_{k=1}^{K} h(z_{jk}) \left[\hat{u}_{ij}^{\text{model}}\left(v_{ij}\left(x_{jk}\right)\right) - \bar{u}_{ij}\left(x_{jk}\right) \right]^2 \right]^{1/2} \tag{56}$$

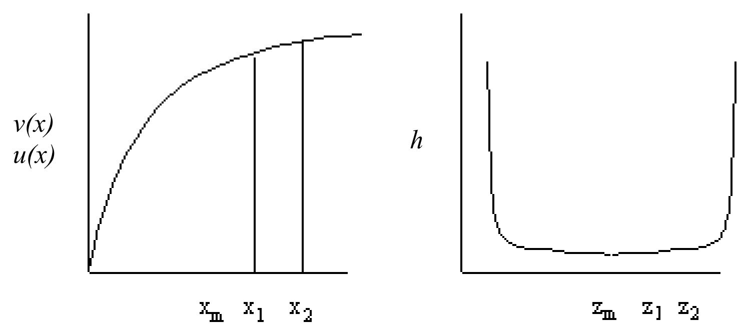


Figure 25. Relationship of selected values of *x* and *z*.

Higher Order Chebyshev Polynomial-like Weighting Functions.

Besides the Chebyshev Polynomial weighting function $h(x)$, functions of higher powers in z meet the criteria expressed above. Other functions similar to the Chebyshev may be

created by modifying the exponent. A family of equations, of which the Chebyshev is a member, may be expressed as

$$t(z,p) = \frac{1}{\left(1-z^{p}\right)^{1/p}} \tag{57}$$

where $t(z,2) = h(z)$. These functions are illustrated in Figure 26 for several values of p. In order to select from this family of curves, it is noted that an additional desirable characteristic for the weighting function is that the rate of change of the weighting function be the same as that of the utility function. This, in conjunction with the other characteristics described above, provides the closest match between the behavior of the weighting function and the preference function.

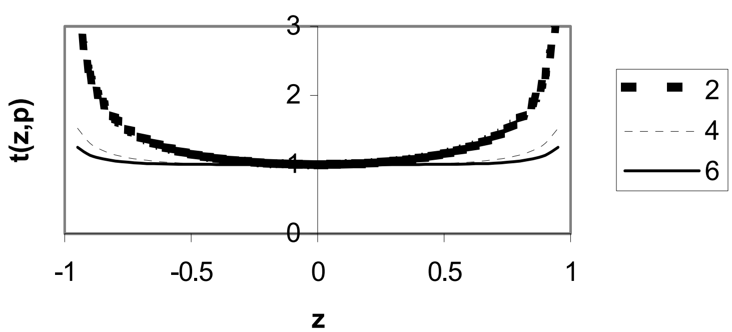


Figure 26. Members of Family of Functions $t(z,p)$ for several levels of p.

Because of the symmetry of the weighting function, the simplest case where the rate of change may be examined is when $K = 5$. This case is analyzed below. This provides two data points in each half domain for slope comparison, x_1, x_2 for $u(x)$ and the corresponding z_1, z_2 for $h(x)$. While the slopes are clearly different between a utility

function and the weighting function, the ratios of the slopes may be maintained. It is desired that $u'(x_2)/u'(x_1) = h'(z_2)/h'(z_1)$. Without loss of generality, assume increased levels of x are preferred. Figure 25 shows the data points and their respective functions.

Unfortunately use of the weighting function will be with utility functions elicited from an unknown subject, so the form of $\hat{u}_{ij}\left(v_{ij}(x)\right)$ in a future analysis is unknown. Kirkwood indicates that the exponential functional form may commonly be employed, and further provides that the exponential constant for the function is generally in the range $[\rho_0, \rho_*] \equiv [-1/10*(x_* - x_0), 1/10*(x_* - x_0)]$. This provides a range of reasonable values for ρ. The exponential function with these values, $[\rho_0, \rho_*]$, as well as for $\rho_{\text{neutral}} = \infty$, is illustrated in Figure 27. Without loss of generality (as the function is symmetric about the ρ_{neutral} curve), consider only $0 < \rho < \rho_*$.

We wish to designate some ρ, call it ρ_m, that is representative of the family of curves evinced by this interval of exponential constants. Selecting a ρ_m is illustrated in Figure 28. The domain values corresponding to $\hat{u}_{ij}\left(v_{ij}(x)\right) = 0.5$ for ρ_* and ρ_{neutral} are x_a and x_m, respectively. Then $x_b = (x_a + x_m)/2$ and an exponential curve fitted through x_b has $\rho = 0.522 = \rho_m$ and represents a measure of central tendency for the interval $0 < \rho < \rho_*$.

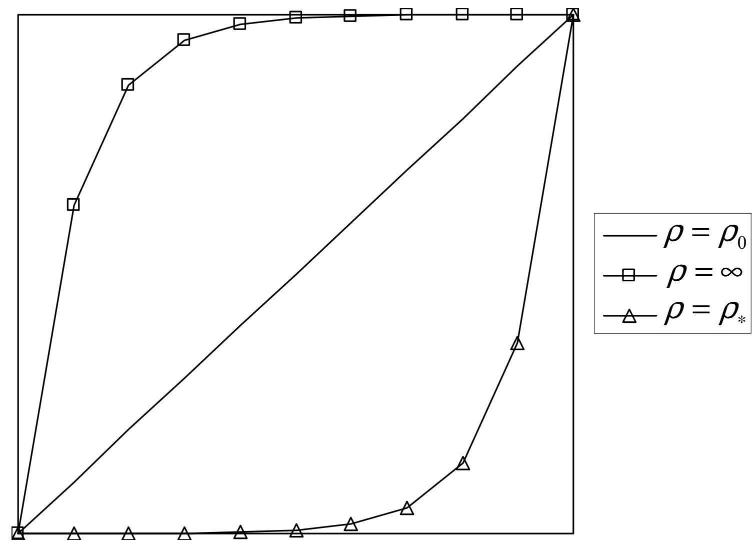


Figure 27. Exponential Constants Effects.

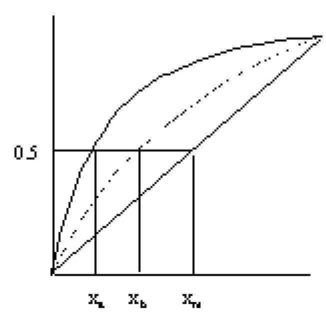


Figure 28. Illustration of Determining ρ_m.

For $u\left(v\left(x\right)\right)$, the function fitted through x_b, the domain is the unit interval and the exponential function is

$$u\left(v\left(x\right)\right)=\frac{1-e^{-v(x)/0.522}}{1-e^{-1/0.522}} \tag{58}$$

and the derivative of Equation (58) provides the slope

$$\frac{d}{dx}u(v(x)) = 2.2465e^{(-1.9157v(x)-0.95785)} \tag{59}$$

The ratio of the slopes of the two equidistant points in the right half-domain is

$\frac{u'(v(x_2))}{u'(v(x_1))} = \frac{u'(0.833)}{u'(0.667)} = \frac{0.388}{0.455} = 0.853$. The ratios of the function $t(z,p)$ for $p = 2,4,6,8$ are 4.02, 10.2, 35.5, and 131.5, respectively. Plots of the curves of these functions are shown in Figure 26. Increasing the parameter *p* moves the weighting function away from the desired slope ratio as the curve center is flattened, providing less discrimination. We conclude that the basic Chebyshev weighting function $h(z)$ is the most appropriate selection from the family provided by Equation (57).

With a method for quantifying the differences between functions established, the next concern is the amount of error inherent in the elicitation process. As with any observed process, there will be error in the captured data. Estimating this error is valuable before making comparisons of the process between functions.

Estimate of the Elicitation Error. In order to examine the fit of various models against the elicited utility function, the inherent error of that elicited utility function must be considered. An opportunity to observe the elicited error is available through the analysis of the elicited single dimensional utility functions. These utility functions were elicited through two different methods, the certainty equivalent method and the probability equivalent method. Each elicitation produced a set of five points, three of which were unique to the subject. (The endpoints are defined.) However the independent variable (evaluation measure) differed for each set of observations. To

observe the differences in the function instantiations, linear interpolation was employed to provide a continuous curve for each set. Line segments joined the known endpoints to the elicited points, in sequence. The mathematical mean, $\bar{u}_{ij}\left(x_k\right)=\left(u_{ij}^{CE}\left(x_k\right)+u_{ij}^{PE}\left(x_k\right)\right)\big/2$, was then determined for the curves for each subject, *i,* and each evaluation measure, *j*, at point x_k at the *k*th point.

For each evaluation measure 100 (50 for ambush) equally spaced observations were made of the two interpolated utility functions. The root mean square error (RMSE) and weighted root mean square error (WRMSE) were then determined using these observations. For this analysis, the RMSE is provided by

$$RMSE_{ij}^{\mathrm{u}}=\left[K^{-1}\sum_{k=1}^{K}\left[u_{ij}^{\mathrm{CE}}\left(x_{jk}\right)-u_{ij}^{PE}\left(x_{jk}\right)\right]^{2}\right]^{1/2} \tag{60}$$

and the WRMSE by

$$WRMSE_{\mathrm{ij}}^{\mathrm{u}}=\left[\frac{1}{\sum_{k=1}^{K}h\left(z_{jk}\right)}\sum_{k=1}^{K}h\left(z_{jk}\right)\left[u_{ij}^{CE}\left(z_{jk}\right)-u_{ij}^{PE}\left(z_{jk}\right)\right]^{2}\right]^{1/2} \tag{61}$$

where

$i\in\{1,2,\ldots,20\}$ is the index for the subject,

$j\in\{1,2,3,4\}$ is the index for the evaluation measure,

$k\in\{1,2,\ldots,K\}$ is the index for the increment,

and *CE* and *PE* indicate elicitation methods of certainty equivalent and probability equivalent, respectively.

An analysis of variance (ANOVA) was performed on the evaluation measures and then the subjects for both RMSE and WRMSE. The intent was to determine if any subset of the errors appeared to have differing means. The RMSE ANOVA for the evaluation measures (all subjects grouped), $\overline{RMSE^{u}_{\bullet j}}$, is shown in Figure 29. The diamonds represent 95 percent confidence intervals on the $\overline{RMSE^{u}_{\bullet j}}$ data. The horizontal bars near the top and bottom of the diamonds define the overlap interval (Sall, Lehman, and Creighton, 2001: 178 – 179). Two diamonds must have the disjoint overlap intervals if the respective distributions differ at the 95 percent confidence level. The lack of a significant difference is apparent. The *p* value for the ANOVA, 0.403, confirms this. The same holds for the $\overline{WRMSE^{u}_{\bullet j}}$ as shown in Figure 30, where the *p* value is 0.560. Trimming the data is discussed below. With the most outlying point removed, the differences between means are still not significant. *P* values are 0.928 and 0.978 for $\overline{RMSE^{u}_{\bullet j}}$ and $\overline{WRMSE^{u}_{\bullet j}}$, respectively. A significance level (alpha) of 0.05 is used in this study.

This indicates that the errors associated with the elicitation process were not different for the separate evaluation measures, regardless of which measure is employed. This is both heartening and helpful. It suggests no biases with respect to evaluation measures. It also supports the pooling of the elicited data so that it may be treated as one large sample when desired. The errors associated with individual subjects is now of interest.

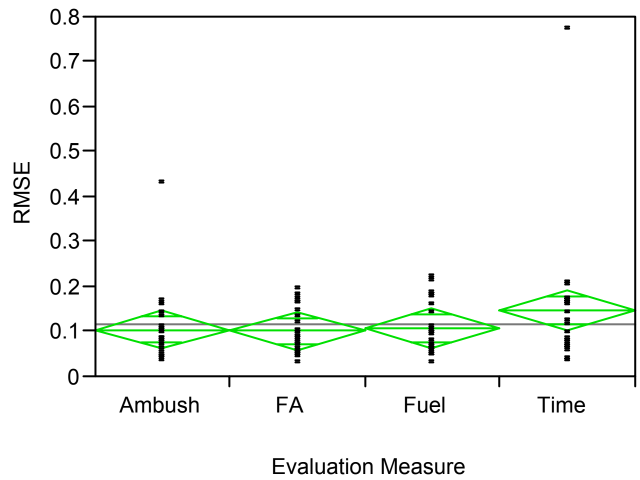


Figure 29. ANOVA of Averaged Utility Function RMSE by Evaluation Measure for All Subjects, $\overline{RMSE^{u}_{\bullet j}}$.

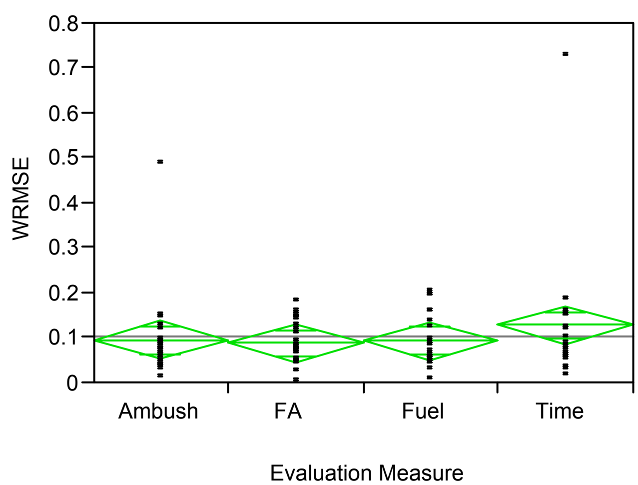


Figure 30. ANOVA of Utility Function WRMSE by Evaluation Measure for All Subjects, $\overline{WRMSE^{u}_{\bullet j}}$.

Analysis based on the subjects provided the $\overline{RMSE^{u}_{i\bullet}}$ graph in Figure 31 and the $\overline{WRMSE^{u}_{i\bullet}}$ graph in Figure 32. The p values were 0.0812 and 0.0330, respectively. This indicates that there are likely significant differences between the means for the subjects.

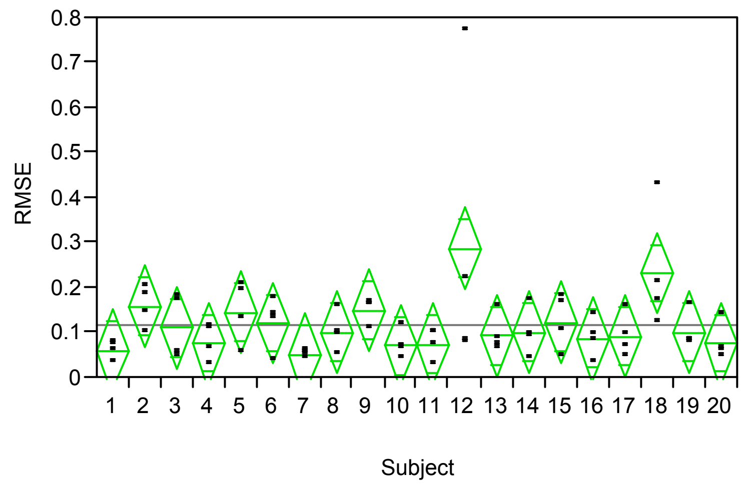


Figure 31. ANOVA of Utility Function RMSE by Subject for All Evaluation Measures, $\overline{RMSE_{i\bullet}^{u}}$.

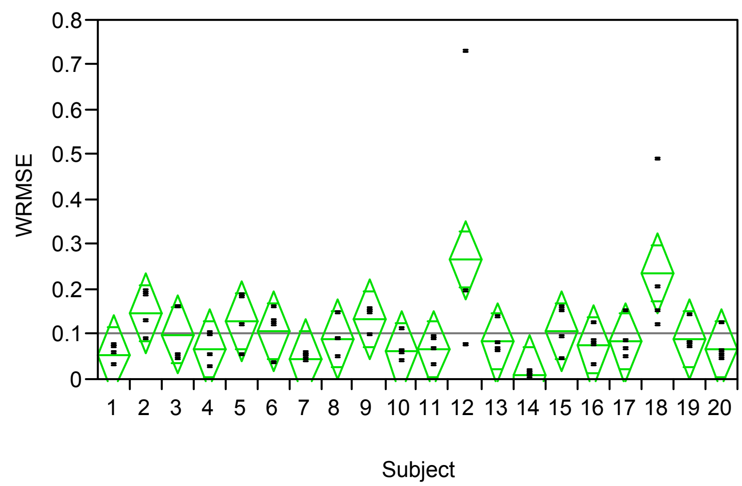


Figure 32. ANOVA of Utility Function WRMSE by Subject for All Evaluation Measures, $\overline{WRMSE_{i\bullet}^{u}}$.

Inspecting Figure 31 and Figure 32, we see that there are two points that may be outliers. The first is one of the elicited evaluation measures of Subject 12. Subject 12 demonstrated very different behavior regarding the time evaluation measure for the two elicitation procedures. When presented with the certainty equivalent lottery, the subject

was extremely risk seeking, preferring the uncertain alternative unless the certain alternative was at the best possible outcome. The subject presented the diametric attitude when presented with the probability equivalent lottery. Then the subject selected the uncertain alternative only when the probability of the more desired outcome was equal to unity. In other words, for the probability equivalent lottery, the subject selected the uncertain outcome only when all uncertainty was removed. Naturally under this condition the “uncertain” alternative provides a more desirable outcome than the certain alternative. This is extreme risk aversion.

Presenting completely opposite risk attitudes for the two elicitation methods is clearly disturbing. A portion of the certainty equivalent data was elicited during a second, separate, session. While the time interval of several weeks between the initial and the follow up session might have contributed to this contradiction, it is not the source. The subject was shown the initial session data and the data were discussed. While the second session data is in consonance with the first session data, within each respective method, the first session data also demonstrates the incompatible risk attitudes. The contradiction was not presented to the subject, as it was not discovered until analysis had begun. Apparently either the subject misunderstood, some other factor affected his risk attitudes, or one or more of these methods fail with this individual.

A possible second outlier is the ambush site evaluation measure for Subject 18. During the certainty equivalent elicitation the subject was completely risk seeking, selecting the certain alternative only when it offered no ambush sites, the best possible outcome on the evaluation measure. During the probability equivalent elicitation, the subject maintained risk-seeking behavior, but to a much less pronounced degree. This

elicitation session was the longest, and the subject often seemed to reconsider the problem and his attitudes seemed to evolve during the session. Other evaluation measures for this subject showed disagreement between methods that is higher than most other subjects as well, suggesting that the subject's views were less firmly established or apparent. The plots of the elicited utility functions for both these subjects are contained in Appendix B.

As Subject 12 shows complete contradiction for time, that datum point will be treated as an outlier. The extreme error point of Subject 18 will be retained, as justification for elimination is not clear. The results are shown in Figure 33 and Figure 34 for both error metrics with the Subject 12 outlier removed. The *p* values then become 0.011 and 0.012 for RMSE and WRMSE, respectively. (Elimination of the Subject 18 potential outlier produces *p* values of 0.049 for RMSE and 0.0012 for WRMSE.) The differences between the means for trimmed data set are significant. We conclude that there are significant differences between the elicitation errors across subjects. Restated, individuals varied with respect to the amount of elicitation error that they exhibited. The subjects provide a sample of the military decision making population. As such, their error statistics provide insight when considering criteria for comparison of preference functions.

As there are no significant differences with the error data based on the evaluation measures, the data will be pooled, less the single outlier, and used for an estimate of the elicitation error for the subjects as a group. The distributions of the RMSE and the WRMSE are shown in Figure 35 and Figure 36, respectively. The moments associated with these distributions are shown in Table 2. Employing a 95 percent confidence two-

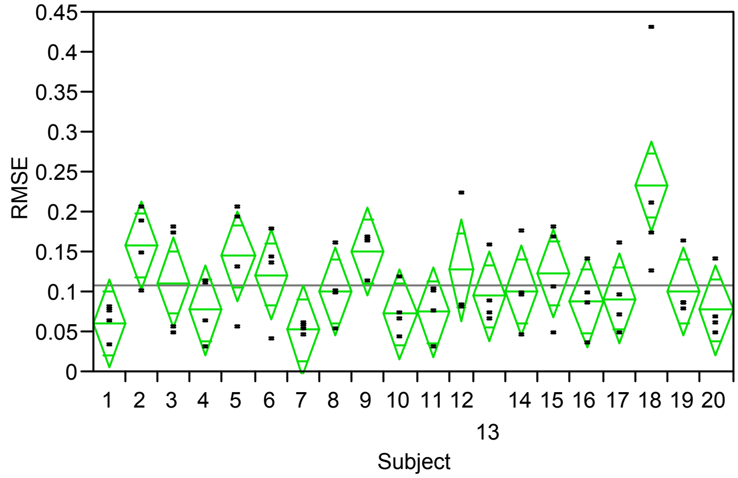


Figure 33. ANOVA of Averaged Utility Function RMSE by Subject, $\overline{RMSE_{i\bullet}^{u}}$, Trimmed Data.

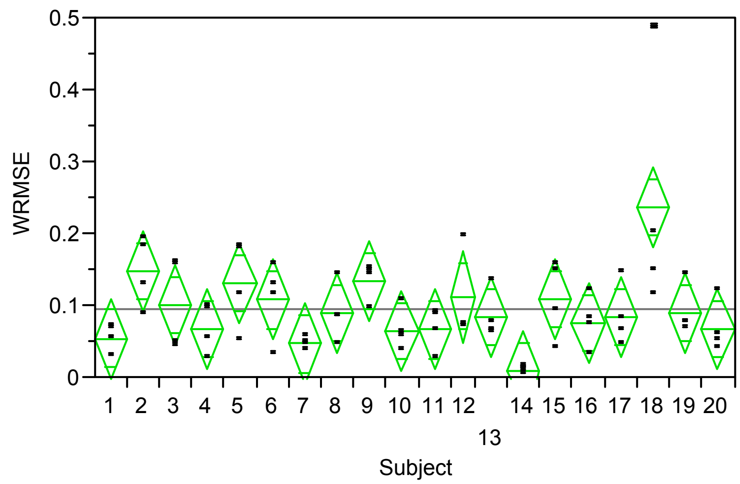


Figure 34. ANOVA of Average Utility Function WRMSE by Subject, $\overline{WRMSE_{i\bullet}^{u}}$, Trimmed Data.

tailed interval, the RMSE for any elicitation should fall between 0.0941 and 0.123, and the WRMSE should fall between 0.0796 and 0.110. Interestingly, for this experiment, the RMSE threshold of 0.05 employed by Keller does not fall within the confidence interval. As error below the confidence interval is not normally a concern, the error

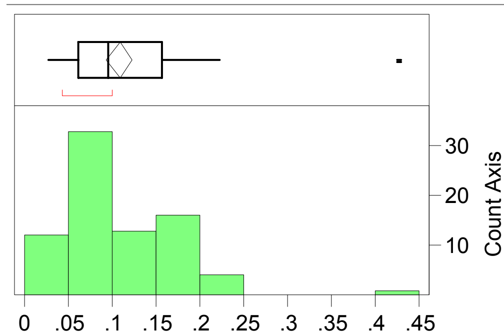


Figure 35. Average Utility $\overline{RMSE^{u}_{\bullet\bullet}}$ Histogram and Box Plot. The Box Plot Rectangle Indicates the Upper and Lower Quartiles, the Internal Bar the Median, and the Diamond the Mean.

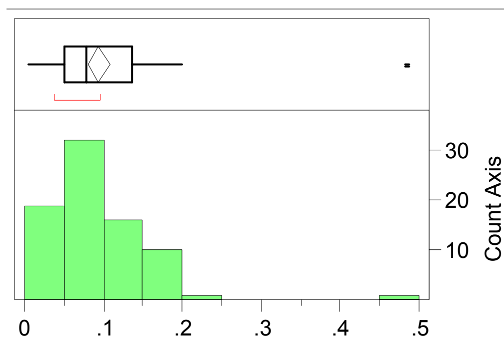


Figure 36. Average Utility $\overline{WRMSE^{u}_{\bullet\bullet}}$ Histogram and Box Plot. The Box Plot Rectangle Indicates the Upper and Lower Quartiles, the Internal Bar the Median, and the Diamond the Mean.

Table 3. $\overline{RMSE^{u}_{\bullet\bullet}}\,/\,\overline{WRMSE^{u}_{\bullet\bullet}}$ Distribution Moments.

	RMSE	WRMSE
Mean	0.10833	0.09554
Std Dev	0.06340	0.066773
Std Err Mean	0.0071329	0.0075125
upper 95%	0.12254	0.10951
lower 95%	0.094137	0.079598
One-sided 95%	0.12004	0.10688

thresholds may be determined using a one-tailed test. This provides the criteria of 0.120 for RMSE and 0.107 for WRMSE for acceptable fit of utility functions.

Returning to the concern of fitting $\hat{u}_{ij}\left(v_{ij}\left(x\right)\right)$ to the corresponding $\bar{u}_{ij}\left(x_{ij}\right)$, the number of model fits for various acceptance criteria for RMSE and WRMSE is shown in Figure 37 and Figure 38. The former shows how many evaluation measures have been fit by at least one model as a function of the RMSE or WRMSE criteria. There are 80 fits for the 20 subjects, each subject with four evaluation measures. The latter figure shows the total number of model fits as a function of the RMSE or WRMSE criteria. There are 400 possible fits, five models tested per four evaluation measures for the 20 subjects. Behavior of the two metrics is similar.

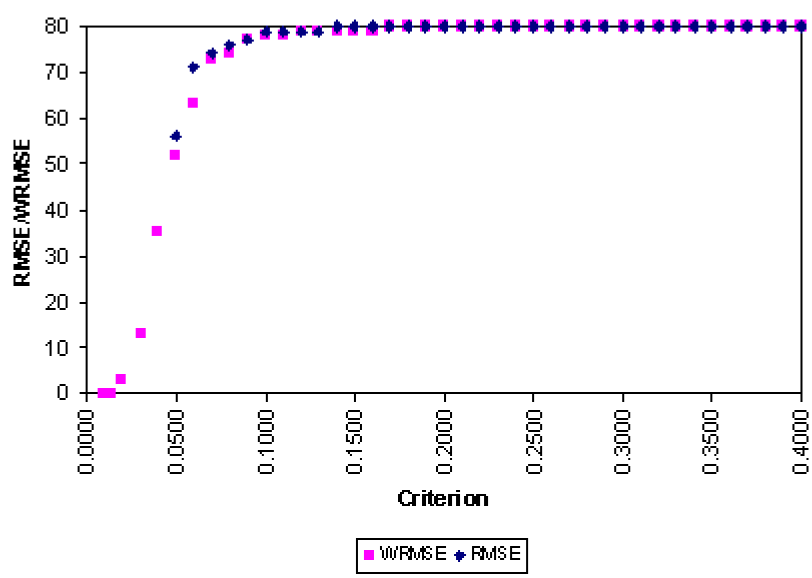


Figure 37. Model Fits of Evaluation Measures for RMSE/WRMSE Criteria.

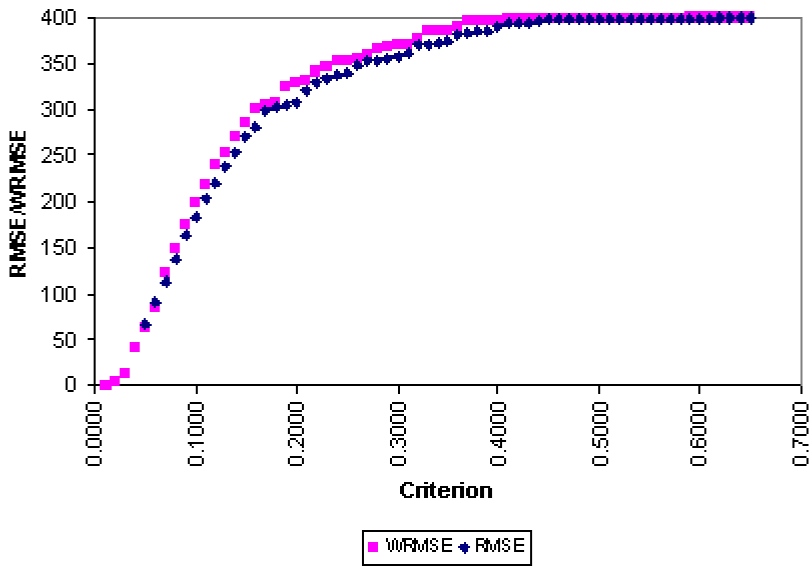


Figure 38. Total Model Fits for RMSE/WRMSE Criteria.

At the acceptance criteria developed above, 0.120 for RMSE and 0.107 for WRMSE, the number of model fits is summarized in Table 4. Almost all categories achieved a fit. Roughly half of the attempts to fit a model were acceptable. The number of fits employing the WRMSE is lower, suggesting that WRMSE is a more discriminating measure than RMSE. To test this we assume that the normal approximation to the binomial is applicable to the total fit data. This assumption is very reasonable because the sample size is so large ($n_{RMSE}, n_{WRMSE} = 400$) for the two error measures. The null hypothesis is that the proportions π for each error metric are equal,

$$H_0 : \pi_{RMSE} = \pi_{WRMSE} \tag{62}$$

and the alternate hypothesis,

$$H_A : \pi_{RMSE} \neq \pi_{WRMSE} \tag{63}$$

Under the assumption the test statistic Z is a standard normal deviate and is defined by

$$Z = \frac{\left(p_{RMSE} - p_{WRMSE}\right)}{\left[\Pi\left(1-\Pi\right)\left(\frac{1}{n_{RMSE}} + \frac{1}{n_{WRMSE}}\right)\right]^{1/2}} \tag{64}$$

where

$$\Pi = \frac{p_{RMSE} n_{RMSE} + p_{WRMSE} n_{WRMSE}}{n_{RMSE} + n_{WRMSE}} \tag{65}$$

and p_{RMSE}, p_{WRMSE} are the proportion of fits for the RMSE and WRMSE metrics, respectively, and n_{RMSE}, n_{WRMSE} is the sample size for the RMSE and WRMSE metrics, respectively (Kanji, 1999: 25).

Table 4. Model Fits at Recommended Criteria.

	RMSE		WRMSE	
Criterion	0.120		0.107	
Categories Fit	79/80	98.8 %	78/80	97.5 %
Total Fits	219/400	54.8 %	211/400	52.8 %

Calculating the proportions, $p_{RMSE} = 219/400 = 0.5475$ and $p_{WRMSE} = 211/400 = 0.5275$. The variable $\Pi = 0.5375$. Calculating Z using Equation (64) provides $Z = 226.91$. Because the critical value $Z_{0.05} = \pm 1.64$, we reject the null hypothesis and conclude that there is a significantly greater acceptance of fits by the RMSE metric. This indicates that the weighted root mean square provides a more conservative measure of the differences than the root mean square.

Fit of Models

The fit of the five models is summarized in Appendix B. The best fits and Keller's G values (see Equation (26), Chapter II) are presented in Appendix B. A summary of the best fits, that met the WRMSE acceptance criterion, is presented in Table 5. Only in a few cases was an acceptable fit not achieved. The sigmoid function was very successful as a model, fitting best in over 80 percent of the cases. No other models achieved more than a four percent rate of best fit. It should also be noted that the linear model, $u_{ij}^{\text{lin}}\left(v_{ij}\left(x\right)\right)=v_{ij}(x)$, failing to fit indicates that utility and value are not identical for any subject for each of the evaluation measures. Finally, Table 5 also shows the number of best fits for each model if RMSE is used rather than WRMSE. Inspection indicates that RMSE provided similar results except the exponential model fit increased to 7.5 percent and that the logarithmic function never fit best. Table 6 shows how often the various models achieved an acceptable (not necessarily best) fit at the selected criteria. Clearly the sigmoid function was very successful. Exponential and power functions had about the same success rate. The logarithmic was less successful and the poorest fit was the linear function. This supports the hypothesis that the value and utility functions are different constructs and different within a professional population of subjects. Further, it indicates that approximating a utility function with the corresponding value function is a poor approximation in the absence of other information.

The good performance of the sigmoid may be in part due to the elicitation over a fixed domain for all individuals. This was done to ensure comparable data between subjects. But inclusion of the domain for which the corresponding codomain is outside

Table 5. Number of Best Acceptable Fits By Model Type.

	RMSE	WRMSE
Linear	0	0
Exponential	6	3
Logarithmic	2	0
Power	3	3
Sigmoid	67	73
None	2	1

Table 6. Total Acceptable Fits by Model Type.

	RMSE		WRMSE	
	Number	Percent	Number	Percent
Linear	19	4.75	17	4.25
Exponential	46	11.5	44	11
Logarithmic	31	7.75	28	7
Power	50	12.5	45	11.25
Sigmoid	78	19.5	77	19.25

of the region where $v(x) = i, i \in (0,1)$ may have introduced an artifact that enhances the probability that the sigmoid would be accepted.

Curve Shape

The utility curve shape is traditionally an indicator of the subject's risk attitude. Concavity is indicative of risk aversion, convexity of risk affinity, and linearity of risk neutrality. Keller (1985b), and others, also categorize some curve shapes into "s-shaped" when they exhibit both concavity and convexity. A summary of the number of differently shaped curves resultant from the elicitations is presented in Table 7. Most subjects were either risk averse or showed a mixture of risk aversion and risk affinity.

Table 7. Utility Function $\underline{u}(v(x))$ Shapes.

	Concave	Convex	Near Linear	S-Shape
Number	39	4	5	32
Percent	48.75	5.00	6.25	40.00

Figure 39 shows this information graphically in a mosaic plot by subject. Mosaic plots are used to depict frequency information for ordinal and nominal data. The column width is proportional to the number of observations for the category and is equal as used in this document. Each vertical bar is partitioned into subregions representing, in this case, curve shapes, and the length of the subregion is proportional to the occurrence of that curve shape. A single bar at the right of the plot provides the overall curve type proportions. It can be seen that instances of risk affinity and neutrality were spread among the subjects. Differences in curve shape between subjects were nonsignificant with a p value of 0.71 when testing the negative log likelihood with the Pearson Chi Square test (Sall, Lehman, and Creighton, 2001: 247 – 248).

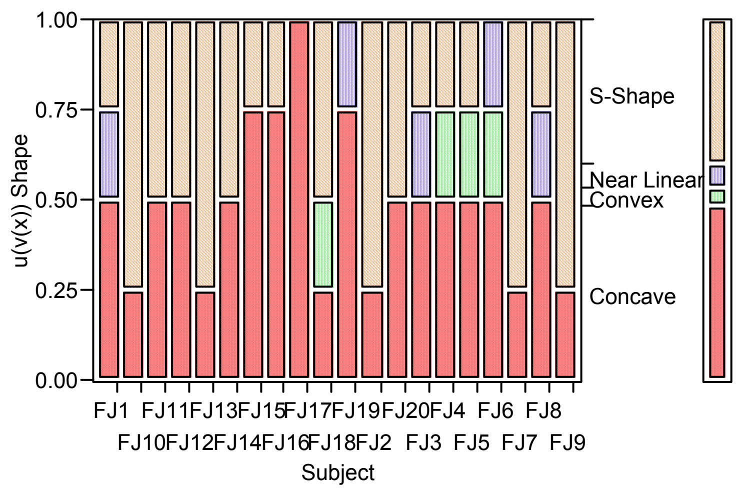


Figure 39. Mosaic Plot of Utility Function u(v(x)) Shapes by Subject.

Figure 40 shows a mosaic plot of the same information by evaluation measure. Interpretation of the risk attitudes is problematic when considering the s-shaped curves, as the relative amounts of risk aversion and affinity are not revealed by this summary. Simple risk aversion is greatest in the case of ambush. Artillery shows the greatest risk neutrality and simple risk affinity. The differences in proportion of curve shapes as a function of evaluation measure is highly significant, with a Pearson Chi Square test *p* value of 0.0087. However the small number of near linear and convex cases makes these categories sparse and below the Cochran criterion (Sall, Lehman, and Creighton, 2001: 245), so the results are suspect.

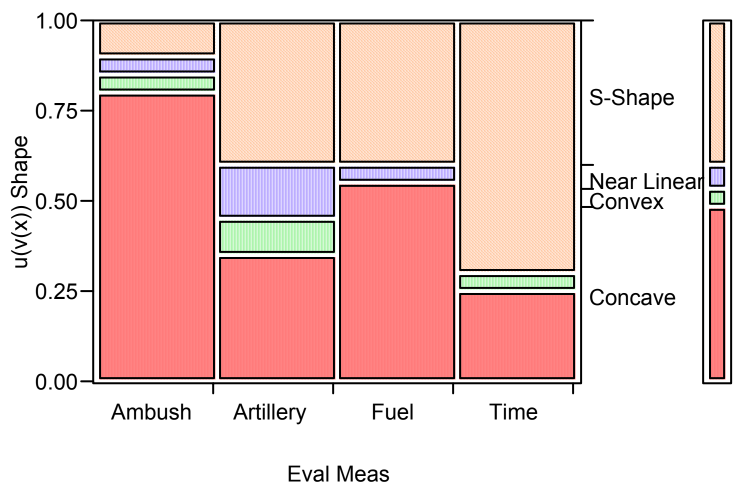


Figure 40. Mosaic Plot of Utility Function $u(v(x))$ Shapes by Evaluation Measure.

Elicitation Error Compared to Value-Utility Differences

It has been argued that elicitation methodology imparts error greater than what is observed between value and utility elicitation approaches, or $\left|u_{CE}(x_i)-u_{PE}(x_i)\right|>\left|\overline{u}(x_i)-v(x_i)\right|$ for most x_i. Since $u_{CE}\left(x^0\right)=u_{PE}\left(x^0\right)=v\left(x^0\right)=0$ and $u_{CE}\left(x^*\right)=u_{PE}\left(x^*\right)=v\left(x^*\right)=1$, the argument presented above for WRMSE is germane.

Employ ε_{ij} as a WRMS measure of the difference between the two elicited utility functions for the ith subject and the jth evaluation measure, defined

$$\varepsilon_{ij}=\left(\frac{1}{\sum_{k=1}^{K}h\left(z_{jk}\right)}\sum_{k=1}^{K}h\left(z_{jk}\right)\left[u_{ij}^{CE}\left(z_{jk}\right)-u_{ij}^{PE}\left(z_{jk}\right)\right]^2\right)^{1/2} \tag{66}$$

Similarly, use δ_{ij} as a WRMS measure of the difference between the mean elicited utility function and the elicited value function, defined

$$\delta_{ij}=\left(\frac{1}{\sum_{k=1}^{K}h\left(z_{jk}\right)}\sum_{k=1}^{K}h\left(z_{jk}\right)\left[\overline{u}_{ij}\left(z_{jk}\right)-v_{ij}\left(z_{jk}\right)\right]^2\right)^{1/2} \tag{67}$$

The null hypothesis may be stated as the elicitation error does not differ from the differences to be observed between the value and utility functions, then becomes

$$H_0:\varepsilon_{ij}\geq\delta_{ij} \tag{68}$$

And the alternate hypothesis

$$H_0:\varepsilon_{ij}<\delta_{ij} \tag{69}$$

The δ_{ij} and ε_{ij} terms are not independent, as the subject's personal error affects both. As the subjects' errors differed significantly, the hypothesis was evaluated with a paired t test. The results are shown in Figure 41. The zero horizontal line in the plot represents the point where there are no differences between δ_{ij} and ε_{ij}. The solid horizontal line enclosed by the dashed lines indicates the mean difference between δ_{ij} and ε_{ij} enclosed by a 95 percent confidence interval. We conclude that there are significant differences. The corresponding p value for the t test is less than 0.0001. The WRMS for δ_{ij} is 0.191 and for ε_{ij} is 0.102. We reject the null hypothesis and conclude that the differences between the value and utility functions are significantly larger than the elicitation error for this sample.

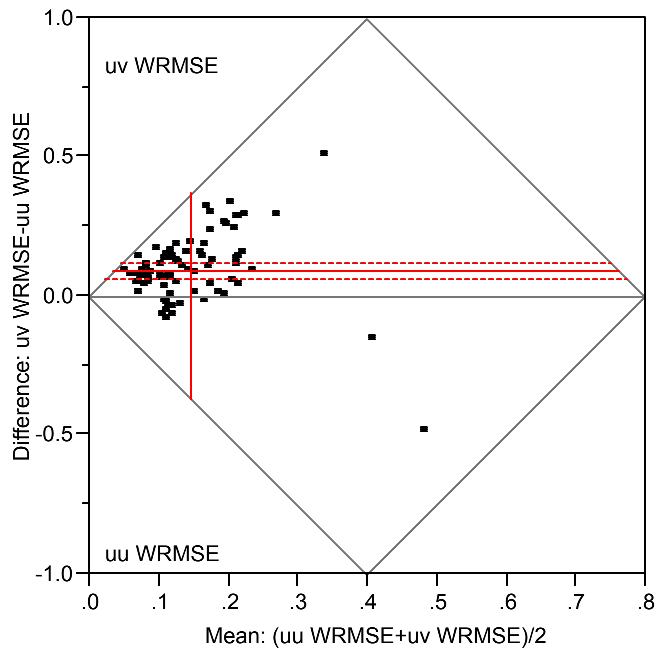


Figure 41. Comparison of uu and uv Elicitations. Paired t Test of Utility Function Elicitation Error (uu) and Differences in Elicited Value and Utility Functions (uv) Employing the Weighted Root Mean Square of the Differences.

Elicitation Learning Effects

The elicitation was performed employing a questionnaire as a guide, with two permutations of the order of elicitations of the preference functions. The two permutations were version A (value, certainty equivalent utility, probability equivalent utility [vuu]) and version B (certainty equivalent utility, value, probability equivalent utility [uvu]). This provides the opportunity to check for learning effects. It was hypothesized that if indeed utility is a function of both the decision maker's preferences and her risk affinity, placing the value function elicitation first should cause her to clarify her thoughts on preferences before the latter two utility elicitations.

This is testable with the null hypothesis

$$H_0 : \varepsilon_{uvu} = \varepsilon_{vuu} \tag{70}$$

and the alternate

$$H_A : \varepsilon_{uvu} \neq \varepsilon_{vuu} \tag{71}$$

Where ε is as defined above and the subscript indicates the elicitation sequence.

The results of an ANOVA of the WRMSE by version are shown in Figure 42. The difference is in the range of significance, with a p value of 0.0210. A similar analysis on the RMSE data provided a p value of 0.0536. Interestingly the version where the value function was elicited first had the higher error. As will be shown below, the officer and enlisted soldiers had differing amounts of WRMSE. The elicitation versions were assigned randomly; however as a check the version was plotted in a mosaic plot against the grade categories of officer and enlisted soldier. This is shown in Figure 43. No pattern is apparent.

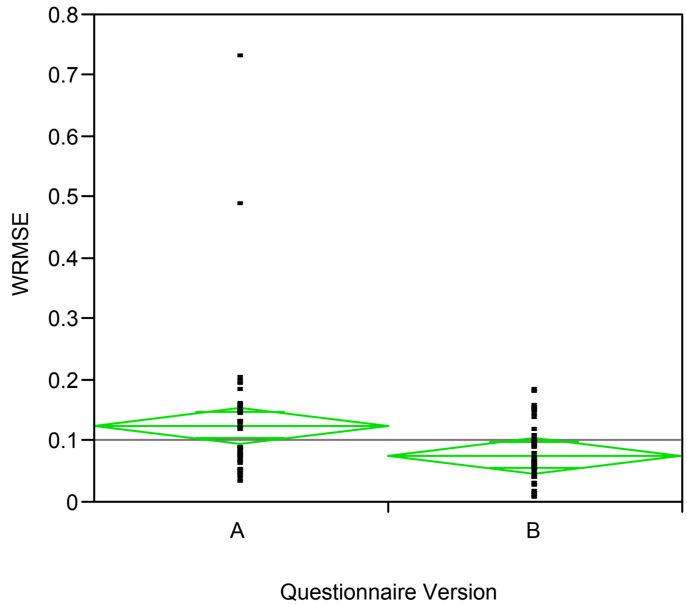


Figure 42. ANOVA on the Preference Function Elicitation Order. The Order of Elicitation for Version A was *vuu*, B was *uvu*.

Figure 43. Mosaic Plot of Questionnaire Version Versus Pay Grade Category.

The results in Figure 42 are clearly affected by the two possible outliers. As discussed above, elimination of one of these points is likely prudent, the other possible

but less justified. Eliminating the most severe outlier produces a p value of 0.0214 forWRMSE (p value of 0.0721 for RMSE). This is illustrated in Figure 44. Even when discarding the potential outlier we reject the null hypothesis and conclude that there are significant effects observed. (Elimination of both outliers produces p values of 0.0299 and 0.139 for WRMSE and RMSE, respectively.)

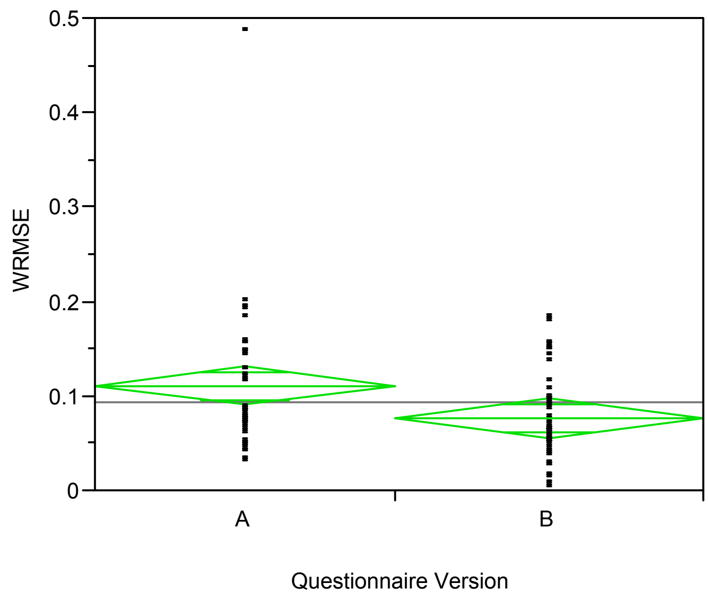


Figure 44. ANOVA on the Preference Function Elicitation Order (Trimmed). The Order of Elicitation for Version A was vuu, B was uvu.

However the one-tail test that a *vuu* elicitation error is less than the *uvu* elicitation, or the null hypothesis

$$H_0 : \varepsilon_{uvu} > \varepsilon_{vuu} \tag{72}$$

and the alternate

$$H_A : \varepsilon_{uvu} \leq \varepsilon_{vuu} \tag{73}$$

provide a more accurate test of learning effects. The *p* value for the untrimmed data set for WRMSE is 0.9893 or 0.9895 for the data set with the most distant outlier removed. Clearly we reject the null hypothesis that eliciting the value function first provides for learning and reduced elicitation error. In fact structuring the reverse construct as a hypothesis would be accepted with this data. This effect remains unexplained. However for these subjects eliciting the value function first led to higher elicitation error. We conclude that learning effects were not present as hypothesized.

Simultaneous Equations

For the multiattribute utility elicitation, the recommended method of Kirkwood (1997: 162 – 163) was employed. The approach is to place the evaluation measures in the certain alternative at either their best or worst levels (x_{0j} or x_{*j} for all *j*) and vary the combinations (a zero-one programming approach) of the certain alternative evaluation measures at best and worst levels until ambivalence is achieved. The reference gamble, the uncertain alternative, is a 50-50 possibility of all evaluation measures at the most or at the least attractive levels. This is depicted in Figure 45 for the tactical situation. The uncertain alternative, labeled *A*, has a probability of 0.5 that the evaluation measures are all at the most preferred levels and a probability of 0.5 that the evaluation measures are all at the least preferred levels. The certain alternative *B* has the evaluation measures undefined, where $x_j = x_{0j}$ or $x_j = x_{*j}$ for all *j*.

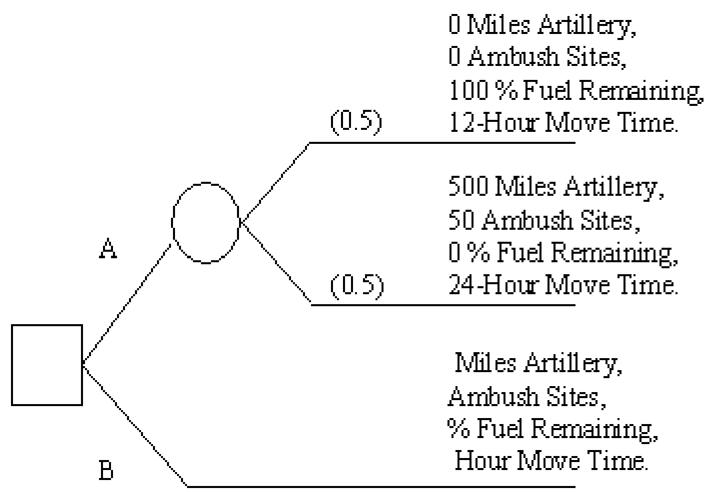


Figure 45. Multiattribute Utility Elicitation Graphical Representation.

For the subjects where no single combination of evaluation measures provided ambivalence to the reference gamble, simultaneous equations were gathered. Each of the nine subjects who were not ambivalent to any of the x_{oj}, x_{*j} combinations was presented four sequential elicitations, where they were offered various combinations of the levels of the four evaluation measures. The levels were adjusted continually in the natural number domain, $x_j \in \left[x_{0j}, x_{*j}\right]$ for each *j*, rather than restricting each $x_j = \left\{x_{0j}, x_{*j}\right\}$, until ambivalence with the reference gamble was achieved.

The elicitation of some combination of evaluation measures for alternative *B* such that the subject is indifferent between *A* and *B* provides a set of evaluation scores that is equivalent in utility to the uncertain alternative *A*. By the expected utility axioms, ambivalence between alternatives *A* and *B* require $E\left[u\left(A\right)\right] = E\left[u\left(B\right)\right]$ where $U\left(\mathbf{X}\right)$ is the objective function – the multiattribute utility function, generally defined by

$$U(\mathbf{X}) = \sum_{j=1}^{J} w_j u_j(x_j) \tag{74}$$

for the *J* evaluation measures, where w_j is the weight of the *j*th evaluation measure and $X = \{x_1, x_2, \ldots, x_J\}^T$. As we have different subjects and four elicitations per subject, further employ the standard definition of the utility of the uncertain outcome

$$U_{ik}(\mathbf{X}) = \sum_{j=1}^{J} p_j u_{ij}(x_{jk}) \tag{75}$$

where *k* indicates the elicitation, one through fours, for the *i*th subject. The utility of alternative A is easily determined as the utility of all evaluation measures at the least preferred levels, $\mathbf{X_0}$, is zero, and at the most preferred, $\mathbf{X}_*$, equal to one. The utility of alternative A is given by

$$U(A) = p_{\mathbf{X}_0} U(\mathbf{X}_0) + p_{\mathbf{X}_*} U(\mathbf{X}_*) = 0.5(0) + 0.5(1) = 0.5 \tag{76}$$

where $p_{\mathbf{X}_0}, p_{\mathbf{X}_*}$ are the respective probabilities of realization of each possible outcome under alternative *A*. As alternative *A* does not change for these four elicitations, $U(\mathbf{X}_{1k}) = U(\mathbf{X}_{2k}) = U(\mathbf{X}_{3k}) = U(\mathbf{X}_{4k}) = 0.5$. Because the expected utility of both alternatives is equal, $E\left[u(B)\right] = 0.5$.

The multiattribute value function *V* is

$$V(X) = \sum_{j=1}^{J} w_j v_j(x_j) \tag{77}$$

and providing for multiple elicitations and subjects becomes

$$V_{ik}(X) = \sum_{j=1}^{J} p_j v_{ij}(x_{jk}) \tag{78}$$

Because $E\left[U\left(B\right)\right]=0.5$ for all four elicitations if there is a functional relationship between value and utility, w, then $U\left(B\right)=\hat{U}\left(V\left(B\right)\right)=0.5$ for all elicitations. Or $\hat{U}\left(V\left(B_1\right)\right)=\hat{U}\left(V\left(B_2\right)\right)=\hat{U}\left(V\left(B_3\right)\right)=\hat{U}\left(V\left(B_4\right)\right)=0.5$ where B_k indicates the *k*th elicitation iteration. This requires that $V\left(B_1\right)=V\left(B_2\right)=V\left(B_3\right)=V\left(B_4\right)=d$ where *d* is some constant.

Each elicitation of a combination of x_j provides an opportunity to estimate *d*. A histogram and q-q plot is shown in Figure 46. The data is well behaved enough to assume normal distribution of the error. A 95 percent confidence interval extends from 0.41 to 0.54, including the nominal value of 0.5. ANOVA results for these subjects are shown in Figure 47. At least one subject has a significantly different error distribution with a *p* value of 0.037.

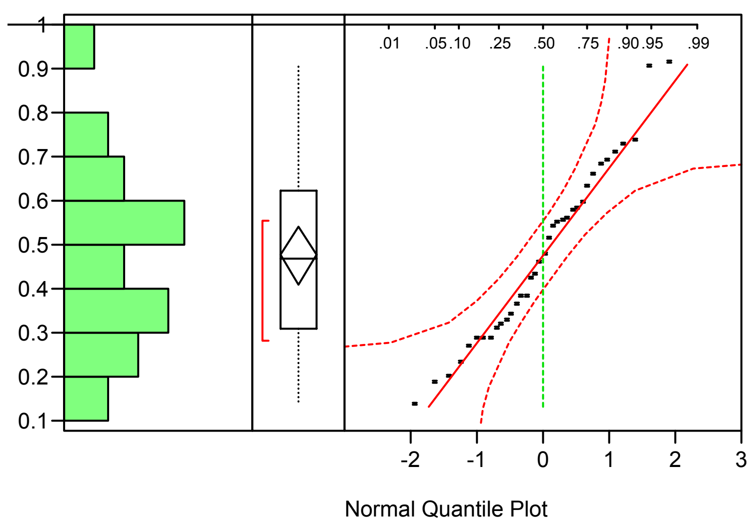


Figure 46. Histogram and Q-Q Plot of the Simultaneous Equation Value Results.

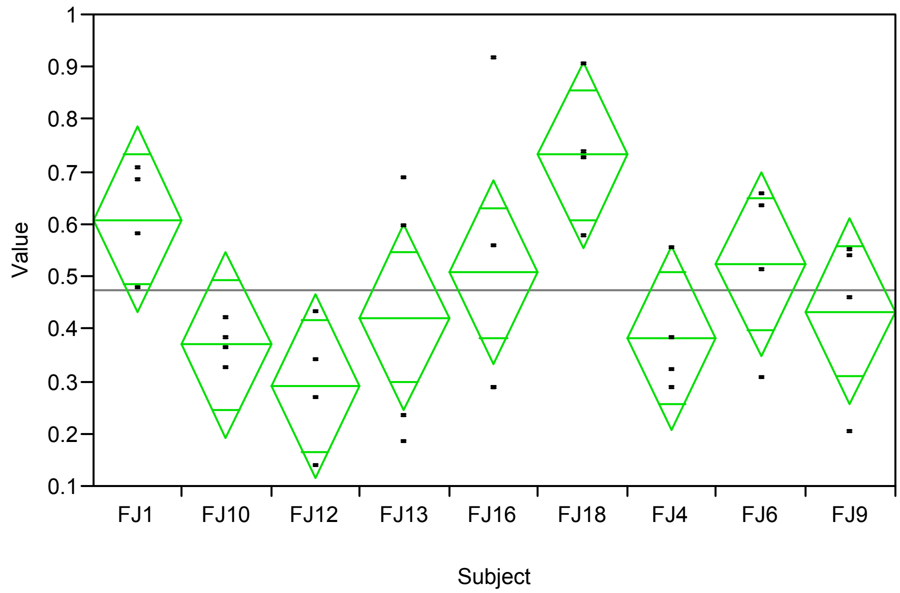


Figure 47. ANOVA of Simultaneous Equation Value Results.

To compare the error in the multiattribute case, ε_M, to that of the single dimensional elicitations, WRMSE is again appropriate. Each of the four B_k for each subject provides a realization of an estimate of the true curve that corresponds to $\underset{\cdot}{U}\left(V\left(B_k\right)\right)$. The four $\hat{\underset{\cdot}{U}}\left(V\left(B_k\right)\right)$ curves were determined by Kirkwood's method where an exponential constant is calculated that causes the curve

$$\hat{\underset{\cdot}{U}}\left(V\left(B\right)\right) = \begin{cases} \dfrac{1-e^{-v(x)/\rho}}{1-e^{-1/\rho}}, & \rho \neq \infty \\ v\left(x\right), & \text{otherwise,} \end{cases} \tag{79}$$

to pass through the particular point $\left(V_M, 0.5\right)$. As it is known that the alternative *B* has a utility equal to 0.5, the point V_M is the point that provides $\hat{\underset{\cdot}{U}}\left(V_M\right) = 0.5$ and is determined as provided by Kirkwood (1997: 162 – 164) using a simple look-up table for ρ. This table is provided in Table 8. Each B_k provides a separate ρ and therefore a

separate estimate of the multiattribute utility function, $\hat{U}(V(z_l))$. The error of the multiattribute utility function for each $\hat{U}(V(z_l))$ is provided by

$$\varepsilon_M = \left[\frac{1}{\sum_{k=1}^{N} h(z_{jk})} \sum_{k=1}^{N} h(z_{jk}) \left[\hat{U}(V(z_l)) - \bar{U}(z_l) \right]^2 \right]^{1/2} \tag{80}$$

where $\bar{U}(z_l) = \dfrac{\sum_{k=1}^{4} \hat{U}_m(z_l)}{4}$. The ε_M for the *k*th elicitation and *i*th subject is denoted $\varepsilon_{M_{ik}}$ The $\varepsilon_{M_{ik}}$ results are summarized in Table 9 along with the single dimensional elicitation error, ε_{ik}, as defined above, for comparison. Results of an ANOVA, blocked on the subjects, are presented in Table 10. In only two cases were the single and multiple dimensional elicitation error significantly different. For one of these two subjects the single dimensional WRMSE was higher, and the reverse was true for the other subject. The data are inconclusive as to whether one method provides superior accuracy.

Decision Analysis Results

The decision analysis models for the preference functions were compared using the DPL programming package. The decision tree and influence diagram for the tactical problem posed in the questionnaire are shown in Figure 48 and Figure 49, respectively. There is only a single decision, that of which route to select. The uncertainties are conditional

Table 8. Exponential Constants, ρ (Kirkwood, 1997b: 69).

$z_{0.5}$	R	$z_{0.5}$	R	$z_{0.5}$	R	$z_{0.5}$	R
0.00		0.25	0.410	0.50	Infinity	0.75	-0.410
0.01	0.014	0.26	0.435	0.51	-12.497	0.76	-0.387
0.02	0.029	0.27	0.462	0.52	-6.243	0.77	-0.365
0.03	0.043	0.28	0.491	0.53	-4.157	0.78	-0.344
0.04	0.058	0.29	0.522	0.54	-3.112	0.79	-0.324
0.05	0.072	0.30	0.555	0.55	-2.483	0.80	-0.305
0.06	0.087	0.31	0.592	0.56	-2.063	0.81	-0.287
0.07	0.101	0.32	0.632	0.57	-1.762	0.82	-0.269
0.08	0.115	0.33	0.677	0.58	-1.536	0.83	-0.252
0.09	0.130	0.34	0.726	0.59	-1.359	0.84	-0.236
0.10	0.144	0.35	0.782	0.60	-1.216	0.85	-0.220
0.11	0.159	0.36	0.845	0.61	-1.099	0.86	-0.204
0.12	0.174	0.37	0.917	0.62	-1.001	0.87	-0.189
0.13	0.189	0.38	1.001	0.63	-0.917	0.88	-0.174
0.14	0.204	0.39	1.099	0.64	-0.845	0.89	-0.159
0.15	0.220	0.40	1.216	0.65	-0.782	0.90	-0.144
0.16	0.236	0.41	1.359	0.66	-0.726	0.91	-0.130
0.17	0.252	0.42	1.536	0.67	-0.677	0.92	-0.115
0.18	0.269	0.43	1.762	0.68	-0.632	0.93	-0.101
0.19	0.287	0.44	2.063	0.69	-0.592	0.94	-0.087
0.20	0.305	0.45	2.483	0.70	-0.555	0.95	-0.072
0.21	0.324	0.46	3.112	0.71	-0.522	0.96	-0.058
0.22	0.344	0.47	4.157	0.72	-0.491	0.97	-0.043
0.23	0.365	0.48	6.243	0.73	-0.462	0.98	-0.029
0.24	0.387	0.49	12.497	0.74	-0.435	0.99	-0.014

probabilities affected by the route selection. These probabilities were modeled as triangular distributions, with the parameters set as listed in Table 11.

The results of the analysis are tabulated in Appendix B. The results were calculated for each subject by route for the various preference functions: value function, $v(x)$; utility function determined by the certainty-equivalent method, $u_{CE}(x)$; utility function determined by the probability-equivalent method, $u_{PE}(x)$; utility function determined by averaging the certainty-equivalent and probability-equivalent methods, $\bar{u}(x)$; and the multiattribute utility method, $u_m(x)$.

The route alternatives ordered in descending preference appear in Appendix B. The mismatches between pairs of various preference functional models are presented in

Table 9. WRMSE for Single and Multiple Dimension Utility Elicitations.

Subject	Single Dimensional Error ε_{ik}	Multi-dimensional Error $\varepsilon_{M_{ik}}$	Subject	Single Dimensional Error ε_{ik}	Multi-dimensional Error $\varepsilon_{M_{ik}}$
FJ1	0.0676	0.0253	FJ12	0.0712	0.0515
FJ1	0.0289	0.0692	FJ12	0.0730	0.0124
FJ1	0.0534	0.0457	FJ12	0.195	0.112
FJ1	0.0717	0.0898	FJ12	0.728	0.160
FJ4	0.0256	0.0734	FJ13	0.0773	0.133
FJ4	0.0964	0.113	FJ13	0.0648	0.184
FJ4	0.0528	0.0417	FJ13	0.136	0.193
FJ4	0.0991	0.0049	FJ13	0.0616	0.124
FJ6	0.116	0.0703	FJ16	0.0822	0.344
FJ6	0.129	0.0122	FJ16	0.0725	0.174
FJ6	0.157	0.0917	FJ16	0.120	0.174
FJ6	0.0333	0.153	FJ16	0.0309	0.0469
FJ9	0.143	0.0747	FJ18	0.148	0.135
FJ9	0.150	0.180	FJ18	0.486	0.184
FJ9	0.0956	0.0252	FJ18	0.200	0.0316
FJ9	0.153	0.0811	FJ18	0.116	0.0248
FJ10	0.107	0.036			
FJ10	0.0373	0.0077			
FJ10	0.0616	0.0065			
FJ10	0.0575	0.0348			

Table 10. Comparison of WRMSE for Multi- and Single Dimensional Utility Elicitations.

Subject	Mean Single Dimensional WRMSE	Mean Multidimensional WRMSE	P Value
FJ1	0.0554	0.0575	0.907
FJ4	0.0685	0.0583	0.737
FJ6	0.109	0.0818	0.522
FJ9	0.135	0.0900	0.245
FJ10	0.0658	0.0213	0.0378
FJ12	0.267	0.0840	0.296
FJ13	0.0848	0.159	0.0241
FJ16	0.0764	0.185	0.140
FJ18	0.237	0.0940	0.175

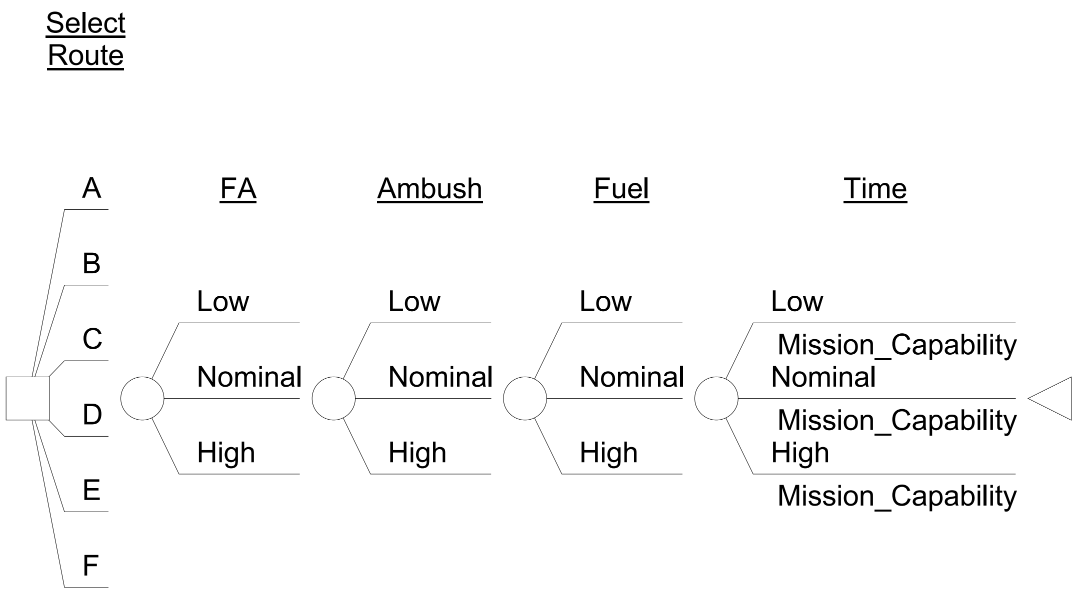


Figure 48. Tactical Scenario Decision Tree.

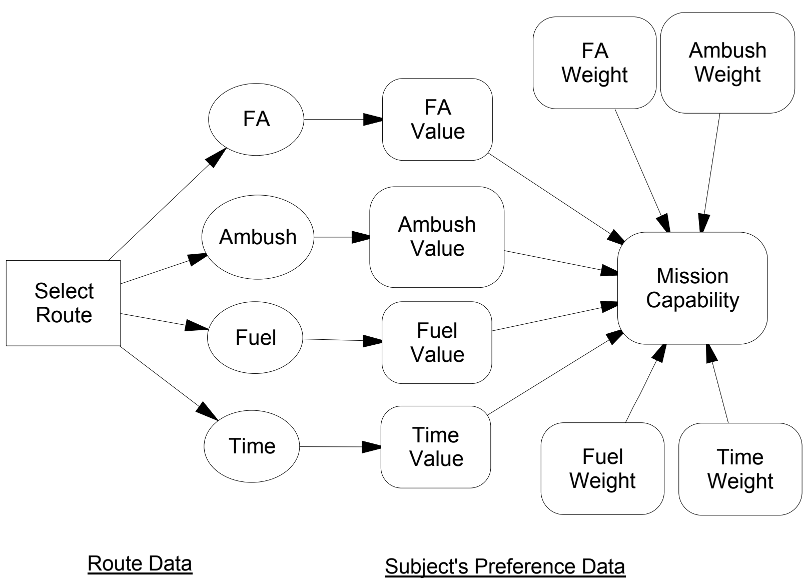


Figure 49. Tactical Scenario Influence Diagram for the Value Function.

Table 11. Triangular Distribution Parameters.

Field Artillery Exposure (miles)			
Route	Minimum	Mode	Maximum
A	0	1	50
B	0	50	300
C	150	300	400
D	200	400	500
E	300	499	500
F	200	499	500
Ambush (sites)			
Route	Minimum	Mode	Maximum
A	15	30	45
B	25	45	50
C	25	49	50
D	20	35	50
E	0	5	10
F	0	1	10
Fuel (percent)			
Route	Minimum	Mode	Maximum
A	0	20	40
B	0	1	40
C	0	20	50
D	0	40	50
E	50	99	100
F	0	50	100
Time (hours)			
Route	Minimum	Mode	Maximum
A	18	21	24
B	16	23.9	24
C	17	22	24
D	14	21	23
E	12	12.1	15
F	12	12.1	16

Table 12. These mismatches occur when the different functional forms produce differing alternatives in corresponding ordinal positions. For example, if the value function model indicates that route C is the third most preferred alternative and the certainty equivalent utility model produces route D for the same position, this is classified as a mismatch. As there are six alternatives and six assignments per subject per model type, the possible number of mismatches per comparison is drawn from the set $\{0,2,3,4,5,6\}$. Also contained in Table 12 is the number of mismatches for a deterministic situation where

Table 12. Mismatches Between Various Preference Functions.

	Stochastic		Deterministic	
Comparison	Mismatches	Percent	Mismatches	Percent
$u_{CE}(x)-u_{PE}(x)$	28	0.23	22	0.18
$v(x)-u_{CE}(x)$	36	0.30	52	0.43
$v(x)-u_{PE}(x)$	33	0.28	49	0.41
$v(x)-\overline{u}(x)$	37	0.31	56	0.47
$v(x)-u_{m_1}(x)$	12	0.10	0	0.00
$u_{m_1}(x)-u_{m_2}(x)$	6	0.11	0	0.00
$u_{CE}(x)-\overline{u}(x)$	18	0.15	8	0.067
$u_{PE}(x)-\overline{u}(x)$	18	0.15	14	0.12
$\overline{u}(x)-u_{m_1}(x)$	29	0.24	56	0.47
$\overline{u}(x)-u_{m_2}(x)$	17	0.32	25	0.46

there is no uncertainty and the route data employed was the modal data from the triangular distributions of the stochastic set of cases.

Normally in employment of decision analysis the interest is in the best (greatest value or utility) alternative, or in those alternatives near the top. Table 13 shows the mismatches for the best (highest) alternative, and the best two alternatives. Mismatches for the latter category are the number of mismatches of the nonordered set $\{a_1, a_2\}$ foreach method being compared, where the a_i are the alternatives and the subscript indicates order of desirability based on the model. The percentage error decreases for the majority of cases when two alternatives are considered. This is expected as the number of possible combinations is reduced.

Examining the deterministic cases first, it is noticed that no differences exist between the value and multiattribute utility comparisons and also for comparisons

Table 13. Mismatches Between Various Preference Functions.

	Best Alternatives		Best Two Alternatives	
Comparison	Mismatches	Percent	Mismatches	Percent
$u_{CE}(x)-u_{PE}(x)$	1	0.05	4	0.10
$v(x)-u_{CE}(x)$	5	0.25	7	0.18
$v(x)-u_{PE}(x)$	5	0.25	5	0.13
$v(x)-\overline{u}(x)$	6	0.30	5	0.13
$v(x)-u_{m_1}(x)$	1	0.05	3	0.075
$u_{m_1}(x)-u_{m_2}(x)$	0	0	2	0.11
$u_{CE}(x)-\overline{u}(x)$	2	0.10	3	0.075
$u_{PE}(x)-\overline{u}(x)$	1	0.05	2	0.05
$\overline{u}(x)-u_{m_1}(x)$	5	0.25	2	0.05
$\overline{u}(x)-u_{m_2}(x)$	3	0.33	2	0.11

between the multiattribute utility functions for the same subject (where they exist). In the deterministic situation, the multiattribute utility merely provides a transform that preserved the ordinality of the value data, so no mismatches are possible. Obviously employing multiattribute utility in a non-aleatory environment does not provide any additional ordinal information over that provided by a value function. Clearly the concept of employing value and single dimensional utility functions interchangeably is not supported by these data. The mismatches between these models almost reached half of the recommendations.

For the stochastic data, the value and single dimensional mismatches occur in about 30 percent of the cases. The two realizations of the single dimension utility curve produced mismatches in 23 percent of the pairs. The averaged single dimensional utility and the multiattribute utility differed almost one-quarter of the time.

Examination of the mismatches only provides a cursory insight, as the binomial result contains no measure of the distance of the differences of the alternatives. One question of interest is whether employing a single elicited realization of the single dimensional utility function significantly affects the results. Figure 50 shows the pairwise comparison of $u_{CE}(x)$ and $u_{PE}(x)$ for the alternatives grouped by subjects. The differences approached significance with a p value of 0.0554. The interpretation is that either preference realization may have been employed, but only barely.

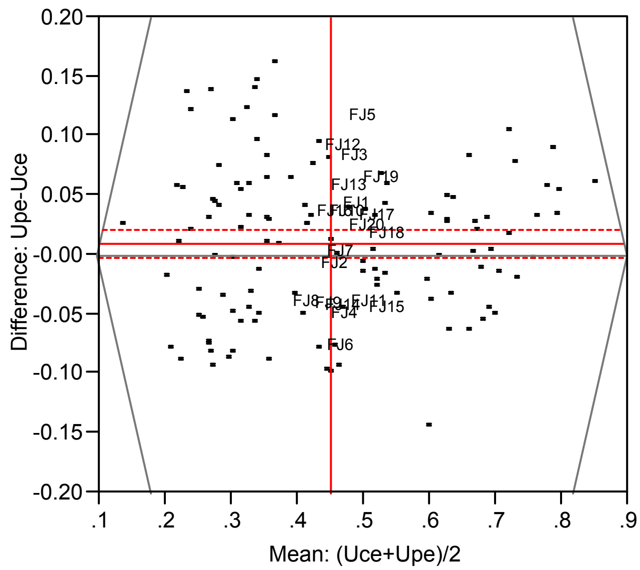


Figure 50. Pairwise Comparison of $u_{CE}(x)$ and $u_{PE}(x)$ by Subject.

Figure 51, illustrating a pairwise comparison between $v(x)$ and $\overline{u}(x)$, grouped by subject, shows that there is a significant difference between them, with a p value of <0.0001. This is no surprise, as the value scores and utility scores were expected to differ. The important consideration is if the scores are coherent, producing similar

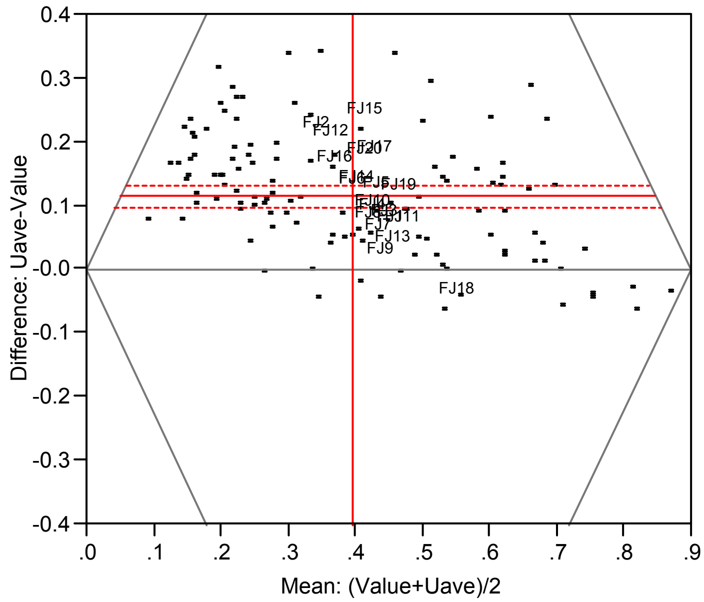


Figure 51, Pairwise Comparison of $v(x)$ and $\bar{u}(x)$.

recommendations (identical ordering, or strategically equivalent recommendations). As discussed above, almost a third of the highest scoring alternatives differed between these preference measures.

Figure 52 illustrates the pairwise comparison between $\bar{u}(x)$ and $u_{m_1}(x)$, grouped by subject, for the eleven subjects who did establish an ambivalence $u_{m_1}(x)$. The two methods did produce significantly different scores, with a p value of 0.0018. This indicates that the multiattribute utility construct is not identical with the single dimensional utility preference model, at least possessing differing means. This result alone does not indicate that the multiattribute utility model (or the single dimension utility model, for that matter) is not acceptable, but that the subjects produced differing constructions for these two utility functions. As indicated above, the multiattribute utility function produced 24 percent mismatches with the averaged single dimension utility

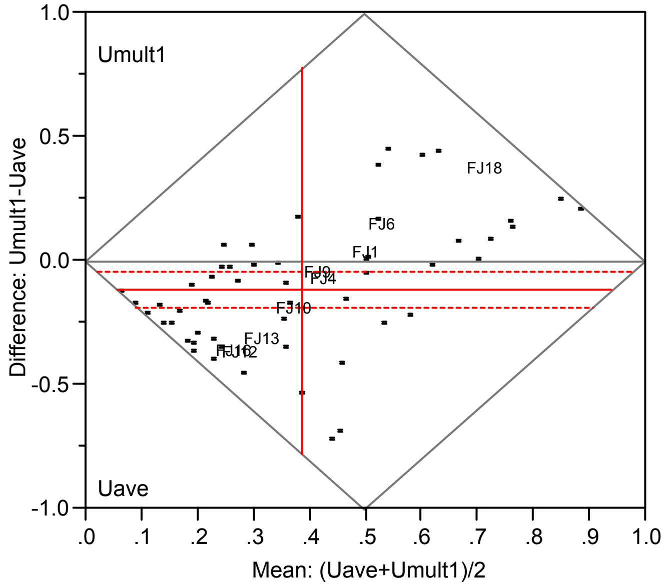


Figure 52. Pairwise Comparison of $\bar{u}(x)$ and $u_{m_1}(x)$.

function model. However the two elicited single dimension utility functions mismatched in 23 percent of the alternatives. For five of twenty of the subjects, the comparison between $\bar{u}(x)$ and $u_{m_1}(x)$ produced differing answers for the best route. When comparing $u_{CE}(x)$ and $u_{PE}(x)$ for the best alternative, there was only a single disagreement. This indicates that the two models are in fact different constructs, measuring the subject's predilections differently, and which produce solution vectors that are not strategically equivalent.

Tactical Scenario Value and Utility Preference Function Summary

The preference function information of 20 subjects, all military professionals, was examined with respect to differences in single dimensional value and utility functions. A new measure of comparing preference functions, the weighted root mean square error (WRMSE), was introduced. The fit of five proposed transforms from value into utility were examined. Acceptance criteria were developed and tested. Previous work had used ad hoc criteria.

The sigmoid function was very successful as a model, fitting best in over 80 percent of the cases, with other models achieving no more than a four percent rate of best fit. The analysis also conclusively confirms that value and utility are different functions and are poor surrogates, when individually considered, for each other, at least for these data. The differences in the value and utility functions were significantly greater than the elicitation error. The methods of probability equivalent and certainty equivalent utility elicitation provide (barely) similar results.

Curve shape is indicative of risk attitudes when risk is included in the assessment protocol. The subjects were risk averse for half of the evaluation measures. They provided s-shaped utility curves in 40 percent of the cases, indicating both risk averse and risk seeking behavior across the evaluation measure domain. The common assumption of risk aversion is poor for this problem and group of subjects, and suggests that this may be common for military decision making in operational settings.

Order of elicitation of single dimensional value and utility information made significant differences in the elicitation error. Eliciting functions in the sequence of

value, utility (certainty equivalent), then utility (probability equivalent) had higher internal error than utility (certainty equivalent), value, and then utility (probability equivalent). If it is accepted that the probability equivalent and certainty equivalent methods produce theoretically identical constructs, then this supports the idea that eliciting value in between the two utility elicitations provides reduced elicitation error than eliciting the utility information sequentially without interruption. Such an idea is counterintuitive, and suggests future work in this area. In any case, typical application of decision analysis involves the use of only one method, and so this has no effect on practical use of DA.

The multiattribute utility approach of Kirkwood is significantly different than the standard single dimension utility approach. Nevertheless, the two methods provide similar levels of accuracy. When eliciting single dimensional utility functions, either the certainty equivalent or the probability equivalent techniques may be employed, but the methods differ significantly with statistical difference approaching significance, as the *p* critical value was 0.055.

Information Operations Value-Utility Data

Information Operations Value-Utility Functions. Doyle (1998) employed a decision analysis value-focused thinking approach to develop evaluation measures for the analysis of information operations. This effort was the first use of decision analysis in information operations. Doyle's published research was based on single dimensional value functions. However both value and utility functions were elicited from participants

in the study and so the data offer the opportunity to compare single dimensional value and utility functions elicited from the same decision makers who are professionals dealing with a operational military subject. (Doyle, 2000) These opportunities are rare, and provide the opportunity to validate the above results with data obtained from a separate researcher and separate subjects. The background information presented in the next section provides a contextual summary as described by Doyle.

Background. *Information Operations* are defined by Joint Publication 3-13 (Department of Defense, 1998: I-9) as those "actions taken to affect adversary information and information systems while safe guarding one's own information and information systems." The intent is to affect the opponent's capabilities and actions through influences on their information and information systems, which provides a more efficient defeat mechanism, at least under some circumstances, than more traditional methods. During periods of conflict, information operations are referred to as information warfare (Department of Defense, 1998: I-11).

Command and control warfare is an application of information warfare in military operations and consists of five components. Operations security is a process of identification of critical information, and execution of corresponding safeguards to reduce enemy exploitation of this information (Department of Defense, 1997: GL-2). Military deception consists of those actions designed to deceive an enemy and causes the enemy to undertake actions, or fail to do so, that facilitate friendly success (Department of Defense, 1996c: I-1). Psychological operations convey selected information to foreign audiences to create conditions favorable to friendly operations (Department of Defense, 1996a: I-1). Electronic warfare is the use of electromagnetic or directed energy devices

to control the electromagnetic spectrum or attack the enemy. Electronic warfare has three subcomponents: electronic attack engages enemy units and facilities to degrade their capability; electronic protection safeguards friendly electromagnetic systems; and electronic warfare support gathers information from electromagnetic emissions to provide information about enemy forces. (Department of Defense, 2000: I-2 – I-3). Physical destruction is the use of physical weapons against enemy targets as an element of command and control warfare (Department of Defense, 1996b: II-7).

While historical examples of what is now known as information operations are as old as military history, the creation of the field of information operations is relatively recent. As information operations span a number of areas of specialization, those charged with conducting them face the challenges of mastering an immature discipline with disparate proficiency requirements. It was desired that a decision support tool be developed to assist in selection between information operations courses of action.

Results. Doyle's analysis provided both an objectives and cost value hierarchy. These are seen in Figure 53 and Figure 54, respectively. As an example, the value and utility cost functions for Minimize Friendly at Risk Personnel are shown in Figure 55. The domain for each evaluation measure differs, but in Doyle's work all preference functions map into $[0,10]$. Plots of all preference functions are contained in Appendix C. Not all preference functions were continuous.

We now move beyond the analysis conducted by Doyle. The continuous functions were normalized by converting their codomain from $[0,10]$ to $[0,1]$. Then analysis proceeded as above for the Measuring Goodness of Fit for the tactical scenario

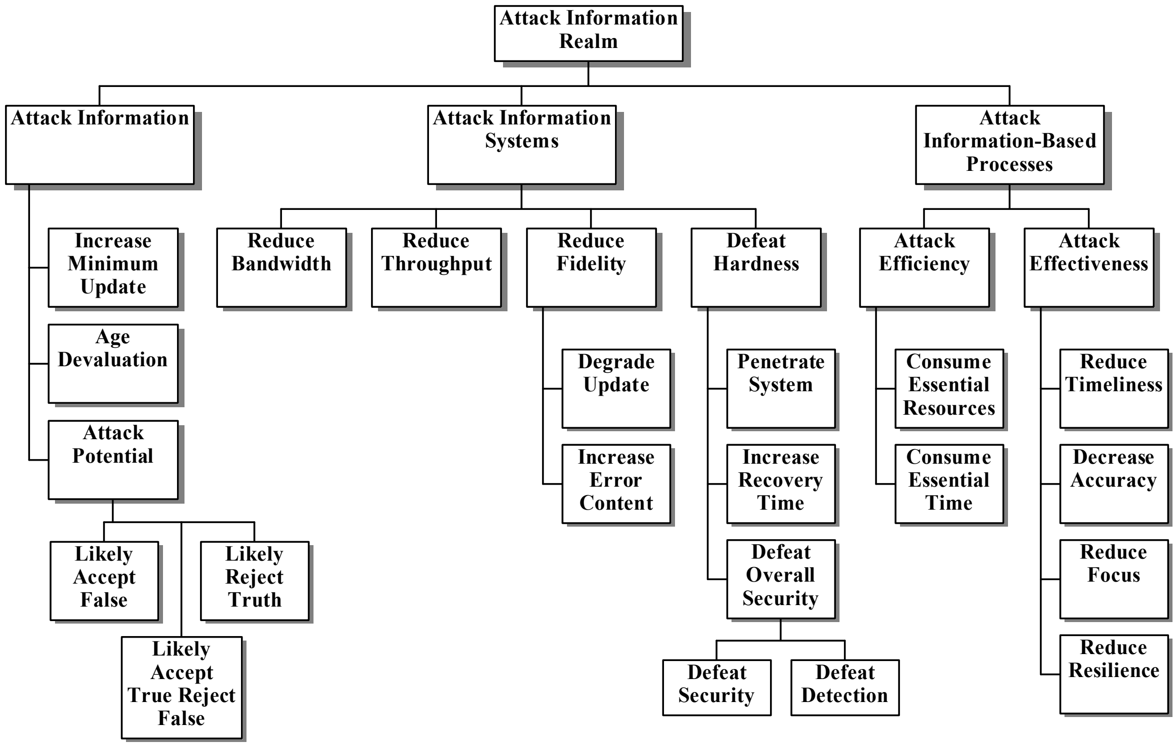


Figure 53. Objectives Hierarchy for Offensive Information Operations, after Doyle (1998: 3-11).

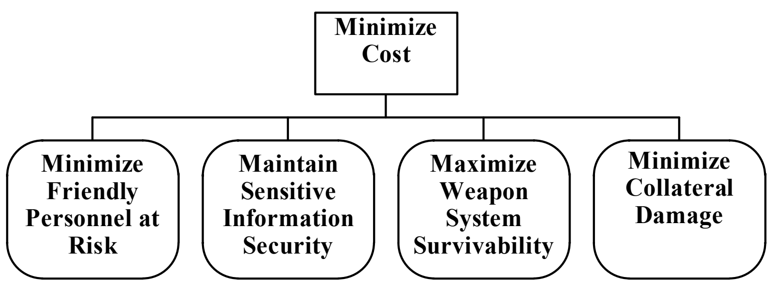


Figure 54. Cost Hierarchy Offensive Information Operations, after Doyle (1998: 3-12).

data. Fits of the five proposed models were examined. As there were no multiple elicitations of the same preference functions, confidence intervals could not be established for WRMSE as done previously. The earlier criterion was employed.

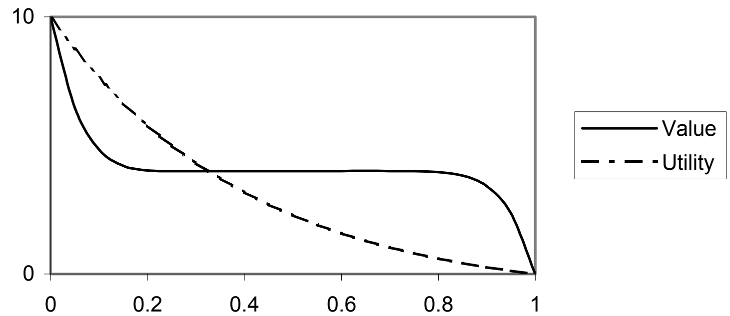


Figure 55. Minimize Friendly at Risk Personnel Functions.

The fit of the five models is tabulated in Appendix C for the twelve continuous evaluation measures. The best fitting models, and other acceptable fits are tabulated in Table 14 along with Keller's measure *G*. The sigmoid function provided the best fit in seven cases and the exponential in the remaining five. Of the twelve continuous evaluation measures, the curve shapes of the assessed utility functions of the subject were not strongly risk averse: two were linear or near linear, four were S-shaped, four were convex, and two were concave. The equivalent results employing RMSE are presented in Appendix C and Table 15. The sigmoid function provided the best fit in eight cases and the exponential in the remaining four. Using either measure, the linear fit was never the best, indicating that value and utility are significantly different in this analysis. One-third of the $\hat{u}(v(x))$ curves, the transform from value to utility, were S-shaped, indicating non-constant risk attitudes.

We conclude that this data set also supports the hypothesis that $u(v(x)) \neq v(x)$. Further, the fit of the sigmoid function indicates that constant risk attitudes are not present across the domain, as well as that the sigmoid is an efficacious representative

Table 14. Summary of Model Fits for $\hat{u}(v(x))$ Employing WRMSE. "Lin" stands for linear, "Log" for logarithmic, "Pow" for power functions.

	Best Acceptable Fit	G	Other Acceptable	Curve Shape
Increase Recovery Time	Expo	0.417267	Lin,Log,Pow,Sig	Linear
Degrade Update	Sigmoid	0.890747	Lin,Exp,Log,Pow	S-Shaped
Increase Error Content	Sigmoid	0.895757	Exp,Pow	Convex
Age Devaluation	Sigmoid	0.863706	Exp,Pow	Convex
Decrease Accuracy	Sigmoid	0.888913	Exp,Pow	S-Shaped
Reduce Timeliness	Expo	0.884586	Lin,Log,Pow,Sig	Near Linear
Consume Essential Resources	Expo	0.98388	Lin,Log,Pow,Sig	Concave
Consume Essential Time	Expo	0.866733	Pow, Sig	Convex
Friendly at Risk Cost	Sigmoid	0.931184	None	S-Shaped
System Survivability Cost	Expo	1	Pow	Convex
Collateral Damage Cost	Sigmoid	0.90562	None	S-Shaped
Sensitive Information Security Cost	Sigmoid	0.991695	None	Concave

Table 15. Summary of Model Fits for $\hat{u}(v(x))$ Employing RMSE. "Lin" stands for linear, "Log" for logarithmic, "Pow" for power functions.

	Best Acceptable Fit	G	Other Acceptable	Curve Shape
Increase Recovery Time	Expo	0.50013	Lin,Log,Pow,Sig	Linear
Degrade Update	Sigmoid	0.931271	Lin,Exp,Log,Pow	S-Shaped
Increase Error Content	Sigmoid	0.93979	Exp,Pow	Convex
Age Devaluation	Sigmoid	0.919279	Exp,Pow	Convex
Decrease Accuracy	Sigmoid	0.93384	Exp,Pow	S-Shaped
Reduce Timeliness	Expo	0.894591	Lin,Log,Pow,Sig	Near Linear
Consume Essential Resources	Expo	0.985921	Lin,Log,Pow,Sig	Concave
Consume Essential Time	Sigmoid	0.888864	Exp,Pow	Convex
Friendly at Risk Cost	Sigmoid	0.954836	None	S-Shaped
System Survivability Cost	Expo	1	Pow	Convex
Collateral Damage Cost	Sigmoid	0.93656	None	S-Shaped
Sensitive Information Security Cost	Sigmoid	0.995113	Pow	Concave

parametric function for $\hat{u}(v(x))$. These results agree with the findings of the analysis of the Army personnel considering the tactical situation described above.

Comparing Preference Functions

The research question of interest is how may differences in risk attitudes as evinced by utility functions be compared? Specifically, how does the risk attitude presented by a subject on one evaluation measure predict risk attitudes on other evaluation measures? If inferences are possible, then efficiencies in elicitation may be achieved. Define *consistent risk attitude* as the condition on the decision maker to maintain risk attitudes on all evaluation measures that are consistent with respect to being risk averse, risk neutral, or risk seeking. If this condition is present, then the decision maker is risk averse (neutral) (seeking) on all evaluation measures if she is risk averse (neutral) (seeking) on any single evaluation measure.

In decision analysis a concave (convex) (linear) utility transform function, $u(v(x))$, indicates risk aversion (affinity) (neutrality). Decision analysis literature suggests that when a decision maker is risk averse (seeking) (neutral) on a single evaluation measure then she will be risk averse (seeking) (neutral) on all evaluation measures. Traditionally these utility functions are assumed to be strictly concave, convex, or linear. However s-shaped curves are possible. Indeed, s-shaped curves are common in the results of the tactical questionnaire data.

When examining the risk attitudes across evaluation measures within subjects, the curves may be grouped by shape. Such a categorical grouping is rendered ineffectual as a risk attitude measure because s-shaped curves demonstrate both risk seeking and risk averse behavior. Grouping all s-curves will place those that are strongly risk averse in the

same taxon as those which are strongly risk seeking. Clearly a quantified metric is desirable over the crude categorization provided by curve shape alone.

As summarized in Chapter II, preference functions may be compared at discrete points or across the domain. Under some circumstances a researcher may be interested in comparing preference functions at a specific point or points, but generally the interest lies in the aggregate. Often a decision maker will be characterized as, say, risk averse, which is in reference to risk attitudes across the domain.

One metric proposed for risk attitudes is the *preference area*. This is the area under the utility curve, $u(v(x))$. A risk neutral DM would have a preference area equal to 0.5. A risk averse DM would have a preference area greater than 0.5, and a risk averse decision maker would have a preference area less than 0.5 When analyzing data, risk neutrality is defined as corresponding to the interval [0.45, 0.55]. (Kimbrough and Weber, 1994: 627)

A shortcoming with the preference area is that for a given preference area, many curves may be constructed that enclose the same area. For example, the curves shown in Figure 56 may be generated with the same area under the curve. Clearly the preference area alone does not uniquely identify s-shaped curves, as Figure 56B and Figure 56C illustrate.

It is desirable to have a measure of risk attitude that clearly distinguishes between risk aversion and risk seeking behavior. Using a sign change to contrast risk seeking versus risk averse behavior provides this. For the utility functions operating in the value domain, $u(v(x))$, we restrict the discussion to the class of functions

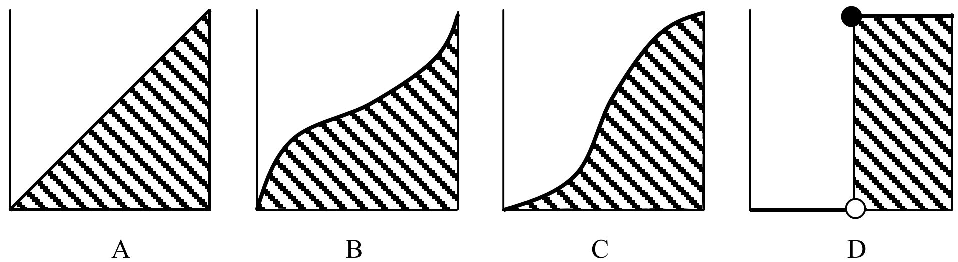


Figure 56. Alternate Utility Curves With Same Preference Area. Graph A, ramp function; Graph B, inverse sigmoid function; Graph C, sigmoid function; and Graph D, step function.

$X = \{ f : [0,1] \to [0,1] \mid f$ is piecewise continuous and nondecreasing on $[0,1], f(0) = 0, f(1) = 1 \}$.

As the preference area, with its single measure, fails to adequately describe risk attitudes, consider that parametric statistics frequently employ two parameters to describe distributions. For example, the normal distribution is completely described by defining the mean and standard deviation. Instead of comparing discrete observations to the mean (a function with slope equal to zero), we are interested in comparing these data to the corresponding risk neutral point.

Define the risk neutral function $n(x) = x$. The difference in preference area to the risk neutral situation is indicated by

$$R(f) = \int_0^1 \left[f(x) - n(x) \right] dx \tag{81}$$

and is illustrated in Figure 57. The shaded area is the region measured by $R(f)$. Because $\int_0^1 n(x) = 0.5$, then $R \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. To gain a more appealing metric on the interval

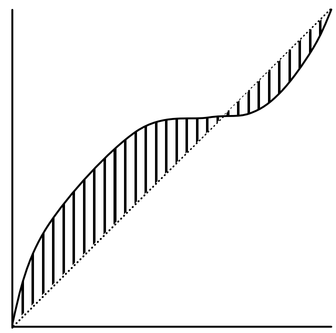


Figure 57. Graphs of *f* and *n*.

$[-1,1]$, equation (81) is modified to

$$R(f) = 2\int_0^1 \left[f(x) - n(x) \right] dx \tag{82}$$

When $R(f)$ is positive, the subject is exhibiting risk averse behavior. When it is negative, risk seeking behavior is present. Accepting the risk neutrality criteria of Kimbrough and Weber (1994: 627), neutrality is present when $-0.1 \le R(f) \le 0.1$. $R(f)$ provides a measure of risk attitude, but like the preference area, is not unique. $R(f)$ provides a measure of central tendency like the mean. A measure of spread away would assist in characterizing the preference functions. Variance would compare $f(x)$ to mean value, a horizontal construct. Such a construct has no interpretation in decision

analysis. Measure of spread with reference to $n(x)$ is straightforward and is a metric that compares to a function, $n(x)$, that has a specific meaning in decision analysis. Define

$$V(f) = 3\int_0^1 \left[f(x) - n(x) \right]^2 dx \tag{83}$$

where the coefficient 3 is employed to provide results on $[0,1]$.

The measures $R(f)$ and $V(f)$ provide different results than the preference area metric. The graphs in Figure 56 all have a preference area of 0.5, but produce generally different results for $R(f)$ and $V(f)$. Examples of representative functions for those in Figure 56 are displayed in Table 16. These metrics do discriminate between these functions in most cases and so this approach is an improvement to the preference area metric. Where the preference area fails to discriminate, so does $R(f)$. However, $V(f)$ provides discrimination when the functions are not reflections about $n(x)$. The metric $R(f)$ does provide the aggregate risk attitudinal information, just as the preference area does.

The functionals $R(f)$ and $V(f)$ would be defined in the region shown in Figure 58. The lower bound of possible data points within the region is the parabola defined by $R(f) = \frac{2}{\sqrt{3}}\sqrt{V(f)}$. Noting the definition of $R(f)$ as provided in Equation (82), we note that

$$R(f) \le |R(f)| = \left| 2\int_0^1 f(x) - n(x)dx \right| = 2\int_0^1 |f(x) - n(x)| dx \tag{84}$$

By the Schwarz inequality (Strang, 1988: 147),

Table 16. Example Functions With Preference Areas Equal To 0.5.

	$f(x)$	$R(f)$	$V(f)$
n(x)	$n(x)=v(x)$	0	0
Convex, concave	$g(v(x))=\begin{cases}\dfrac{1-e^{-v(x)/\rho}}{1-e^{-0.5/\rho}}, \rho=-0.15; & x\in(0,0.5)\\ \dfrac{1-e^{-(v(x)-0.5)/\rho}}{1-e^{-0.5/\rho}}, \rho=0.15; & \text{otherwise}\end{cases}$	0	0.051
Concave, convex	$h(v(x))=\begin{cases}\dfrac{1-e^{-v(x)/\rho}}{1-e^{-0.5/\rho}}, \rho=0.15; & x\in(0,0.5)\\ \dfrac{1-e^{-(v(x)-0.5)/\rho}}{1-e^{-0.5/\rho}}, \rho=-0.15; & \text{otherwise}\end{cases}$	0	0.051
Step	$k(v(x))=\begin{cases}0, & v(x)<0.5\\ 1, & v(x)\geq 0.5\end{cases}$	0	0.25

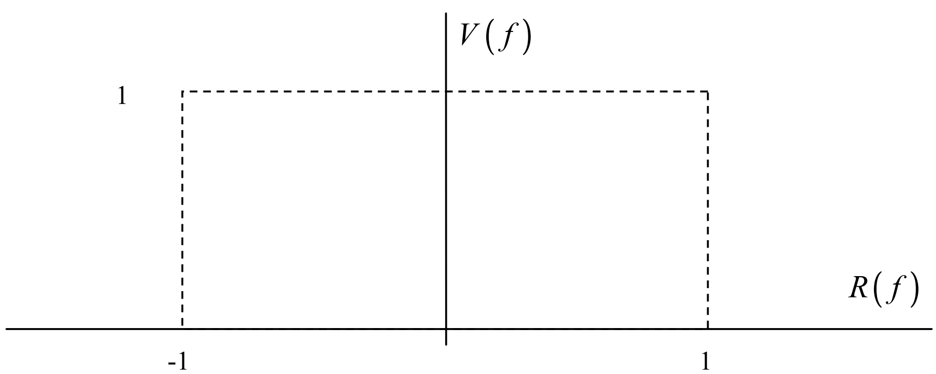


Figure 58. Region Defined by $R(f)$ and $V(f)$.

$$2\int_0^1 |f(x)-n(x)|dx \leq 2\sqrt{\int_0^1 1^2 dx}\sqrt{\int_0^1 |f(x)-n(x)|^2 dx} \tag{85}$$

$$2\int_0^1 |f(x)-n(x)|dx \leq 2\sqrt{\int_0^1 |f(x)-n(x)|^2 dx} \tag{86}$$

$$2\int_0^1 |f(x)-n(x)|dx \leq 2\sqrt{\frac{3}{3}\int_0^1 |f(x)-n(x)|^2 dx} \tag{87}$$

$$2\int_0^1 \left|f(x)-n(x)\right|dx \le \frac{2}{\sqrt{3}}\sqrt{3\int_0^1 \left|f(x)-n(x)\right|^2 dx} = \frac{2}{\sqrt{3}}\sqrt{V(f)} \tag{88}$$

So

$$R(f) \le \frac{2}{\sqrt{3}} V(f)^{1/2} \tag{89}$$

It may be that a single subject may provide elicited responses that are grouped with in the region. If so, it may be possible to draw inferences about subject's responses. For example, after sampling a number of evaluation measures for a subject, it may be inferred at some level of confidence that the additional measures will lay within some radius about the centroid of the current observations. If this is correct, then this may assist in efficiency of elicitations.

The data from the tactical questionnaires was analyzed to determine the $R(f)$ and $V(f)$ for each subject's evaluation measures. As the various $\hat{\underset{.}{u}}(v(x))$ functions are not standard functions, the analysis was performed numerically in Microsoft® Excel. The domain was divided into 100 intervals (except for the ambush evaluation measure, which was treated as integer units) and the functions assumed to be linear over the interval of each subdomain. The results are contained in Table 17. Of the 80 $\hat{\underset{.}{u}}(v(x))$ functions (four for each of the 20 subjects), 50 exhibited risk aversion, 15 were risk neutral, and 15 were risk seeking. Clearly the common assumption that risk aversion is present does not hold for these data where for almost two-fifths of the evaluation measures the subjects were not risk averse. A two-way analysis of variance provides *p* values of 0.058 for the subjects and 0.016 for the evaluation measures. The subjects could be pooled and treated as a group, but the difference of at least one subject

Table 17. Tactical Questionnaire Results for $R(f)$ and $V(f)$.

Subject Number	Artillery		Ambush		Fuel		Time	
	R(f)	V(f)	R(f)	V(f)	R(f)	V(f)	R(f)	V(f)
FJ1	-0.142	0.022	0.102	0.030	0.380	0.132	0.260	0.071
FJ2	0.143	0.039	0.939	0.930	0.373	0.551	-0.393	0.187
FJ3	-0.043	0.014	0.205	0.093	0.353	0.147	0.438	0.226
FJ4	-0.154	0.030	0.422	0.204	0.247	0.058	0.235	0.134
FJ5	-0.281	0.083	0.413	0.201	0.113	0.030	0.091	0.112
FJ6	0.373	0.149	0.767	0.591	-0.097	0.022	-0.276	0.074
FJ7	-0.083	0.036	0.418	0.182	-0.441	0.235	-0.039	0.052
FJ8	0.095	0.028	-0.035	0.015	0.310	0.140	-0.122	0.057
FJ9	-0.089	0.031	0.530	0.295	-0.238	0.075	-0.127	0.053
FJ10	0.138	0.079	0.239	0.049	0.031	0.031	0.080	0.048
FJ11	-0.197	0.069	0.424	0.206	0.640	0.425	0.357	0.170
FJ12	0.683	0.470	0.370	0.133	0.233	0.137	0.102	0.163
FJ13	0.065	0.065	-0.059	0.041	-0.158	0.043	-0.074	0.042
FJ14	0.016	0.065	0.568	0.336	0.175	0.030	0.228	0.136
FJ15	0.684	0.453	0.604	0.351	0.142	0.170	0.606	0.348
FJ16	0.314	0.108	0.599	0.368	0.077	0.033	0.204	0.075
FJ17	0.447	0.195	0.357	0.122	0.773	0.601	0.474	0.268
FJ18	-0.279	0.088	-0.594	0.342	0.436	0.186	-0.167	0.040
FJ19	-0.147	0.023	0.655	0.474	0.633	0.404	0.550	0.288
FJ20	0.388	0.143	0.526	0.282	0.251	0.144	0.377	0.200

approached significance. The evaluation measures differed significantly, indicating that the subjects' risk attitudes were not shared according to evaluation measure. For example, while the risk attitudes of the subjects were (barely) similar, no conclusion may be drawn that they were, for example, risk averse to a certain degree regarding the enemy artillery threat. A plot of $R(f)$ versus $V(f)$ appears as Figure 59.

As we have seen that knowledge of one subject's risk attitudes cannot provide information on another subject's attitude regarding that evaluation measure for this data set, we turn to examining the within subject risk attitudes. If the risk attitudes are known for a number of evaluation measures for a specific subject, what inferences may be drawn about the subject's risk attitudes on the remaining evaluation measures?

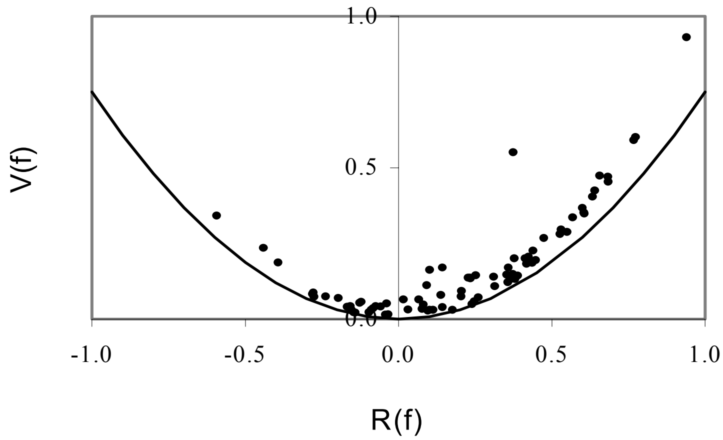


Figure 59. Plot of $R(f)$ versus $V(f)$ for the Tactical Questionnaire. Limit shown as solid line.

First examining this issue employing $R(f)$ alone, the ranges of the $R(f)$, $r = R^*(f) - R^0(f)$, where $R^*(f) = \max\left\{R(f_1), R(f_2), \ldots, R(f_j)\right\}$ and $R^0(f) = \min\left\{R(f_1), R(f_2), \ldots, R(f_j)\right\}$, for the subjects of the tactical questionnaire are listed in Table 18. As the ANOVA results permit pooling of the subjects' data, assume that the ranges *r* are identically independently distributed. Then the Central Limit Theorem provides that normality is present for sufficiently large samples. Using the tactical questionnaire sample of *r*, with 20 samples, it is determined that *r* is distributed with a mean of 0.63 and a standard deviation of 0.28. Constructing a 95 percent confidence interval for such a distribution provides a length of 1.10.

The interpretation of this calculation is that, for subjects for which these are representative, presented with the tactical questionnaire, once three utility functions have been elicited, the fourth lies within 0.505 of the arithmetic mean of the known $R(f)$

Table 18. Values of r, N, and Δ for Tactical Questionnaire Subjects, Ascending Order.

Subject	r	Subject	N	Subject	Δ
FJ10	0.21	FJ13	0.10	FJ13	0.06
FJ13	0.22	FJ10	0.14	FJ10	0.07
FJ20	0.28	FJ8	0.15	FJ20	0.09
FJ17	0.42	FJ1	0.23	FJ8	0.15
FJ8	0.43	FJ5	0.25	FJ3	0.17
FJ3	0.48	FJ9	0.27	FJ1	0.18
FJ1	0.52	FJ7	0.28	FJ4	0.18
FJ16	0.52	FJ3	0.29	FJ14	0.19
FJ15	0.54	FJ4	0.29	FJ16	0.20
FJ14	0.55	FJ14	0.29	FJ5	0.20
FJ4	0.58	FJ16	0.33	FJ17	0.20
FJ12	0.58	FJ18	0.41	FJ15	0.20
FJ5	0.69	FJ12	0.42	FJ12	0.23
FJ9	0.77	FJ20	0.43	FJ7	0.26
FJ19	0.80	FJ6	0.44	FJ9	0.27
FJ11	0.84	FJ11	0.46	FJ11	0.28
FJ7	0.86	FJ19	0.58	FJ19	0.32
FJ18	1.03	FJ17	0.60	FJ18	0.33
FJ6	1.04	FJ15	0.61	FJ6	0.43
FJ2	1.33	FJ2	0.64	FJ2	0.53

values with a probability of 0.95. The extreme length of this confidence interval, extending across almost one-half of the domain space, renders it rather useless.

An improved analysis would include $V(f)$. Two approaches are considered, one with respect to the origin and the other with respect to the average $R(f)$ and $V(f)$ for a subject. The former is a measure of the distance from overall risk neutrality for the subject. As we are working in two-dimensional Cartesian space, the total distance from the origin, N, is straightforward and is defined

$$N = \frac{1}{J}\sum_{j=1}^{J}\left[R(f_j)^2 + V(f_j)^2\right]^{0.5} \tag{90}$$

The distance from the average $R(f_\bullet) = \frac{1}{J}\sum_{j=1}^{J} R(f_j)$ and $V(f_\bullet) = \frac{1}{J}\sum_{j=1}^{J} V(f_j)$ is determined utilizing

$$\Delta = \frac{1}{J}\sum_{j=1}^{J}\left[\left(R(f_j) - R(f_\bullet)\right)^2 + \left(V(f_j) - V(f_\bullet)\right)^2\right]^{0.5} \tag{91}$$

The values of N and Δ are presented in Table 18 in ascending order. Inspection shows that these measures differ from each other as well as *r*. As measures, N and Δ provide more information and so are more appealing. Similarly as was done for *r*, it is determined that Δ is distributed with a mean of 0.23 and a standard deviation of 0.11, producing a 95 percent confidence interval of length 0.44. A method of utilizing this information would be for an individual sharing the characteristics of this group of subjects, elicit utility functions for three of the evaluation measures. The fourth utility function will possess $R(f)$ and $V(f)$ within a radius 0.22 of the mean of the three known functions with a probability of 95 percent. Unfortunately, this radius is also rather large, making this approach inoperable as a useful approach.

The idea that subjects are risk attitude consistent is not supported by the tactical questionnaire data. Thirteen of the 20 subjects exhibit $R(f)$ that change sign. Removing the two subjects for whom the sign changes take place within the risk neutral region, eleven subjects were risk seeking for at least one evaluation measure and risk averse on at least one other.

It is possible that future work with larger sample sizes and different decision situations could produce a confidence interval that is diminutive enough to be practical. Barring such improvements, the idea that a decision maker's known risk attitudes may provide insight into risk attitudes on other evaluation measures is not operational. The data of this study suggest that the idea is not sound, but rather that risk attitudes are

completely independent for different evaluation measures for a single individual, particularly with respect to risk aversion and risk seeking behavior.

We conclude this section by observing that $R(f)$ and $V(f)$ provide improved an improved metric for evaluating risk attitudes over both curve shape and preference area. Metrics for the assessment of risk attitude consistency are developed: r, N and Δ. Risk attitude consistency is not demonstrated by the tactical questionnaire data. Instead, the research subjects exhibit wide differences in risk attitude by evaluation measure.

Conclusions

This research supports the hypothesis that single dimensional value and utility functions are not equivalent. This conclusion was confirmed for two separate military decision making situations. No single functional relationship defined the relationship of value and utility functions. Certainty equivalent and probability equivalent and Multiattribute elicitation methodologies provided equivalent results. Value and utility functions differed significantly more than can be explained by elicitation error.

A new metric for assessing preference function fit, WRMSE, was developed. The frequent assumption of risk aversion was a poor one for this subject group. Risk attitudes were not found to be consistent between evaluation measures for the subjects. New metrics for assessing risk attitudes were presented.

V. Hybrid Value-Utility Decision Analysis Models

General

As discussed previously, decision analysis models may be utility-function-based, or value-function-based. Research in Chapter IV showed that the two models differ significantly. The utility-based model is appropriate when outcomes are not certain, as is typical for difficult decisions. As utility functions incorporate the risk attitudes of an individual, they are specific to an individual. Therefore construction of utility models requires a significant amount of the decision maker's time. For large decision problems, this personal commitment on the part of the decision maker is onerous and is a barrier to the use of multiple objective decision analysis. Value-based models, appropriate under conditions of certainty, may be constructed through consultation with appropriate experts rather than the decision maker. In fact, consultation with subject matter experts is generally required in any complex problem, regardless of methodology applied.

If a utility-based model may be approximated by modification of a value-based model, reduction of the time demands on the decision maker may be realized. For large, important decisions, these time demands are significant. Reducing the time required of the decision maker improves the acceptance of the procedure and improves the probability that decision analysis will be employed. The objective of this portion of the research is to determine when single dimensional value and utility functions differ significantly, and use this information to construct a hybrid value-utility model that may be used to estimate the performance of the utility model.

This chapter will first review pertinent background information. An algorithm will be presented for construction of the hybrid model. The algorithm will be tested on a simple example problem. Next, the algorithm will be used to extend work preformed for a client to select an automatic target recognition classification system. Finally the chapter is summarized and potential future work presented.

Background

Decision analysis models consist of a set of evaluation criteria for which preference functions are used to assess the decision maker's strength of preference for various levels of the criteria, and often also attitudes regarding the uncertainty of the decision situation. In a typical value-focused approach (Keeney, 1992a), the evaluation criteria are determined through decomposition of the decision maker's strategic priorities until measurable criteria, the evaluation measures, are obtained. This inverted dendritic structure, the value hierarchy, is shown in the upper portion of Figure 60. Each evaluation measure is assessed using a single dimensional preference function. Under conditions of certainty, the preference function is referred to as a value function and measures the decision maker's strength of preference. Under uncertainty, the preference function is referred to as a utility function and measures both the decision maker's strength of preference and her attitude toward the risk inherent in the uncertain decision context. The preference functions are contained in the second from bottom row of Figure 60. The various possible levels of the evaluation measure variables are represented as the bottom row of Figure 60. A multiattribute (multidimensional) function employing

relative weights (relative importance) of the various evaluation measures combines the single dimensional preference functions and completes the model, permitting evaluation of alternatives. Figure 60 represents a common view of decision analysis models, which assumes perfect information. (In fairness, it must be observed that sensitivity analysis is often used to examine the assumption of perfect information.)

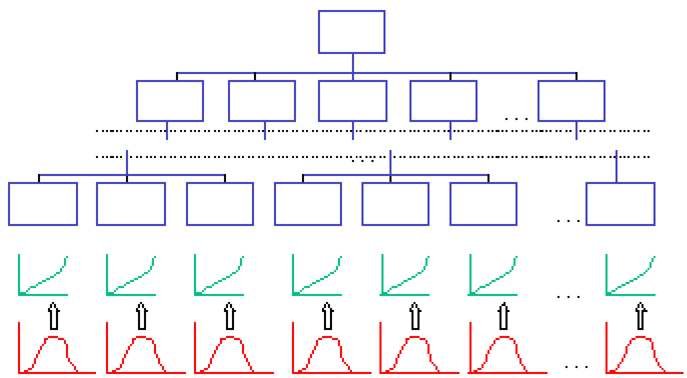


Figure 60. Common Decision Analysis Model Structure. The bottom tier represents the evaluation measures and the next tier the preference functions. The hierarchy represents the decomposed values of the decision maker.

Decision analysis models are actually more complex. This complexity is often largely ignored for tractability. Figure 61 shows the taxonomy of variation in a DA model. The bottom tier represents the evaluation measure levels. These levels may be fixed, under certainty when the evaluation measure levels are known, or stochastic, under uncertainty or risk. Under the latter condition the evaluation measure levels are distributed in a known or unknown way, respectively (usually referred to as under risk or uncertainty). Additionally, the evaluation measure levels are often a function of an alterative, with different alternatives possessing differing distributions. These distributions make the utility model a random variable.

Only one distribution is shown for each evaluation measure in the figure. The figure shows continuous distributions, but discrete distributions and simple scalars (no variation) are possible. The next higher tier represents the estimates of the true distributions. The third tier from the bottom illustrates the value functions. Above them are the value probability density function distributions created by transforming the evaluation measure distributions by the value functions. Above the value distributions are the utility functions. Two forms of utility functions are employed. The domain of one form is the evaluation measure level domain. The domain of the other is the value function codomain. Depending which form is used, differing prior information is required, as indicated by the arrows on the right. The utility functions provide utility probability density function distributions when the inherent uncertainty derived from the evaluation measures is transformed. Note that both value and utility functions produce an associated preference distribution. Each evaluation measure has an associated elicited weight. Elicitation of these weights involves elicitation error, so that the weights, w_i, are associated with some distribution and represented by an estimate, $\hat{w}_i$. For our purposes, in this document such error is disregarded, and $\hat{w}_i = w_i$. Notation is indicated on the right hand side of Figure 61.

Finally, a function is employed to aggregate the decision maker's strength of preference and risk attitudes across the various dimensions of the decision. A multiattribute function for a utility model is often written in the additive form. This is

$$U_j = \sum_{i=1}^{I} E\left[w_i u_i \left(x_i \right) \right] \tag{92}$$

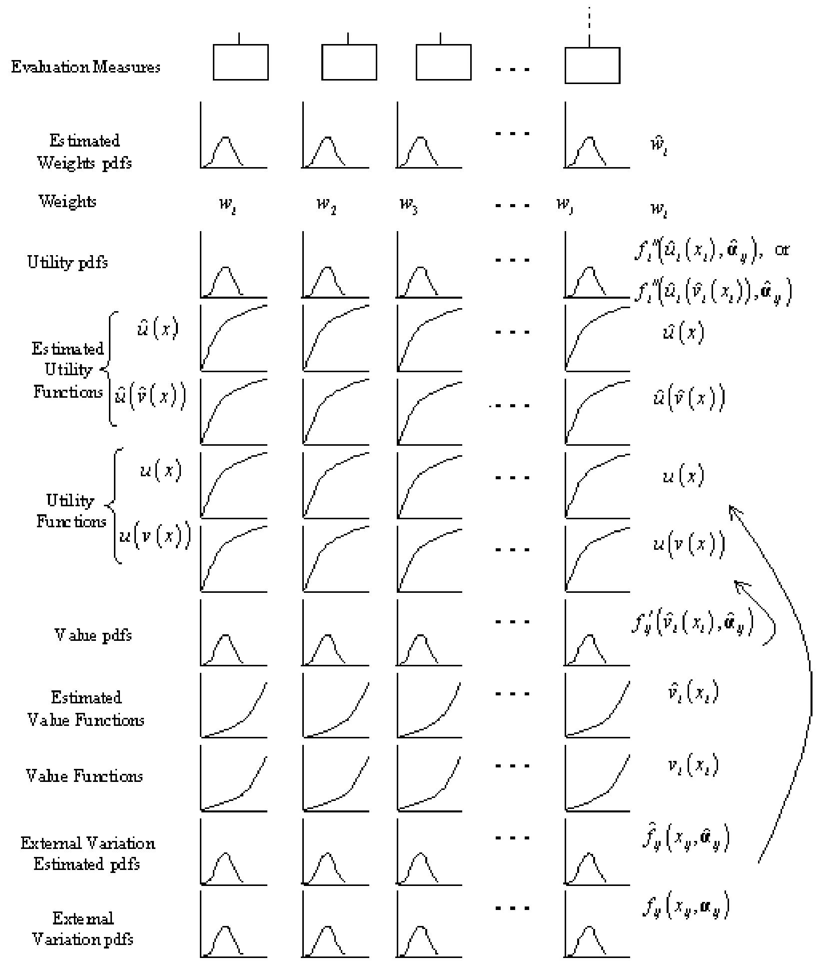


Figure 61. Taxonomy of Variables in a Decision Analysis Problem.

where U_j is the utility of the *j*th alternative, w_i is the weight (relative importance) of the *i*th evaluation measure, and $u_i(x_i)$ is the single dimensional utility function for the *i*th evaluation measure, x_i is the random variable for the *i*th evaluation measure, and I is the number of evaluation measures.

Equation (92) may be rewritten in terms of the utility probability density function, or

$$E\left[U_j\right]=E\left[\sum_{i=1}^{I} w_i f_i''\left(u_i\left(x_i\right),\hat{\boldsymbol{\alpha}}'_{ij}\right)\right]=\sum_{i=1}^{I} w_i E\left[f_i''\left(u_i\left(x_i\right),\hat{\boldsymbol{\alpha}}'_{ij}\right)\right] \tag{93}$$

where f_i'' is the probability density function of the ith utility function, and $\hat{\boldsymbol{\alpha}}'_{ij}$ is a vector of parameters of the estimated evaluation measure distributions. The DA problem may be stated as maximizing $E\left[U_j\right]$ through the selection of the jth alternative. More formally, we take the maximum

$$\text{Max } E\left[U_j\right] \tag{94}$$

such that

$$U_j \in \left\{U_1, U_2, \ldots, U_J\right\} \tag{95}$$

$$E\left[U_j\right]=E\left[\sum_{i=1}^{I} w_i f_i''\left(u_i\left(x_i\right),\hat{\boldsymbol{\alpha}}''_{ij}\right)\right] \tag{96}$$

$$i=1,\ldots,I \tag{97}$$

$$j=1,\ldots,J \tag{98}$$

Figure 62 illustrates an example where there are two alternatives, *A* and *B*. Choosing *A* or *B* is the only decision and is followed by a differing uncertain event. These events have distinct distributions of overall utility associated with it. In Figure 62, choosing *B* will provide a higher expected value of utility. Or to avoid the confusion associated with the statistical term “expected value,” one may say choosing *B* will provide a higher mathematical expectation of utility. More often, this is shortened to stating that choosing *B* will provide a higher expected utility.

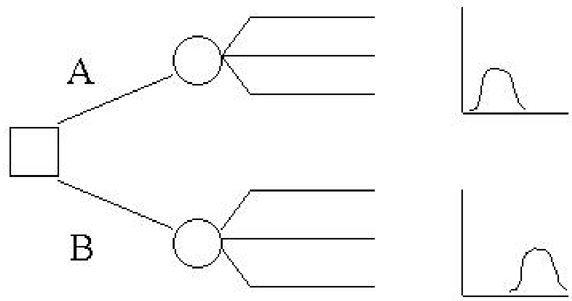


Figure 62. Example Decision Tree. Squares represent decisions, circles represent discretized uncertain events. The utility PDF for each choice is shown at right. These distributions are typically discretized for representation in decision analysis.

For a value-based model, the additive multiattribute function is

$$V_j = \sum_{i=1}^{I} w_i v_i \left(x_i \right) \tag{99}$$

where V_j is the value of the jth alternative and $v_i \left(x_i \right)$ is the single dimensional value function for the ith evaluation measure. Equation (99) contains no expectation operator, as it is intended for us under conditions of certainty. However, it may be seen from Figure 61 that using value functions under uncertainty produces a value pdf that behaves similarly to the utility pdf. Equation (99) then becomes

$$V_j = \sum_{i=1}^{I} E\left[w_i v_i \left(x_i \right) \right] \tag{100}$$

In analogous fashion, an objective value function may be constructed

$$\text{Max } E\left[V_j \right] \tag{101}$$

such that

$$V_j \in \{V_1, V_2, \dots, V_J\} \tag{102}$$

$$E\left[V_j\right] = E\left[\sum_{i=1}^{I} w_i f_i'\left(v_i\left(x_i\right), \hat{\boldsymbol{\alpha}}_{ij}'\right)\right] \tag{103}$$

$$i = 1, \dots, I \tag{104}$$

$$j = 1, \dots, J \tag{105}$$

where f_i' is the probability density function of the *i*th value function, and $\hat{\boldsymbol{\alpha}}_{ij}'$ is a vector of parameters of the estimated evaluation measure distributions. The DA problem may be stated as maximizing $E\left[V_j\right]$ through the selection of the *j*th alternative.

In uncertain decision contexts Equation (93) is appropriate but for large problems of great import the $u_i\left(x_i\right)$ are difficult to elicit from the decision maker because of the time involved. As risk attitudes vary within an organization, it is not acceptable to use a surrogate for the decision maker to elicit utility functions. Arguably, an organization should have a consensus for values in a decision situation, so parameters for Equation (99) are may be elicited from subject matter experts. Modifying Equation (99) provides an approach to estimate Equation (92) to within some specified accuracy. The advantage of employing a different model that adequately approximates Equation (92), but is based on Equation (99), is the reduction of the elicitation burden on the decision maker. This onerous burden sometimes either precludes employing a utility approach or causes the selection of non-decision analysis techniques, and has been identified as a limitation of this methodology. It is possible to do so as both f' and f'' map from the real numbers into the unit interval, as was presented above in the discussion of Figure 61.

The proposed procedure is depicted graphically in Figure 63. Starting with the value model, a hybrid model is created by replacing certain single dimension value functions with the corresponding utility functions. The resultant hybrid model is then used as would the utility model to analyze the decision situation.

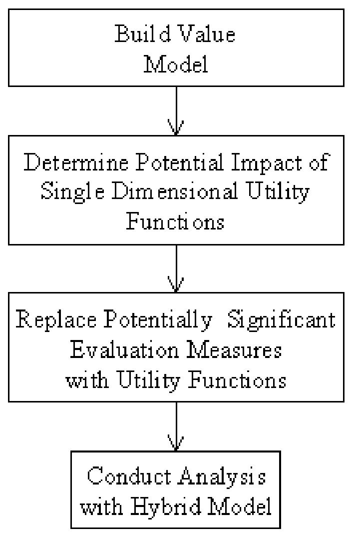


Figure 63. Basic Hybrid Value-Utility Model Approach.

Typically, the initial step of building the value model is done through interview of the decision maker. As we wish to minimize the time required of this individual, we will construct the value model using appropriate subject matter experts. The decision maker then should validate the value model. As we are concerned with complex decision situations, often we would expect the number of evaluation measures to be large. Group screening is a method that efficiently examines large numbers of variables. We anticipate that group screening will be employed as part of the step determining significance of the

evaluation measures. Further development of the methodology outlined in Figure 63 is provided below.

A Hybrid Value-Utility Algorithm

Algorithm Development. A method of approximating Equation (92) is desired. Beginning with Equation (99), one evaluation measure's value function, $v_k\left(x_k\right)$, is replaced with the appropriate utility function, $u_k\left(x_k\right)$. The model becomes

$$E\left[\hat{U}_j^{(1)}\right]=E\left[\sum_{i=1,i\neq k}^{I} w_i f_i'\left(v_i\left(x_i\right),\hat{\boldsymbol{\alpha}}_{ij}'\right)+w_k f_k''\left(u_k\left(x_k\right),\hat{\boldsymbol{\alpha}}_{ij}''\right)\right] \tag{106}$$

where the kth evaluation measure value function has been replaced with the corresponding utility function and $\hat{U}_j^{(1)}$ is an estimate of U_j after one substitution.

Equation (106) then becomes, after m substitutions,

$$E\left[\hat{U}_j^{(m)}\right]=E\left[\sum_{i=1}^{I}\left[\left(1-K_i\right)w_i f_i'\left(v_i\left(x_i\right),\hat{\boldsymbol{\alpha}}_{ij}'\right)+K_i w_i f_i''\left(u_i\left(x_i\right),\hat{\boldsymbol{\alpha}}_{ij}''\right)\right]\right] \tag{107}$$

where K_i is an indicator variable corresponding to the substitutions defined for the ith evaluation measure as

$$K_i=\begin{cases}0 & \text{when } v_i\left(x_i\right) \text{remains,}\\ 1 & \text{when } u_i\left(x_i\right) \text{substituted}\end{cases} \tag{108}$$

This substitution protocol continues in an iterative fashion until the estimated utility, $\hat{U}_j^{(m)}$ is sufficiently close to U_j. This may be stated as $\left|E\left[\hat{U}_j^{(m)}\right]-E\left[U_j\right]\right|\leq\delta$, where

δ is the level of accuracy desired. Then $E\left[\hat{U}_j^{(m)}\right] \approx E\left[U_j\right]$. Equation (107) is a hybrid value-utility model that estimates the true utility model behavior.

Typically decision analytic studies proceed using either a value or a utility based approach. Figure 64 illustrates that after the evaluation measure variation is estimated, the analyst adopts a preference model of value or utility. Using the selected preference function for each evaluation measure, the evaluation measure scores are transformed into the value or utility space. Typically these evaluation measures have uncertainty associated with them, so the various levels of value and utility have some associated probability. Figure 64 shows probability density functions for value and utility levels. As shown at the top of Figure 64, Equation (107) bridges the division between value and utility regions. In all cases in Figure 64, an additive value model is shown as an example.

Substitution order. Two additional considerations must be addressed in order to develop an algorithm. These are the order of the substitutions of single dimensional utility for single dimensional value functions, and when the substitutions may be halted. The interview session time cost of each substitution (each evaluation measure) is roughly equal because the procedure is identical. Some evaluation measures may require more reflection or definition, and so require longer elicitation periods. This time cost is not considered in this analysis. With this simplification, the substitutions should be made in order of decreasing impact upon the hybrid model (107). The substitutions may be stopped when sufficient accuracy is achieved, as discussed below.

Several candidates exist for prioritizing the substitutions. The relative weight, w_i, is an obvious possibility. However the contribution of each evaluation measure is the

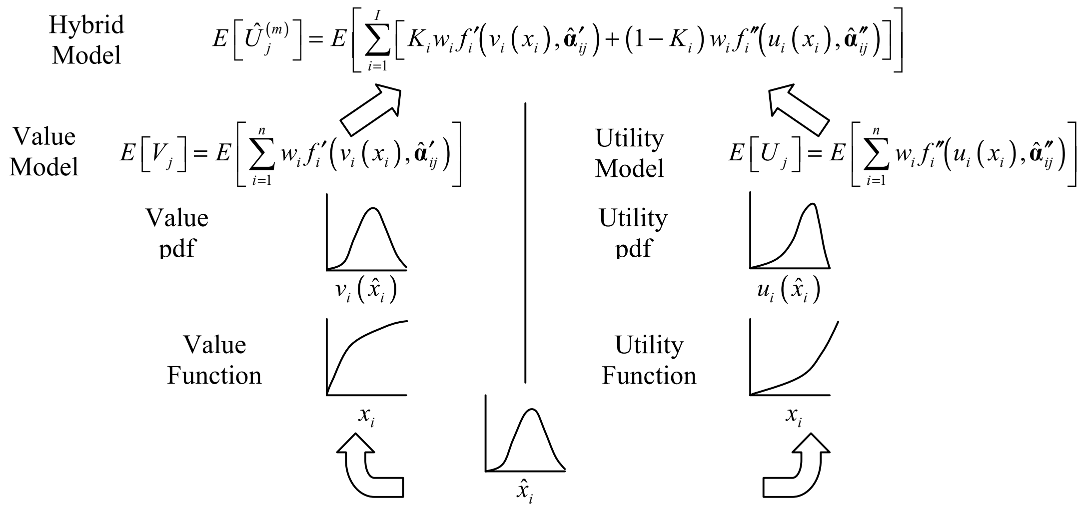


Figure 64. Bifurcation of Preference Functions.

product of the weight multiplied by the corresponding preference function. Evaluation measures with high relative importance may not be significant contributors to the overall value or utility. That is, an ordering established by w_i may be very different than an ordering based on $w_i v_i\left(x_i\right)$. While the product $E\left[w_i v_i\left(x_i\right)\right]$ may be calculated for each evaluation measure for the alternatives, this is not a good predictor of $E\left[w_i u_i\left(x_i\right)\right]$. As was shown in the last chapter, generally $v_i\left(x_i\right) \neq u_i\left(x_i\right)$, so value is a poor predictor of utility.

While assumptions like consistent risk attitude across evaluation measures may be used to improve the estimate of $E\left[w_i u_i\left(x_i\right)\right]$, response surface methodology (RSM) provides a means to assess the potential relative contributions of $E\left[w_i u_i\left(x_i\right)\right]$ without introducing additional constraints of the decision maker’s risk attitudes. The coefficients

of the regression model produced by RSM indicate the potential contribution of each model variable and permit the prioritization of substitution of the evaluation measures.

The single dimensional preference functions transform the inherent problem variation into value and utility probability density functions. While usually not directly considered, these derived PDFs affect the decision analysis model. Differing single dimensional utility functions produce differing utility PDFs, as shown in Figure 65.

Frequently, utility functions are approximated employing parametric functions fitted to the data (Kirkwood, 1997: 65). When the exponential function,

$$\hat{u}_i\left(v_i\left(x_i\right)\right)=\frac{1-e^{-v_i(x_i)/\rho_i}}{1-e^{-1/\rho_i}} \tag{109}$$

is used to approximate a utility function the exponential constant, ρ, may be modified to examine an envelope of utility curves. Setting $\rho_i=\infty$ for $\hat{u}_i\left(v\left(x_i\right)\right)$ establishes that there are no risk attitudes and therefore $\hat{u}_i\left(v\left(x_i\right)\right)=v_i\left(x_i\right)$. Varying ρ_i permits employment of response surface methodology to determine whether each $\hat{u}_i\left(x_i\right)$ potentially exerts a significant effect on the hybrid model. Kirkwood (1997b: 66) indicates that the absolute value of ρ is typically greater than one-tenth of the evaluation measure domain, or $\rho_i \geq \left|\frac{\max x_i-\min x_i}{10}\right|$. The region bounded this relationship is illustrated in Figure 66. Accepting the limits proposed by Kirkwood means that if we examine the two $\hat{u}_i\left(x_i\right)$ for $\rho=\pm 0.1$, and they do not significantly affect the hybrid model, then the preference function for the i th evaluation measure may remain a value

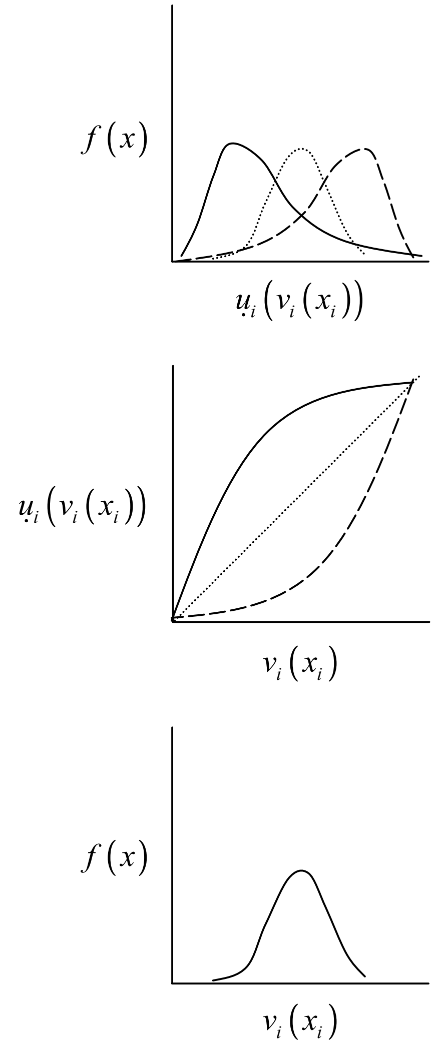


Figure 65. Affect of Various Utility Functions of the Utility PDF.

function. We do not expect the utility function for that evaluation measure to significantly affect the hybrid model.

The limits proposed by Kirkwood, coupled with RSM, provide an excellent way to assess the potential significance of the utility functions. Therefore RSM is a desirable methodology to use to order the substitution of the evaluation measures for the hybrid utility model algorithm.

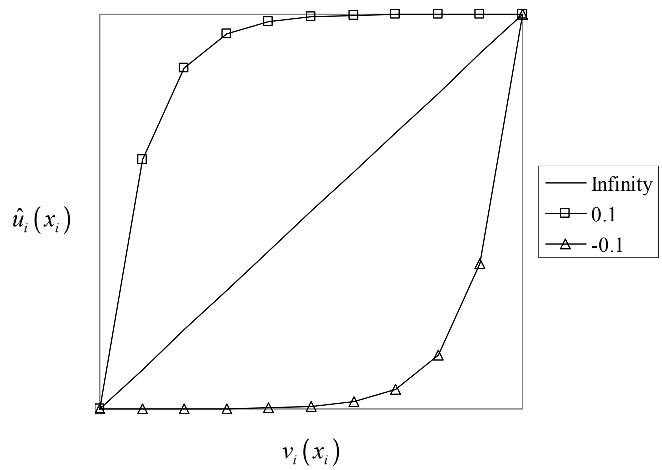


Figure 66. Exponential Utility Function Limits. The straight line, where $u_i(x_i) = v_i(x_i)$, occurs when $\rho = \infty$. The other curves show $u_i(x_i)$ when $\rho = \pm 0.1$.

Stopping Criteria. Replacing all potentially significant evaluation measures will ensure that the hybrid model adequately represents the utility model. However, it is desired to halt the iterative substitutions when $\left| E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right] \right| \le \delta$, which may occur before all the evaluation measures have been converted. We may write the problem as a mathematical programming problem. The optimization model is:

$$\min\ m = \sum_{i=1}^{I} K_i \tag{110}$$

subject to

$$\left| E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right] \right| \le \delta \ \forall j \tag{111}$$

$$\delta \in [0,1] \tag{112}$$

However, $E\left[U_j\right]$ is unknown, so it is not directly possible to determine when $E\left[\hat{U}_j^{(m)}\right]$ is within δ of $E\left[U_j\right]$. When there is consistent risk aversion (risk affinity) then the set

of all possible $E\left[\hat{U}_j^{(m)}\right]$, S_j, is bounded below (above) by V_j and above (below) by $E\left[U_j\right]$. See Figure 67 for examples of behavior of preference function under various risk attitudes. When risk aversion is present for the *i*th evaluation measure, $u_i\left(v_i\left(x_i\right)\right) \geq v_i\left(x_i\right)$. The reverse is found under risk seeking behavior. Mixed risk attitudes, as typically encountered in this research (see Chapter IV), preclude such general conclusions. Figure 68 shows iterative ordered substitutions of decreasing impact under conditions of risk aversion and risk seeking behavior.

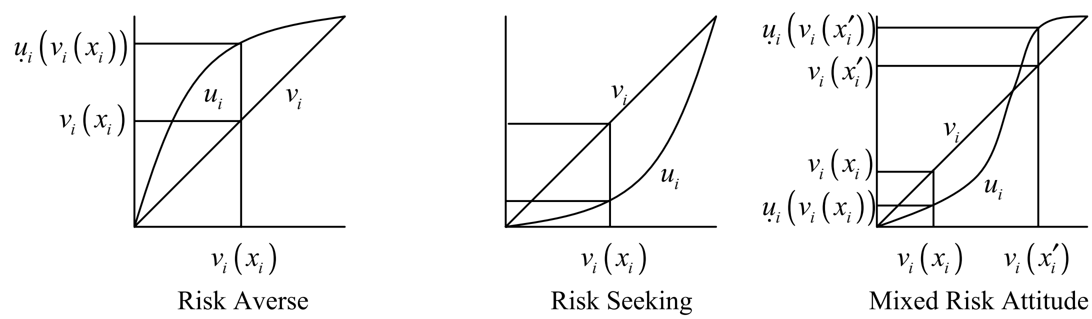


Figure 67. Risk Attitudinal Effects on the Relationship of Value and Utility Functions.

Definition (Substitution Distance). Given a series of substitutions to create hybrid value-utility models for the *j*th alternative, $S_j = \left\{V_j, \hat{U}_j^{(1)}, \hat{U}_j^{(2)}, \ldots, \hat{U}_j^{(m)}\right\}$, the distance between successive substitutions is given by $\lambda_j^{(m)}$, defined as

$$\lambda_j^{(m)} = \left|E\left[\hat{U}_j^{(m)}\right] - E\left[\hat{U}_j^{(m-1)}\right]\right| \tag{113}$$

where *m* indicates the number of substitutions, and $E\left[\hat{U}_j^{(0)}\right] = E\left[V_j\right]$.

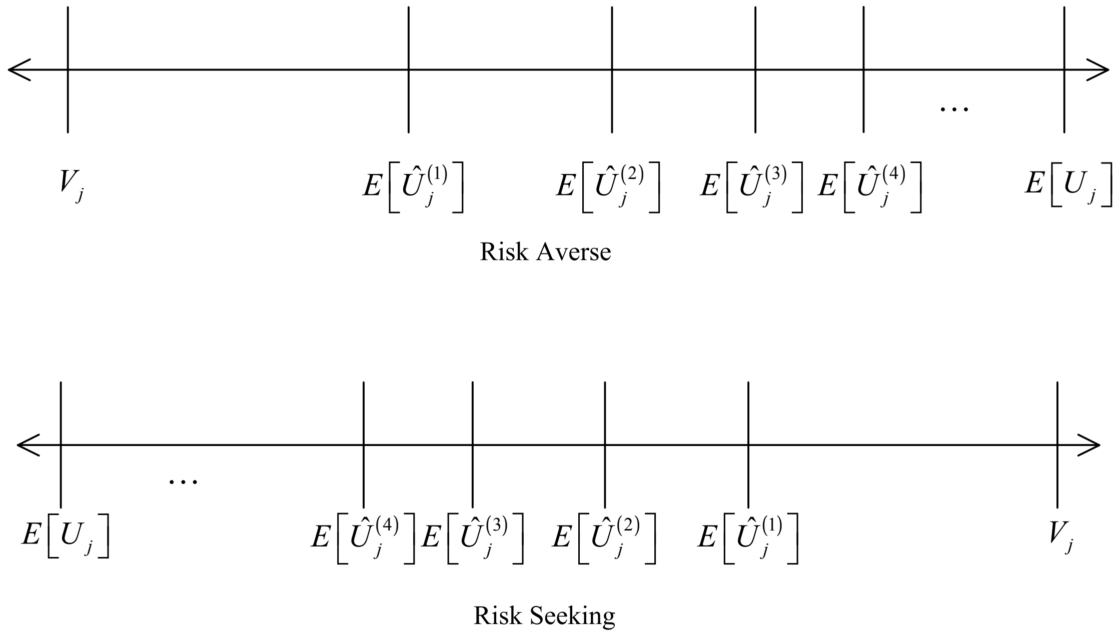


Figure 68. Iterative Decreasing Ordered Substitutions. Shown are the substitutions for a single alternative, *j*.

When the set S_j is ordered with decreasing intervals between iterative estimates, $\lambda_j^{(m)} \le \lambda_j^{(m-1)}, \forall m \in \{2,3,\ldots,I\}$, then S_j is a partially ordered set. Under this condition, when $(I-m)\lambda_j^{(m)} \le \delta$, then $\left|E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right]\right| \le \delta$. When $(I-m)\lambda_j^{(m)} \le \delta$ is encountered the substitutions of single dimensional utility functions for value functions is halted and the current $E\left[U_j^{(m)}\right]$ accepted as accurately estimating $E\left[U_j\right]$.

There are a number of potential alternatives of interest, not just alternative *j*. Define the vectors $\mathbf{U}$ and $\hat{\mathbf{U}}^{(m)}$

$$\mathbf{U} = \begin{bmatrix} U_1 \\ \vdots \\ U_j \\ \vdots \\ U_J \end{bmatrix} \text{ and } \hat{\mathbf{U}}^{(m)} = \begin{bmatrix} \hat{U}_1^{(m)} \\ \vdots \\ U_j^{(m)} \\ \vdots \\ U_J^{(m)} \end{bmatrix} \qquad (114)$$

Clearly it is not necessary to continue iterations, and so to increment *m*, when $\left|E\left[\hat{\mathbf{U}}_j^{(m)}\right]-E\left[\mathbf{U}_j\right]\right| \le \delta\ \forall j$. What is of interest is, naturally, the alternative with the greatest utility. Therefore we are satisfied when the alternative with the greatest utility is alone within the accepted tolerance. Define $\hat{U}_*^{(m)} = \max\left\{\hat{U}_1^{(m)}, \hat{U}_2^{(m)}, \cdots, \hat{U}_J^{(m)}\right\}$ where *m* is the most recent iteration. Define $\lambda_*^{(m)} = \left|E\left[\hat{U}_*^{(m)}\right]-E\left[\hat{U}_*^{(m-1)}\right]\right|$ where * represents the evaluation measure with the greatest utility in the last, *m*th, iteration. The mathematical programming formulation now becomes:

$$\min\ m = \sum_{i=1}^{I} K_i \tag{115}$$

subject to

$$\left(I-m\right)\lambda_*^{(m)} \le \delta \tag{116}$$

$$\delta \in \left[0,1\right] \tag{117}$$

When S_j is partially ordered, *m* is minimized when equation (116) is first found to hold. Stop the algorithm when $\left(I-m\right)\lambda_*^{(m)} \le \delta$ and accept $E\left[U_*^{(m)}\right]$ as accurately estimating $E\left[U_*\right]$ and the alternative corresponding to $U_*^{(m)}$ as being the optimal choice.

Group Screening. As stated previously, the class of decision analysis problems considered on which these techniques are likely to be beneficial are those with a large number of evaluation measures. For such problems, group screening is likely to be required. The group screening employment is as traditionally employed as presented in Chapter II. Once the screening is completed, the information must be re-aggregated, which is a new contribution.

Employing groups screening permits examination of a large number of variables through decomposing the independent variables into subsets. Once the groups screening is completed, it is desired to reassemble the variables into a single set, rank ordered by the degree of affect each has on the model. The utility model is

$$\underset{\cdot}{\hat{U}} = \sum_{i=1}^{n} w_i \underset{\cdot}{\hat{u}}_i \left(v_i \left(x_i \right) \right) \tag{118}$$

where, with the assumption of the exponential single dimensional model, the ith evaluation measure is determined by

$$\underset{\cdot}{\hat{u}}_i \left(v_i \left(x_i \right) \right) = \frac{1 - e^{-v_i(x_i) / \rho_i}}{1 - e^{-1 / \rho_i}} \tag{119}$$

When the *k*th element of (118) is examined separately, the model becomes

$$\underset{\cdot}{\hat{U}} = \sum_{i=1}^{k-1} w_i \underset{\cdot}{\hat{u}}_i \left(v_i \left(x_i \right) \right) + w_k \underset{\cdot}{\hat{u}}_k \left(v_k \left(x_k \right) \right) + \sum_{i=k+1}^{n} c_i x_i \tag{120}$$

As equation (118) may be represented by a fit of a linear model based on the exponential constant

$$\hat{U} = c_0 + \sum_{i=1}^{n} c_i \rho_i \tag{121}$$

the *k*th element of (120) may be decomposed from (121) and modeled by a least squares model

$$u_k = c_0 + \sum_{i=1}^{k-1} c_i \rho_i + c_k \rho_k + \sum_{i=k+1}^{n} c_i \rho_i \tag{122}$$

The coefficient c_k provides a measure of importance of ρ_k relative to the other ρ_i.

If the *k*th element has been selected to be treated as a group of variables for screening, then it becomes necessary to decompose the *k*th group when it is determined to be significant. Because the *k*th group may also be modeled by

$$u_k = c_{k0} + \sum_{i=1}^{n} c_{ki}\rho_{ki} \tag{123}$$

the contribution to the overall model for the ρ_{km} th variable is $c_k c_{km}$. Similar combinations are made for all grouped variable coefficients to permit comparison of evaluation measure priorities. The relative importance of utility functions for the evaluation measures is provided by the corresponding $c_k c_{km} \cdots c_{k(r-1)} c_{kr}$ where *r* is the number of screening sets in which the *k*th evaluation measure has been grouped. Rank ordering the evaluation measures by relative importance provides the order for the substitutions from single dimensional value function to single dimensional utility functions when the hybrid model is created.

The Detailed Algorithm. Refining the basic concept provided in Figure 63 by minimizing the interactions required with the decision maker, incorporating group screening, and using response surface methodology to assess potential significance provides the algorithm presented in Figure 69.

The steps of this protocol are:

1. Build Value Model.
   a. Establish the multiattribute functional form. Typically the simple additive form is used, but the algorithm is independent of the

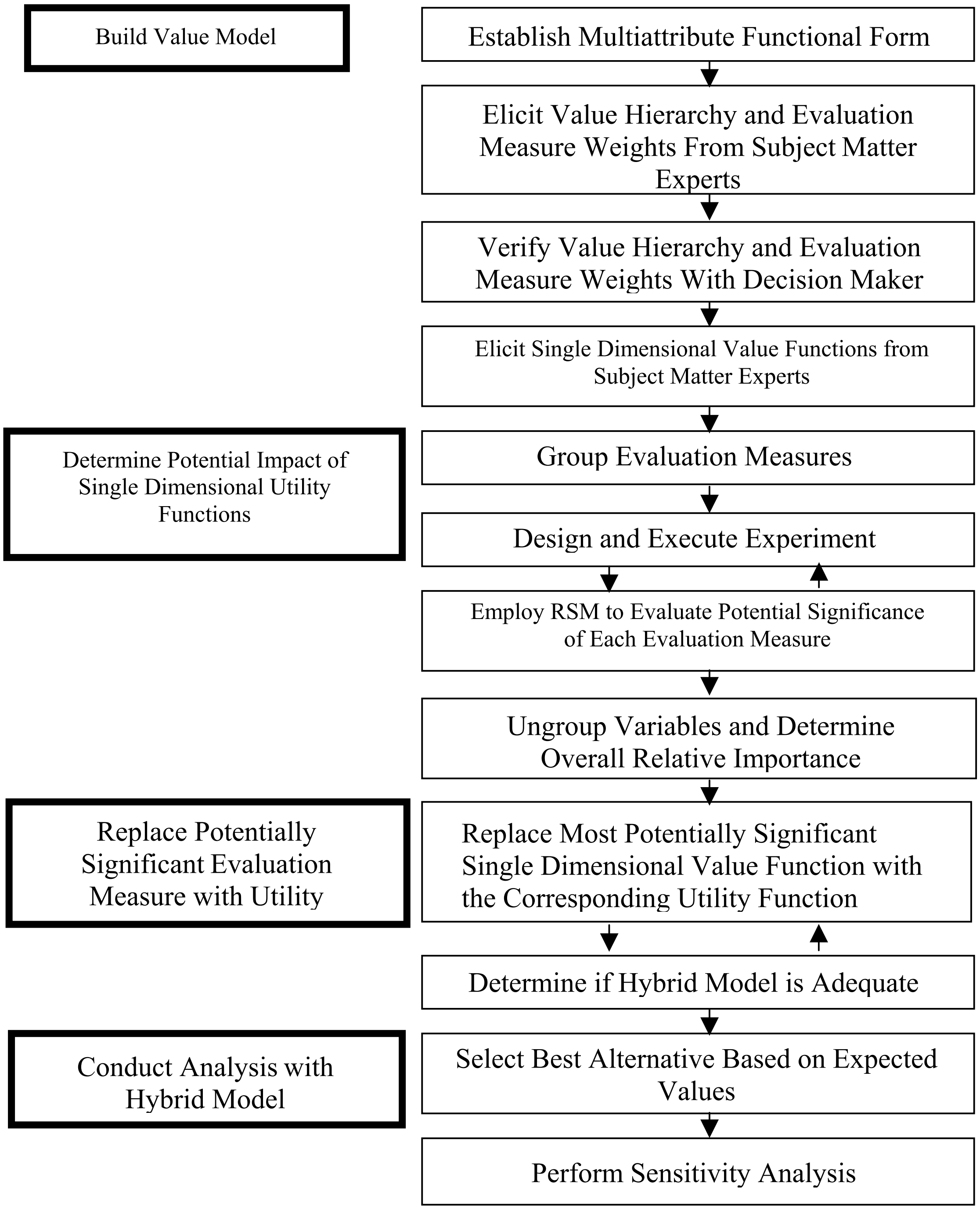


Figure 69. Steps to Create Hybrid Value-Utility Model.

multiattribute functional form. A constraint on the algorithm is that the value model and the utility model must be of the same form.

b. Elicit value hierarchy and evaluation measure weights, w_i, from subject matter experts. This avoids a significant time burden being placed upon the decision maker.

c. Verify the value hierarchy and evaluation measure weights with decision maker. Clearly the model must be acceptable to the decision maker. Verification at this point provides efficient elicitation of the single dimensional value functions.

d. Elicit $v_i\left(x_i\right)$ for all *i* from subject matter experts.

2. Determine potential impact of single dimensional utility functions.

   a. Group $v_i\left(x_i\right)$. Grouping is done to permit group screening, which allows examination of large numbers of evaluation measures. The value hierarchy provides a ready framework for grouping of variables. An obvious approach is to group variables at the highest partitioning. Each group may be subsequently examined, employing group screening as required. An additional consideration is that if any interactions are believed to be present, the groupings should not separate those variables.

   b. Develop and execute experimental design. Design of experiments permits the significance of the predictor variables to be assessed

with the minimum number of experimental runs. A run in this sense, is setting the variables at some level and observing the expected utilities of the various alternatives. The variables of interest with respect to the hybrid model are the exponential constants of the $\hat{\underset{.}{u}}_i\left(v_i\left(x_i\right)\right)$ functions. As the exponential constants assume values of $\rho = \{-0.1, \infty, 0.1\}$, the fractional factorial with center point designs are attractive. The experimental design is complicated by the consideration of the set of alternatives. The alternative choice may be included as a variable in the experimental design. If the set is large, the number of available experimental designs is small. This step and the following step are repeated for each grouping of variables.

c. Employ RSM. Examine potential contribution of $u_i\left(x_i\right)$ in the current model in place of $v_i\left(x_i\right)$ by employing response surface methodology to examine $\hat{\underset{.}{u}}_i\left(v_i\left(x_i\right)\right)$. This analysis provides regression coefficients showing the contribution of each variable as well as significance information. This provides data that facilitates determining the evaluation measures that need to be considered for conversion into utility, at some defined alpha level, as well as straightforward prioritization information. This step is done for each grouping of variables.

d. Ungroup $v_i(x_i)$. The RSM information from the previous step is ungrouped to provide a common comparison for all evaluation measures. Linear regression coefficients are multiplied by the coefficients for all parent groups to provide a single rank-ordered list of the potentially significant evaluation measures.

3. Replace potentially significant evaluation measures with utility functions. In priority order, perform iterative substitutions. This is done by:
   a. Set iteration counter $m = 1$
   b. Select the remaining $v_i(x_i)$ from the list of potentially significant evaluation measures with the greatest coefficients from the RSM model.
   c. Elicit $u_i(x_i)$ for this evaluation measure.
   d. Determine $\hat{U}_j^{(m)}$ for all *j* by substituting the $u_i(x_i)$ from the previous step.
   e. If $m = 1$ or if either stopping criteria is not met, increment $m = m + 1$ and return to step b. Otherwise stop and $E\left[\hat{U}_*^{(m)}\right] \approx E[U_*]$. The stopping criteria are:
      i. All significant evaluation measures, as determined by RSM, have been converted from $v_i(x_i)$ to $u_i(x_i)$.
      ii. $E\left[\hat{U}_j^{(m)}\right]$ is sufficiently close to $E\left[U_j\right]$.
4. The hybrid model is now used as would a standard utility model to:

a. Select the best alternative, $\hat{U}_*^{(m)}$, from an expected value standpoint, and

b. Perform a sensitivity analysis.

Example

Problem Statement. This example provides a simple situation with two alternatives and two evaluation measures. Consider a project where a parking lot must be constructed. As a lot must be constructed for some reason, the alternative of the status quo is not acceptable. The decision maker's value hierarchy is provided in Figure 70. There are only two evaluation measures, cost and time. Both are uncertain. Two alternatives have been identified. Alternative *A* is to employ in-house construction assets. Alternative *B* is to hire a contractor. Alternative *A* is less expensive, but will take considerably longer. Evaluation measure distributions are as provided by Law and Kelton (1991, 331 – 334). The cost of Alternative *A*, measured in \$k, is estimated to behave in accordance with the gamma(3,2) distribution. The cost of Alternative *B* is estimated by the Weibull(5,10). Time for project completion, measured in weeks, is estimated to follow the Weibull(3,8) distribution for Alternative *A* and the gamma (2,1) for Alternative *B*. The relative weights of cost and time have been determined to be 0.667 and 0.333, respectively. The ranges for cost and time are [0, 20] for both evaluation measures. These data and the preference functions are summarized in Table 19. As per the class of problems under discussion, the utility functions are unknown,

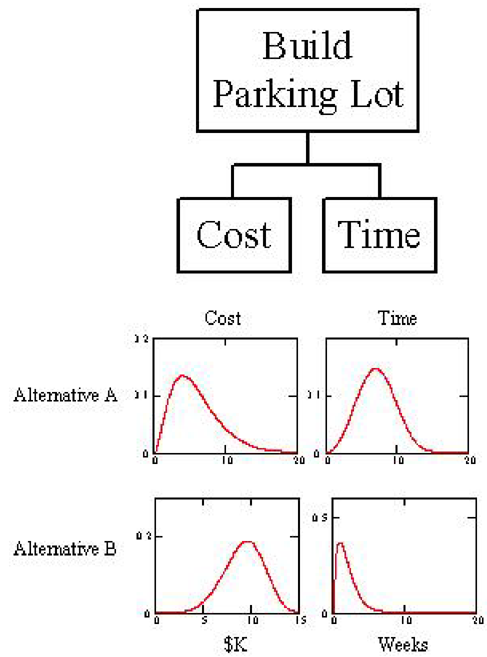


Figure 70. Value Hierarchy and Evaluation Measures for Parking Lot Example.

however they are listed in Table 19 for this example. They are exponential in form, with $\rho_1 = 8$ and $\rho_2 = 2$. The balance of the information is known. The issue is which, if any, of the utility functions must be discerned in order to adequately model the problem within the desired accuracy. The value/utility estimate should agree to within 0.10 of the true problem utility.

While this problem is small, with only two evaluation measures, the class of problems of interest consists of problems with many evaluation measures. Often the influential evaluation measures are a small subset. This is recognized in decision analysis literature as well as response surface methodology literature. In RSM, the Scarcity-of-Effect Principle indicates that a system is usually significantly influenced by a few main effects and low-order interactions (Myers and Montgomery, 1995: 134). For efficiency

Table 19. Parking Lot Example Information.

	Cost, x_1	Time, x_2
Alternative A, in-house	gamma(3,2)	Weibull(3,8)
Alternative B, contractor	Weibull(5,10)	gamma(2,1)
Range	[0, 20]	[0, 20]
Value Function	$v_1(x_1) = 1 - x_1/20$	$v_2(x_2) = \frac{1-e^{-(20-x_2)/10}}{1-e^{-20/10}}$
Utility Function (unknown)	$u_1(x_1) = \frac{1-e^{-(20-x_1)/8}}{1-e^{-20/8}}$	$u_2(x_2) = \frac{1-e^{-(20-x_2)/2}}{1-e^{-20/2}}$
Weight	$w_1 = 0.667$	$w_2 = 0.333$

in analysis it is desirable to screen the variables to reduce those under consideration by elimination of those that fail to influence the problem.

In RSM, screening experiments typically fit a first order polynomial that approximates the response to the predictor variables. Screening experiments reduce the number of variables by identifying those that do not significantly affect the system under consideration. These nonsignificant variables may be eliminated from consideration. Typically in RSM, additional analysis is conducted employing higher order polynomials to better fit the response surface. Such modeling is not required for the hybrid value-utility algorithm.

For this problem a pre-screen was used where some stochastic variables were treated in a deterministic fashion as an excursion. Normally variables are not treated deterministically. Additionally, the small number of variables in this example problem does not require group screening techniques, so they are omitted.

Pre-Screen Phase. Prior to screening the variables, a pre-screening experiment was performed as an excursion. Rather than examining the stochastic variables, they

were treated deterministically as an approximation. The intent was to compare the results of this deterministic approximation to a normal screening approach.

Because this example problem has few variables, no group screening is required. Typical experimental designs are sufficient. The pre-screen design selected was a 2^k plus center point. This design consists of each variable set at two levels, high and low, plus the center point, the mid point between all variables. The design is referred to as a 2^k design as the number of required runs is equal to 2^k where *k* is the number of variables, plus the center point runs. The cost and time variables were treated in a deterministic fashion for the prescreen analysis. The high and low values were those associated with probabilities of 0.4 and 0.6, respectively. That is, each x_i were selected such that $\int_0^{x_i} f(y)\,dy = 0.4$ for the low point (represented by "-") and $\int_0^{x_i} f(y)\,dy = 0.6$ for the high (represented by "+") point. The midpoint (represented by "0") was x_i such that $\int_0^{x_i} f(y)\,dy = 0.5$. These probabilities produce different x_i for the two alternatives, so an indicator variable, *a*, was used to designate which alternative pertained to each run. The utility functions were varied by changing the coefficient of the exponential function, ρ, for each evaluation measure. For the high point, $\rho_i = 0.1$. For the low point, $\rho_i = -0.1$ At the midpoint, $\rho_i = \infty$. The input data is displayed in Table 20. The symbols "+" indicate that the high level of the variable is to be used, "-" the low level, and "0" indicates the midpoint or nominal datum.

For each row of the experimental design matrix, the variables are set at either the high or low level, or at the center point. The variables are inserted in the utility function

Table 20. Pre-Screen Experiment Input Data.

		Alternative, a	
		A	B
Cost, x_1	0	5.3905	9.285
	+	6.211	9.827
	-	4.57	8.743
Time, x_2	0	7.0825	1.67
	+	7.77	2.022
	-	6.395	1.376
Utility, ρ_1, ρ_2	0	∞	∞
	+	0.1	0.1
	-	-0.1	-0.1

where the value function is as indicated in Table 19 and the utility function is

$$\underset{\cdot}{u}_i\left(v_i\left(x_i\right)\right)=\frac{1-e^{-\left(1-v_i\left(x_i\right)\right)/\rho_i}}{1-e^{-1/\rho_i}} \tag{124}$$

and the objective function is

$$\hat{\underset{\cdot}{U}}=\sum_{i=1}^{I} w_i \cdot \underset{\cdot}{u}_i\left(v_i\left(x_i\right)\right). \tag{125}$$

The design matrix and the results are presented in Table 21. The variable settings are indicated in order of the cost, time, cost utility exponential coefficient, time utility exponential coefficient, and alternative. As above, the variable setting of high is indicated by a plus sign, low by a minus sign, and center points by a zero.

A least squares model was fitted to the $\underset{\cdot}{U}$ data in Table 21. Figure 71 shows the observed data, $\underset{\cdot}{U}$, plotted against the predicted values, $\hat{\underset{\cdot}{U}}$. The two points that lie outside of the 95 percent confidence bounds are the center points, and indicate lack of fit of the model to the data. Despite this, the R^2 value for the model was 0.99, the adjusted

Table 21. Pre-Screening Experimental Design and Results.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	-----	0.068	18	+---+	0.00488
2	----+	0.00840	19	+--+-	0.337
3	---+-	0.375	20	+--++	0.12
4	---++	0.123	21	+-+--	0.667
5	--+--	0.667	22	+-+-+	0.663
6	--+-+	0.665	23	+-++-	0.974
7	--++-	0.974	24	+-+++	0.778
8	--+++	0.78	25	++---	0.03
9	-+---	0.68	26	++--+	0.00488
10	-+--+	0.00841	27	++-+-	0.351
11	-+-+-	0.389	28	++-++	0.163
12	-+-++	0.166	29	+++--	0.667
13	-++--	0.667	30	+++-+	0.663
14	-++-+	0.667	31	++++-	0.988
15	-+++-	0.988	32	+++++	0.821
16	-++++	0.823	33	0000+	0.414
17	+----	0.03	34	0000-	0.625

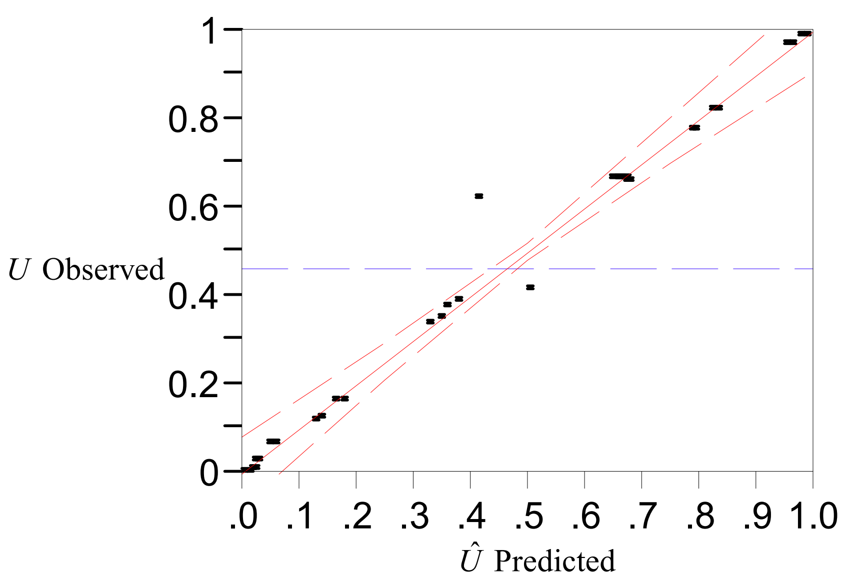


Figure 71. Prediction Versus Actual Data Plot for Pre-Screen Experiment.

R^2 was 0.97. Despite the lack of fit at the center points, analysis was continued. The analysis of variance (ANOVA) results are provided in Table 22. The *p* value for the F test indicates that the model is highly significant. This means that the model provides a significantly better estimate of the problem than does the overall mean. Parameter estimates are contained in Table 23. The two utility coefficients were significant, indicating that the utility functions are important, at least at the margins of the utility function envelope. Had this been a large model, likely not all utility functions would have been significant, permitting removing them from consideration for the hybrid utility model. The alternatives were significant. Only one interaction, the time utility and the alternative were significant.

Table 22. ANOVA Results from Model Fitted to Pre-Screen Data.

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	15	3.80	0.253	80.285
Error	18	0.0569	0.00316	Prob > F
C. Total	33	3.86		<.0001

Screening Experiment. A standard screening experiment was conducted. As mentioned in the prescreen section, the few variables of this example mean that no groups screening is required. Typical experimental designs where the variables are considered individually are sufficient. The experimental design was a full factorial plus center point replications. The center point was repeated twice with each alternative, for a total of four center point runs. The cost and time evaluation measures were handled normally as random variables. The variables in the experiment are the exponential

Table 23. Parameter Estimates. Variable x_1 represents cost, x_2 time, x_3 the cost utility coefficient, x_4 the time utility coefficient, x_5 and the alternative, a.

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.463	0.00964	48.02	<.0001
Cost	-0.00541	0.00994	-0.54	0.593
Time	0.00713	0.00994	0.72	0.483
ρ_{Cost}	0.319	0.00994	32.09	<.0001
ρ_{Time}	0.113	0.00994	11.33	<.0001
a	0.0462	0.00964	4.80	0.0001
Cost * Time	3.13e-8	0.00994	0.00	1.0000
Cost * ρ_{Cost}	0.00491	0.00994	0.49	0.628
Time * ρ_{Cost}	-3.44e-7	0.00994	-0.00	1.0000
Cost * ρ_{Time}	0.0000328	0.00994	0.00	0.997
Time * ρ_{Time}	0.00712	0.00994	0.72	0.483
ρ_{Cost} * ρ_{Time}	0.000018	0.00994	0.00	0.999
Cost * a	-0.00409	0.00994	-0.41	0.685
Time * a	-0.00363	0.00994	-0.36	0.719
ρ_{Cost} * a	-0.00986	0.00994	-0.99	0.334
ρ_{Time} * a	0.0444	0.00994	4.47	0.0003

coefficients for the evaluation measures, the cost weight variable, and a variable assigned for the alternative. Only a single weight variable may be employed, as the weights must sum to one. Including the time weight causes colinearity problems. The high and low levels for the cost weight were ± 10 percent of the nominal (elicited) value, and the time weight adjusted accordingly. Table 24 displays the variable values for the screening experiment.

The experimental design is illustrated in Table 25, along with the results of each run. The experiment was not randomized, as is typically done in an experiment involving physical systems, as the results are completely from mathematical expressions and random number generation. The pattern column of Table 25 indicates the settings of the

Table 24. Screening Experiment Input Data.

Variable	Position	Value
Utility, ρ_1, ρ_2	Center point	∞
	+	0.1
	-	-0.1
Cost Weight, w_1	Center point	0.667
	+	0.7337
	-	0.6003
Alternative	+	B
	-	A

Table 25. Screening Experimental Design and Results. Row indicates experimental design points. Pattern indicates the condition of the variables ρ_1, ρ_2, the alternative, and w_1. The observed utility is U, the predicted utility $\hat{U}$.

Row	Pattern	U	$\hat{U}$	Row	Pattern	U	$\hat{U}$
1	----	0.0000356	0.0298	11	+-+-	0.595	0.690
2	---+	0.000693	0.0885	12	+-++	0.731	0.768
3	--+-	0.00720	0.0444	13	++--	0.938	0.962
4	--++	0.00613	0.0908	14	++-+	0.858	0.952
5	-+--	0.229	0.323	15	+++-	0.727	0.770
6	-+-+	0.268	0.282	16	++++	0.67	0.749
7	-++-	0.048	0.126	17	00+0	0.71	0.508
8	-+++	0.019	0.0723	18	00-0	0.677	0.414
9	+---	0.591	0.669	19	00+0	0.758	0.508
10	+--+	0.729	0.759	20	00-0	0.657	0.414

four variables in the order of ρ_{Cost}, ρ_{Time}, alternative, and weight. A minus sign indicates that the variable for that position is at the low setting. The high setting is indicated by a plus sign. A zero indicates that the variable is to be placed at the center point setting. Table 25 also contains the results of each experimental run, the observed utility U. For ease of comparison, the utility estimated by the fitted least squares model, $\hat{U}$ is also contained in Table 25.

The predicted versus observed utility data is shown in Figure 72. The R^2 and adjusted R^2 values for the model were 0.863 and 0.712, respectively. The four points outside of the 95 percent confidence bounds are the four center points. The two center points for alternative A are the vertical pair on the right. The other pair is from center point runs with alternative B. ANOVA results are presented in Table 26. The model is highly significant when compared to the overall mean response.

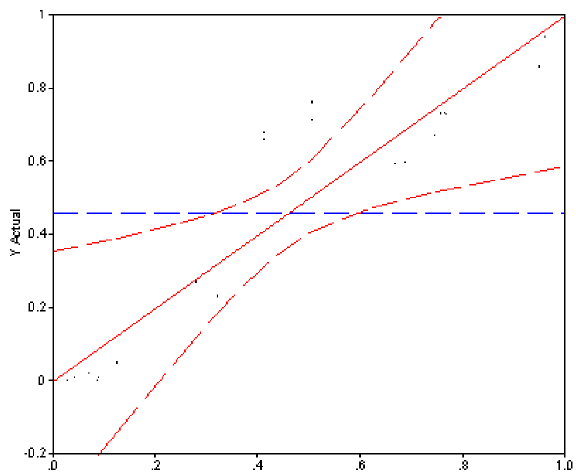


Figure 72. Prediction Versus Actual Data Plot for Screening Experiment.

Table 26. ANOVA Results from Model Fitted to Screening Data.

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	1.9071229	0.190712	5.6922
Error	9	0.3015372	0.033504	Prob > F
C. Total	19	2.2086602		0.0076

Table 27 contains the lack of fit statistics. The lack of fit is significant, indicating that the model likely could be improved through the addition of higher order terms of the model predictor variables. This is not unusual for screening experiments.

Table 27. Lack of Fit Statistics.

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	7	0.30018524	0.042884	63.4373
Pure Error	2	0.00135200	0.000676	Prob > F
Total Error	9	0.30153724		0.0156
				Max RSq
				0.9994

The estimates of the parameters are provided in Table 28. The exponential constant for the cost utility function is highly significant. This indicates that the true cost utility function has the potential to be significant in the hybrid model. In contrast, the time utility function is not significant. This indicates that within the range of exponential constant values normally encountered, the time utility function may be represented by the time value function.

The parking lot example was analyzed using Logical Decisions® software. In succession the single dimensional preference functions used in the model were both value functions. Then the significant cost function was replaced with the appropriate single dimension utility function. This completes the hybrid model. For comparative purposes, the other two possible model combinations were examined. These are the full utility model, with utility functions used for both single dimensional preference functions, and the remaining possible hybrid model combination, with cost modeled with a value

Table 28. Screening Experiment Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.461	0.0409	11.26	<.0001
ρ_{Cost}	0.329	0.0458	7.19	<.0001
ρ_{Time}	0.0686	0.0458	1.50	0.168
Alternative	0.0472	0.0409	1.15	0.278
Weight	0.00916	0.0458	0.20	0.846
$\rho_{Cost} * \rho_{Time}$	-0.000184	0.0458	-0.00	0.997
ρ_{Cost} *Alternative	-0.00153	0.0458	-0.03	0.974
ρ_{Time} *Alternative	0.0530	0.0458	1.16	0.277
ρ_{Cost} * Weight	0.00796	0.0458	0.17	0.866
ρ_{Time} * Weight	-0.0250	0.0458	-0.55	0.598
Alternative*Weight	0.00305	0.0458	0.07	0.948

function and tine modeled with an utility function. These final two models are not called for in the algorithm and so would not be considered when employing the methodology for a complex decision. These results are presented in Table 29. In each case Alternative *A* is the best choice.

Table 29. Results for Various Combinations of Value and Utility Functions for the Parking Lot Example.

Sequence	Preference Functions	Alternative A		Alternative B	
		Model Results	Difference from Utility Model	Model Results	Difference from Utility Model
1	Value: Cost and Time	0.742	0.177	0.682	0.183
2	Utility: Cost Value: Time	0.862	0.057	0.853	0.012
Not Applicable	Utility: Cost and Time	0.919	0	0.865	0
Not Applicable	Utility: Time Value: Cost	0.799	0.12	0.694	0.171

Examination the sensitivity of the alternatives to the relative importance (weight) of the evaluation measures is shown in Figure 73 through Figure 76. Figure 73 depicts the sensitivity of the exclusively value model. Figure 74 illustrates the sensitivity of the algorithm's hybrid model. Figure 75 shows the sensitivity of the model using only utility functions. The remaining possible combination, which is not called for in the algorithm, is shown in Figure 76. Figure 76 most closely resembles the full utility model. The vertical line in each graph indicates the nominal cost weight value, $w_{Cost} = 0.667$. The RSM examination only varied w_{Cost} by ten percent. The alternative choice is not sensitive within this range for the value model (Figure 73) and the utility model (Figure 75). However the hybrid model (Figure 74) is sensitive to the weights. While the hybrid model incorporates the significant utility aspects, the response to other perturbations is not representative of the utility model (or the value model either). This is a shortcoming in the methodology, but it can be mitigated through performance of sensitivity analysis on both the hybrid model and the value model.

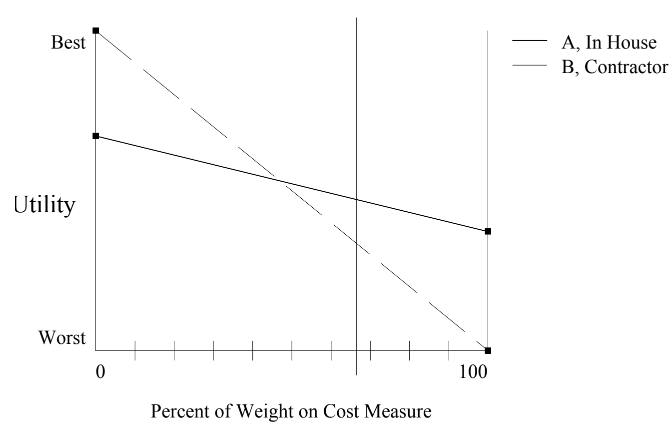


Figure 73. Weight Sensitivity for Parking Lot Example with Single Dimensional Value Functions.

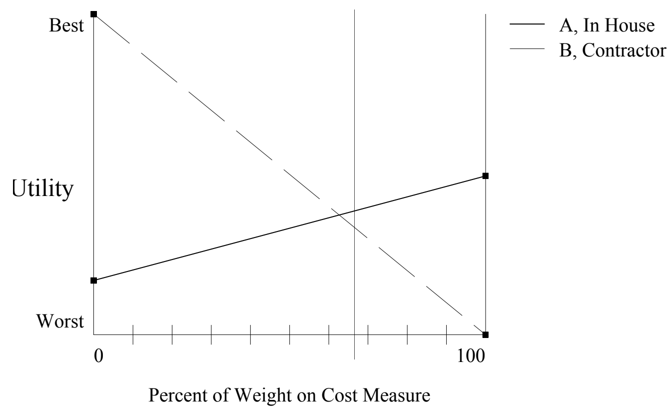


Figure 74. Weight Sensitivity for Parking Lot Example with Single Dimensional Cost Utility and Time Value Functions.

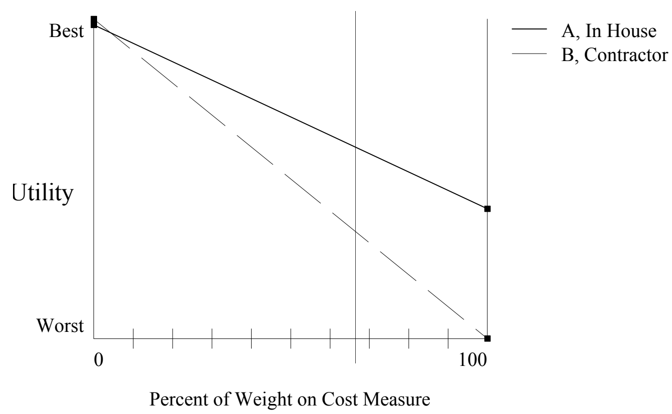


Figure 75. Weight Sensitivity for Parking Lot Example with Single Dimensional Utility Functions.

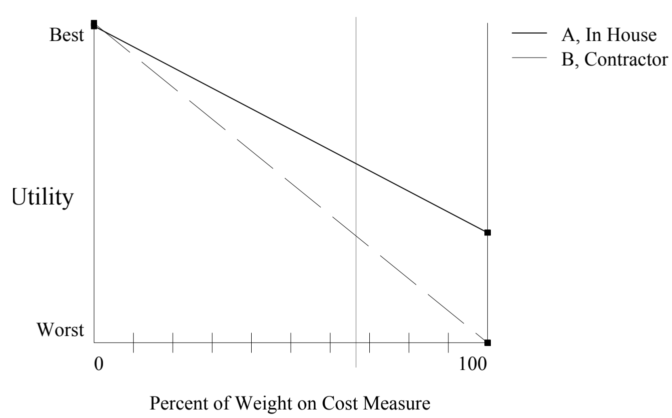


Figure 76. Weight Sensitivity for Parking Lot Example with Single Dimensional Cost Value and Time Utility Functions.

Example Summary. The parking lot example problem has been investigated using both a prescreen approach, where the variables were treated deterministically, and with a stochastic screening methodology. The two approaches are depicted graphically in Figure 77. The prescreen indicated that both the utility functions may be significant aswell as the alternative and one pair of interactions. The screening experiment identified only the cost utility function as being potentially significant. The screening experiment was the more accurate approach. The optimum alternative never did change, as predicted by the screening experiment in contrast with the prescreen. The hybrid approach did perform potentially poorly with respect to the sensitivity of the hybrid model compared to the true utility models. However the performance from an expected value perspective was satisfactory.

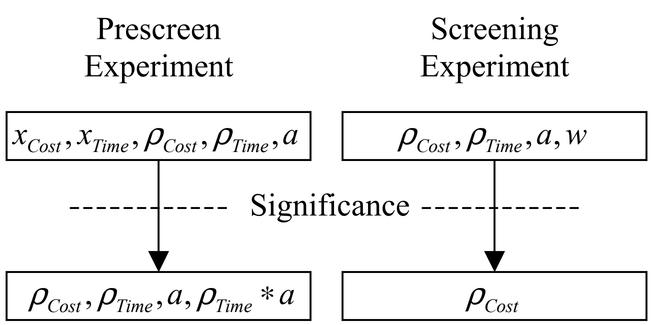


Figure 77. Prescreen and Screening for Parking Lot Example.

Demonstration of Value-Utility Hybridization

Klimack, Bassham, and Bauer (2002) provide an example of application of decision analysis to an Automatic Target Recognition (ATR) problem. This application of decision analysis included eliciting both single dimensional value and utility functions from the subject, an expert in the area. This provides an opportunity to employ the Value-Utility Hybridization methodology. The basic DA analysis is provided in Appendix D. A brief summary is provided for background here.

Background. Automatic Target Recognition (ATR) is a processing problem where an image is examined to identify or otherwise characterize targets. All ATR classification systems (CSs) have a number of desirable traits that may be used as evaluation measures when selecting from among candidate systems. In general, these measures have not applied in total, but specific measures have been selected when considering alternatives for a specific program. A decision analysis approach was employed. A value model was constructed. The value hierarchy is presented in Figure 78, and captures the motivators important to the decision maker. Besides the values, the problem is affected by the two mission profiles (combat identification [CID] and intelligence/surveillance/reconnaissance [ISR]) under which the ATR CS would be employed. Additionally there are three alternatives under consideration. Employing single dimensional value functions and single dimensional utility functions produced differing results, indicating that the problem was sensitive to preference function construction.

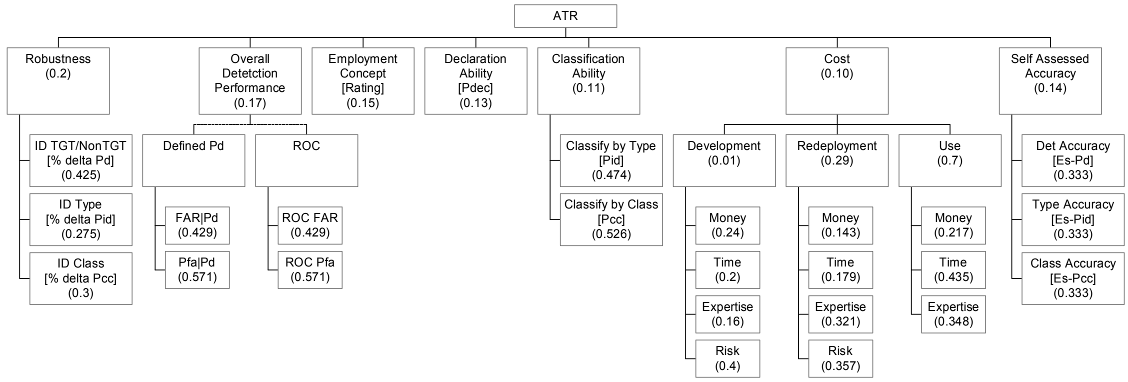


Figure 78. The Value Hierarchy for the ATR System. The dashed line under Overall Detection Performance indicates that either the Defined P_d or the ROC measures are employed. The parenthetical numbers indicate the relative weights for a value, within the parent value.

Employing RSM to Prioritize Model Parameters. The ATR DA problem has 23 evaluation measures, 23 corresponding weights, two mission profiles (combat identification [CID] and intelligence/surveillance/reconnaissance [ISR]), and three alternatives. This is a moderately large number of parameters. As the weights sum to unity, they may not simply be perturbed in the RSM process. The procedure provided by Kirkwood (1997b: 83 – 84) and implemented by Bauer, Parnell, and Meyers (1999: 168) was employed. The relative weight deemed most important, w_i, (generally the largest in magnitude) was varied by a factor of 0.1, or $w_i^+ = 1.1w_i$ and $w_i^- = 0.9w_i$. The remaining weights were adjusted to meet the summation constraint while retaining their relative ratios. The weights then become a single variable in the regression to find the response surface.

The remaining 25 variables (the 23 evaluation measure variables, the mission profile, and the alternative variables) are still a large number of parameters. To reduce the number to a more tractable amount, the group screening technique was employed.

Bauer, Parnell, and Meyers (1999: 165 – 166) introduced this approach for the decision analysis context. In this method, the input variables are grouped in some fashion. Perturbations are made uniformly within each group, that is, all group members are set at their high or low setting simultaneously. If a variable group is found not to be significant, then all group members are not considered significant. The group screening results are independent of how the grouping is performed, but in a DA context the value hierarchy offers an obvious cladistical grouping tool.

In this application, the number of evaluation measures suggests that group screening will be employed iteratively. Here the first decomposition of the value hierarchy, the second row from the top, will be used as an initial groups screening. RSM will be employed on these groups to determine their significance. RSM will then be employed separately for each decomposition of the value groups from the second row. The evaluation measure least square coefficients from the RSM analyses will then be combined to produce a single list of significant evaluation measures. The list will be prioritized based on least squares model coefficients. This prioritized list will then be used for construction of the hybrid value-utility model, which is then used to analyze the decision in a typical manner.

Initial Group Screening. For the ATR analysis, the grouping of the continuous variables was selected at the second level of the value hierarchy (see Figure 79). This provides seven groups:

1. Robustness,
2. Overall Detection Performance,

3. Employment Concept,
4. Declaration Ability,
5. Classification Ability,
6. Cost, and
7. Self-Assessed Accuracy.

Counting the weight variable, there were eight continuous variables. There are also two categorical variables: the mission profile, with two levels, CID and ISR; and the alternative, with three levels, ATR 33, ATR 55, and ATR 89. The design was developed in the JMP® software package, which recommended a L36 Taguchi Design. The categorical variables reduce the flexibility in choosing a design. For this example only first order effects, without interactions, will be examined. As no interactions are under consideration, aliasing concerns are avoided.

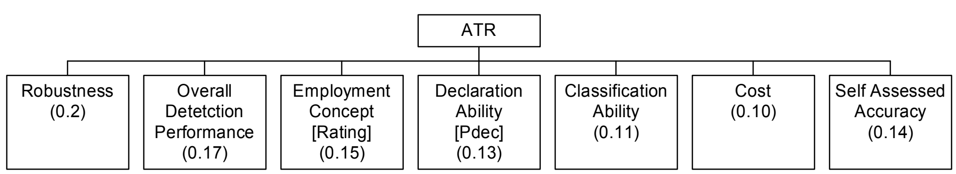


Figure 79. First Group Screening, Second Tier of Value Hierarchy.

An alternative approach to using an experimental design that includes categorical variables would be to select a specific combination of categories. For this problem, perhaps the CID profile and the ATR 55 alternative. The problem is then reduced to continuous input variables, which lend themselves to traditional fractional factorial designs. Working strictly with continuous variables also permits examination of

interactions, which was not considered here, because of the increased availability of orthogonal experimental designs. Six center points were added to the L36 design, permitting the various combinations of profile and alternative to be examined.

The L36 design employs the continuous variables at a high or low setting, indicated by a plus or minus sign, respectively, in the pattern notation. The patterns for the matrix rows are shown in Table 30. For the continuous variables, the minus sign indicates the variable of interest is set to the lower level, a plus sign indicates it is set to the higher level. The number zero identifies center points. For the categorical variables, which are handled by JMP with dummy variables, the minus, plus, and zero indicate different categories. For example, the lead minus sign in row 1 indicates that the robustness variable is at the lower setting. The order of the variables is as presented in the list above, then the profile and alternative variables. The data for these setting are presented below in Equations (127) through (131). More detailed information is available in Appendix E. Also shown in Table 30 are the results for each combination (row), $\hat{U}$.

To determine the response, the decision problem was modeled in the DPL® software package. The various evaluation measure levels, x_i, were passed to an Excel spreadsheet where $v_i(x_i)$ was calculated. This intermediate result was used to determine $u_i(v_i(x_i))$ from

$$u_i(v_i(x_i)) = \frac{1 - e^{-v_i(x_i)/\rho_i}}{1 - e^{-1/\rho_i}} \tag{126}$$

Table 30. Initial Group Screening Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	----------	0.278	15	+-++-+---+	0.627	29	---+++-++0	0.360
2	---------0	0.302	16	++-++-+---	0.610	30	---+++-+++	0.376
3	---------+	0.292	17	++-++-+--0	0.623	31	+---+++-+-	0.524
4	+-+---+++-	0.535	18	++-++-+--+	0.602	32	+---+++-+0	0.515
5	+-+---+++0	0.505	19	+++-++-+--	0.772	33	+---+++-++	0.531
6	+-+---++++	0.506	20	+++-++-+-0	0.806	34	-+---+++--	0.414
7	++-+---++-	0.480	21	+++-++-+-+	0.795	35	-+---+++-0	0.433
8	++-+---++0	0.544	22	-+++-++-+-	0.512	36	-+---+++-+	0.414
9	++-+---+++	0.546	23	-+++-++-+0	0.565	37	00000000--	0.509
10	-++-+---+-	0.601	24	-+++-++-++	0.583	38	00000000-0	0.556
11	-++-+---+0	0.645	25	--+++-++--	0.500	39	00000000-+	0.525
12	-++-+---++	0.647	26	--+++-++-0	0.545	40	00000000+-	0.497
13	+-++-+----	0.574	27	--+++-++-+	0.541	41	00000000+0	0.531
14	+-++-+---0	0.622	28	---+++-++-	0.381	42	00000000++	0.497

where

$$\rho_i = \begin{cases} 0.1 & \text{for pattern "+"} \\ \infty & \text{for pattern "0"} \\ -0.1 & \text{for pattern "-"} \end{cases} \tag{127}$$

A zero indicates nominal setting, when $u_i\left(v_i\left(x_i\right)\right) = v_i\left(x_i\right)$.

The weight variable $\tilde{\mathbf{w}}$ is a vector. The nominal value for the largest weight is that of the Robustness evaluation measure with a weight of 0.2. However no data was available for the robustness for identification of class. Because of this, the next largest evaluation measure weight was selected. This was the Overall Detection Performance evaluation measure, denoted w_2, equal to 0.17. Perturbing this by 0.1, or $w_2 = w_2 \pm 0.1 w_2$, and adjusting the remaining weights produces two weight vectors as called for by the high and low pattern indicators. As with ρ_i, a zero indicates nominal setting, or the elicited weights, $\tilde{\mathbf{w}}^0$.

These vectors are

$$\tilde{\mathbf{w}} = \begin{cases} \tilde{\mathbf{w}}^{+} & \text{for pattern "+"} \\ \tilde{\mathbf{w}}^{0} & \text{for pattern "0"} \\ \tilde{\mathbf{w}}^{-} & \text{for pattern "-"} \end{cases} \tag{128}$$

where

$$\tilde{\mathbf{w}}^{+} = \begin{bmatrix} 0.1959 \\ 0.187 \\ 0.1469 \\ 0.1273 \\ 0.1077 \\ 0.098 \\ 0.1371 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{0} = \begin{bmatrix} 0.2 \\ 0.17 \\ 0.15 \\ 0.13 \\ 0.11 \\ 0.1 \\ 0.14 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{-} = \begin{bmatrix} 0.2041 \\ 0.153 \\ 0.1531 \\ 0.1327 \\ 0.1123 \\ 0.1020 \\ 0.1429 \end{bmatrix} \tag{129}$$

The categorical variables, which were handled by employing dummy variables, were set according to

$$\text{Profile} = \begin{cases} \text{ISR} & \text{for pattern "+"} \\ \text{CID} & \text{for pattern "-"} \end{cases} \tag{130}$$

and

$$\text{Alternative} = \begin{cases} \text{ATR 89} & \text{for pattern "+"} \\ \text{ATR 55} & \text{for pattern "0"} \\ \text{ATR 33} & \text{for pattern "-"} \end{cases} \tag{131}$$

The response variable, utility $\hat{U}$, is also presented in Table 30. A plot of actual versus predicted values is contained in Figure 80. The linear model fit well with $R^2 = 0.978$ and $R^2_{adj} = 0.970$. The p value was less than 0.0001. The parameters estimates and the significance information for the model are contained in Table 31. The Declaration Ability and Self-Assessment groups are clearly not significant. They may be

dropped from consideration. The weights are also not significant, which agrees with the sensitivity analysis performed as part of the basic DA analysis of the problem, and will be eliminated from further consideration. The other groups are clearly significant. Note that the coefficient estimates for the least squares model agree with the significance information. The significant variables have coefficients that are an order of magnitude larger than those found to be non-significant. The coefficients also provide a measure of importance. Repeating the model fit with these groups removed would provide the same information (although the p-values would change slightly because of changes in the number of degrees of freedom) and so is not required.

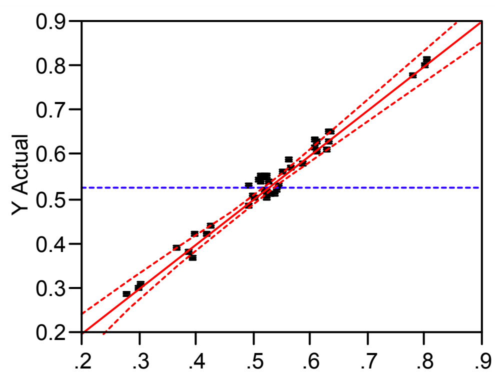


Figure 80. ATR Group Screening Model Actual versus Predicted Values.

The significance information in Table 31 may be used to prioritize the order of converting the single dimensional value functions to their utility equivalents. However the group screening has only eliminated five evaluation measures. Further, four groups have their significance levels indicated as equal. This fails to prioritize between eight

Table 31. ATR Group Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.529	0.00316	167.50	<.0001
Robustness	0.0647	0.00341	18.95	<.0001
Detection Performance	0.0577	0.00341	16.92	<.0001
Employment Concept	0.0738	0.00341	21.62	<.0001
Declaration Ability	0.00211	0.00341	0.62	0.541
Classification Ability	0.0456	0.00341	13.37	<.0001
Cost	0.0139	0.00341	4.09	0.0003
Self Assessment Accuracy	-0.00528	0.00341	-1.55	0.132
Weights	-0.00556	0.00341	-1.63	0.114
CID Profile	0.0109	0.00316	3.46	0.0016
ISR Profile	-0.0109	0.00316	-3.46	0.0016
ATR 33 Alternative	-0.0157	0.00447	-3.52	0.0014
ATR 55 Alternative	0.0104	0.00447	2.32	0.0274
ATR 89 Alternative	0.00536	0.00446	1.20	0.240

evaluation measures. For further refinement the next step is to decompose the groups, to determine the significance of the group members.

Second Group Screening. The candidate groups for decomposition and examination with RSM are: Robustness, Detection Performance, Classification Ability, and Cost. The Employment Concept criterion is not decomposable, so cannot be examined further. Employment Concept should be converted to utility in the hybrid model. (However of note is that all three candidates have equal scores for this criterion. If no additional candidates are introduced with a differing score for Employment Concept, and no sensitivity analysis is performed on the Employment Concept scores, it may be retained as a value function. Typically the sensitivity analysis restriction is onerous, but this point might be valuable where many utility elicitations are involved.) Eliminated from the immediate requirement to convert from value to utility functions are Declaration Ability and Self-Assessment. Choosing the first subgroup to screen should

be done with consideration as to the potential for eliminating criteria. The Cost measure provides this, but is complex with two sub-levels and eleven evaluation measures. For simplicity Robustness is chosen; other groups will be addressed later. The Robustness portion of the value hierarchy is shown in Figure 81.

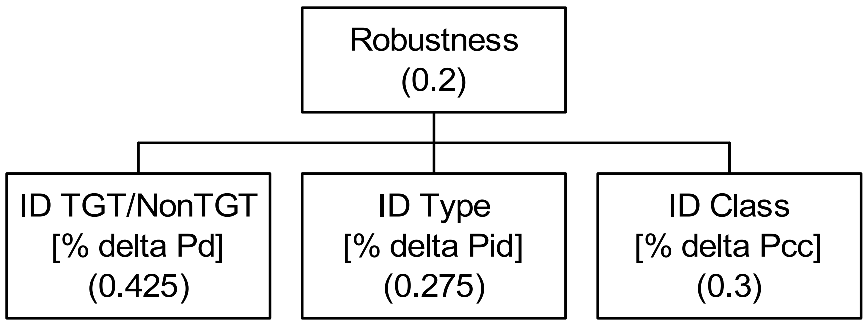


Figure 81. Second Group Screening, Robustness Portion of Value Hierarchy.

Robustness has three evaluation measures: Robustness in Detection (ΔP_d), Robustness in Typification (ΔP_{id}), and Robustness in Classification (ΔP_{cc}). The variables under consideration are the exponential constants for these measures. Refer to them as ρ_1, ρ_2, and ρ_3, respectively, for this portion of the analysis. They take on values as indicated in equation (127). The weights will be perturbed as above with the largest weight, that of Robustness in Detection, being perturbed ten percent about the nominal value. The equivalent of equation (129) becomes

$$\tilde{\mathbf{w}}^+ = \begin{bmatrix} 0.4675 \\ 0.2547 \\ 0.2778 \end{bmatrix}, \quad \tilde{\mathbf{w}}^0 = \begin{bmatrix} 0.425 \\ 0.275 \\ 0.3 \end{bmatrix}, \quad \tilde{\mathbf{w}}^- = \begin{bmatrix} 0.3825 \\ 0.2953 \\ 0.3222 \end{bmatrix} \tag{132}$$

The profile and alternative information remains unchanged. Categorical data again restricts the possible experimental designs available. A Hunter L18 (SAS Institute, Inc., 2001: 57) design was selected. This guarantees orthogonality and there are no aliasing concerns, as we do not investigate interactions here. The Robustness screening experimental design is illustrated in Table 32, along with the results. The pattern information indicates variables levels as explained above, with a variable order of Robustness in Detection, Robustness in Typification, and Robustness in Classification, weight, profile, and alternative. Analysis of these data provides the coefficient estimates in Table 33.

Table 32. Robustness Screening Experimental Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	---++-	0.0289	9	--+-+0	0.331	17	-----+	0.324
2	+++---	0.949	10	++-+-0	0.995	18	++++++	0.987
3	-++---	0.577	11	-+-+-0	0.534	19	000011	0.618
4	+--++-	0.745	12	+-+-+0	0.708	20	000012	0.671
5	-+--+-	0.577	13	-+++++	0.526	21	000013	0.584
6	+-++--	0.745	14	+----+	0.701	22	000021	0.618
7	--++-0	0.286	15	-----+	0.324	23	000022	0.671
8	++--+0	0.996	16	++++++	0.987	24	000023	0.584

As can be seen from the data in Table 33, Robustness in Detection and Robustness in Typification are significant while Robustness in Classification is not significant. The first two evaluation measures are candidates for conversion from $v_j\left(x_j\right)$ to $u_j\left(x_j\right)$ while the last is excluded from consideration. The mission profile is not significant. Any other result would be surprising, as the input data for robustness was

Table 33. ATR Robustness Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.628	0.0127	49.38	<.0001
Robustness in Detection	0.226	0.0149	15.10	<.0001
Robustness in Typification	0.140	0.0149	9.39	<.0001
Robustness in Classification	0.0113	0.0149	0.75	0.461
Weights(-1,1)	-0.0214	0.0149	-1.43	0.171
Profile[CID]	0.0109	0.0129	0.84	0.411
Profile[ISR]	-0.0109	0.0129	-0.84	0.411
Alternative[ATR 33]	-0.0207	0.0180	-1.15	0.267
Alternative[ATR 55]	0.0213	0.0180	1.18	0.254
Alternative[ATR 89]	-0.000597	0.0180	-0.03	0.974

not conditioned on mission profile. The alternatives are not significant. This indicates that the most desired alternative changes over the range of the perturbation introduced in this analysis.

Third Group Screening. Turning to consider cost, there are eleven evaluation measures. The screening design must include twelve continuous variables to account for the evaluation measures and the weights (see Figure 82), plus a three-level categorical variable for the alternatives. The profile may be neglected, as costs are not conditioned upon it.

To handle the weights, the largest of the weights for the tier immediately below the cost category was selected – the redeployment cost weight. Use has the highest relative weight and was varied 0.10, providing as before the weight vectors,

$$\tilde{\mathbf{w}}^{+} = \begin{bmatrix} 0.0077 \\ 0.2223 \\ 0.77 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{0} = \begin{bmatrix} 0.01 \\ 0.29 \\ 0.7 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{-} = \begin{bmatrix} 0.0123 \\ 0.3577 \\ 0.63 \end{bmatrix} \tag{133}$$

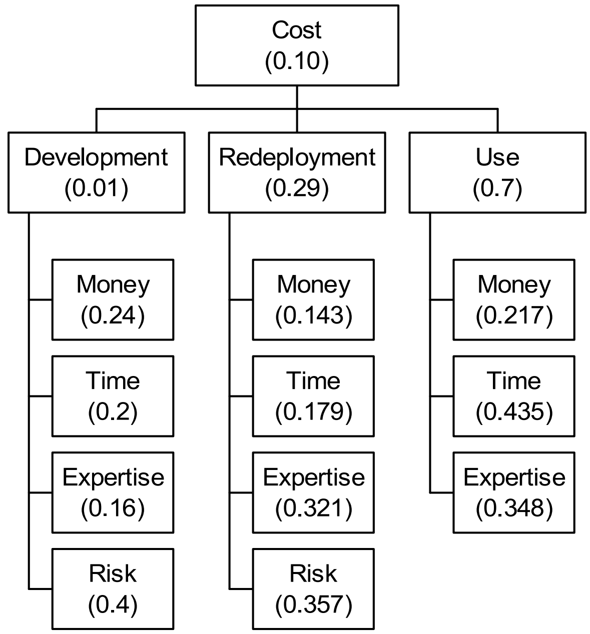


Figure 82. Third Group Screening, Cost Portion of Value Hierarchy.

The weights for the subcategories within development, redeployment, and use were not altered. The Cost screening experimental design is illustrated in Table 34, along with the results. The pattern information indicates variables levels as explained above, with a variable order of Development variables (Money, Time, Expertise, and Risk), Redeployment variables (Money, Time, Expertise, and Risk), Use variables (Money, Time, and Expertise), weight, and alternative. Profile is not included as a variable as the data is identical for both mission profiles.

The least squares fit of the data is provided in Figure 83 and Table 35. The Cost Use Time is highly significant, but the model fit is mediocre, with $R^2 = 0.657$ and $R^2_{adj} = 0.523$. Because of the fit, Cost Redeployment Time, Cost Use Experience, and Cost Redeployment Risk will be retained in the list of significant variables for the moment.

Table 34. Cost Screening Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	--++--++-+---	0.642	27	+-++--++-++-+	0.747
2	---+++++++-+0	0.617	28	-++--------++	0.445
3	+-+--++-+-+--	0.612	29	+++-++-+-++-0	0.477
4	-------+----+	0.370	30	--+++-------+	0.370
5	+----++--+---	0.439	31	-++++-+++----	0.549
6	+++-+--++---0	0.496	32	-----++-+---0	0.522
7	++++-++++--+-	0.703	33	++-+-+++--+++	0.552
8	--+-+++++--+0	0.617	34	+-++----++++-	0.778
9	++-----+++++-	0.778	35	-+-------++-0	0.312
10	++-++-+--+-+0	0.443	36	-+--++-++++-+	0.703
11	+---++++-+++0	0.546	37	-++-+++--+++-	0.817
12	++++++-+++--+	0.694	38	++-++-+-----0	0.425
13	-++-++-+--++-	0.703	39	+--++---+-+++	0.456
14	-++---+--+-+0	0.443	40	-+--+-----++-	0.664
15	++-+-+---+-++	0.804	41	-+++-+--++-+0	0.468
16	---+---++-++-	0.664	42	-+-+-+++-++-+	0.807
17	--++-+----+-0	0.351	43	+---++-------	0.612
18	--+-+-+-+++-+	0.756	44	---+-+-+--++0	0.475
19	-+-+++--++---	0.705	45	--+-+-+--++++	0.837
20	+---+--+-+---	0.642	46	---++-++++-+-	0.778
21	+-+--+--++-++	0.815	47	+-+++--+--+-0	0.438
22	+++---+++--++	0.526	48	-+-++++-+-+-+	0.553
23	+-+++---++++0	0.444	49	000000000000-	0.719
24	+---+++-+--++	0.456	50	0000000000000	0.500
25	+++++++---+--	0.612	51	000000000000+	0.670
26	++----+-+-+-0	0.483			

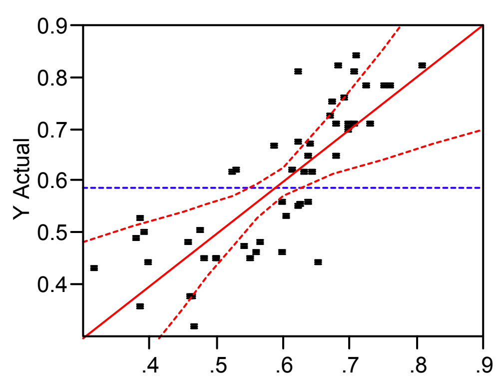


Figure 83. ATR Cost Model Actual versus Predicted Values.

Table 35. ATR Cost Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.590	0.0140	42.27	<.0001
Development Money(-1,1)	-0.00393	0.0143	-0.28	0.785
Development Time(-1,1)	0.00171	0.0143	0.12	0.906
Development Expertise(-1,1)	0.0131	0.0143	0.91	0.368
Development Risk(-1,1)	-0.00183	0.0144	-0.13	0.899
Redeployment Money(-1,1)	0.0101	0.0144	0.70	0.489
Redeployment Time(-1,1)	0.0239	0.0144	1.66	0.106
Redeployment Expertise(-1,1)	0.0170	0.0144	1.18	0.244
Redeployment Risk(-1,1)	0.0231	0.0144	1.60	0.118
Use Money(-1,1)	0.0169	0.0144	1.17	0.249
Use Time(-1,1)	0.0569	0.0143	3.97	0.0003
Use Expertise(-1,1)	0.0233	0.0144	1.62	0.115
Weights(-1,1)	0.0261	0.0144	1.81	0.0787
ATR[ATR 33]	0.0805	0.0196	4.10	0.0002
ATR[ATR 55]	-0.114	0.0196	-5.83	<.0001
ATR[ATR 89]	0.0340	0.0196	1.73	0.0924

Fourth Group Screening. The Detection Performance taxon will now be examined. This decomposes into two evaluation measures, False Alarm Rate given a Probability of Detection and the Probability of False Alarm given a Probability of Detection. (Alternatively, it could have been structured on the Required Operating Curve False Alarm Rate and the Required Operating Curve Probability of False Alarm.) This is depicted in Figure 84.

The weight for Probability of False Alarm will be varied ten percent, with the weight of False Alarm Rate adjusted according so that the pair sum to one. Coupled with the weights, there are three continuous variables plus the profile and alternative categorical variables. The weight vectors are

$$\tilde{\mathbf{w}}^{+} = \begin{bmatrix} 0.3719 \\ 0.6281 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{0} = \begin{bmatrix} 0.429 \\ 0.571 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{-} = \begin{bmatrix} 0.4861 \\ 0.5139 \end{bmatrix} \tag{134}$$

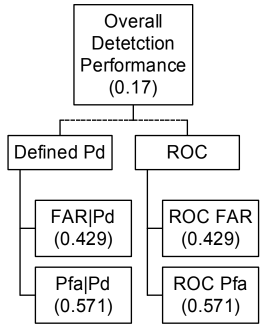


Figure 84. Fourth Group Screening, Robustness Portion of Value Hierarchy.

The experimental design and the response variables are contained in Table 36 and the least square variable significance and parameter estimates are in Table 37. The pattern information in Table 36 indicates variables levels as explained above, with a variable order of $FAR|P_D$, $P_{FA}|P_D$, weight, and alternative. Profile data is again identical for both mission profiles. As seen in Table 37, both the False Alarm Rate and the Probability of False Alarm were found to be significant.

Table 36. Detection Performance Screening Experimental Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	---+-	0.0850	13	-++++	0.626
2	+++--	0.680	14	+---+	0.486
3	-++--	0.317	15	----+	0.383
4	+--+-	0.0850	16	+++++	0.991
5	-+---	0.260	17	----+	0.383
6	+-++-	0.104	18	+++++	0.991
7	--++0	0.0133	19	000--	0.204
8	++--0	0.919	20	000-0	0.570
9	--+-0	0.331	21	000-+	0.464
10	++-+0	0.996	22	000+-	0.468
11	-+-+0	0.522	23	000+0	0.562
12	+-+-0	0.373	24	000++	0.480

Table 37. ATR Detection Performance Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.471	0.0247	19.02	<.0001
FAR(-1,1)	0.1293	0.0290	4.46	0.0003
Pfa(-1,1)	0.214	0.0290	7.38	<.0001
w(-1,1)	-0.0207	0.0290	-0.71	0.486
Profile[CID]	0.00384	0.0250	0.15	0.880
Profile[ISR]	-0.00384	0.0250	-0.15	0.880
ATR[ATR 33]	-0.195	0.0350	-5.58	<.0001
ATR[ATR 55]	0.0653	0.0350	1.86	0.0796
ATR[ATR 89]	0.130	0.0350	3.72	0.0017

Fifth Group Screening. Examining Classification Ability (see Figure 85), the weight vectors are established by selecting the larger weight, that of Classification by Class, to vary by ten percent.

$$\tilde{\mathbf{w}}^{+} = \begin{bmatrix} 0.4734 \\ 0.5216 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{0} = \begin{bmatrix} 0.526 \\ 0.474 \end{bmatrix}, \quad \tilde{\mathbf{w}}^{-} = \begin{bmatrix} 0.5786 \\ 0.4214 \end{bmatrix} \tag{135}$$

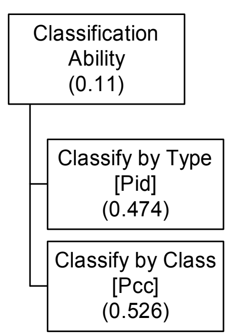


Figure 85. Fifth Group Screening, Classification Ability Portion of Value Hierarchy.

Table 38 contains the experimental design information and the response. The pattern information indicates variables levels as explained above, with a variable order of P_{ID}, P_{CC}, weight, profile, and alternative. The coefficients and significance statistics are

Table 38. Classification Ability Screening Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	---+-	0.00106	13	-++++	0.552
2	+++--	1.000	14	+---+	0.529
3	-++--	0.684	15	----+	0.0436
4	+--+-	0.508	16	+++++	0.967
5	-+---	0.603	17	----+	0.0436
6	+-++-	0.410	18	+++++	0.967
7	--++0	0.0150	19	000--	0.811
8	++--0	0.995	20	000-0	0.769
9	--+-0	0.110	21	000-+	0.655
10	++-+0	0.990	22	000+-	0.293
11	-+-+0	0.484	23	000+0	0.557
12	+-+-0	0.455	24	000++	0.374

contained in Table 39. Both Classify by Type and Classify by Class evaluation measures are highly significant.

Table 39. ATR Classification Ability Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.534	0.0204	26.12	<.0001
Type(-1,1)	0.215	0.0240	8.97	<.0001
Class(-1,1)	0.267	0.0239	11.2	<.0001
w(-1,1)	0.00705	0.0239	0.29	0.772
Profile[CID]	0.0649	0.0207	3.14	0.0060
Profile[ISR]	-0.0649	0.0207	-3.14	0.0060
ATR[ATR 33]	0.00477	0.0289	0.16	0.871
ATR[ATR 55]	0.0129	0.0289	0.45	0.661
ATR[ATR 89]	-0.0177	0.0289	-0.61	0.550

Sixth Group Screening. Finally the Self Assessed Accuracy category is considered. This is depicted in Figure 86. The data and both value and utility functions are identical for the three evaluation measures in this category. The input data are all

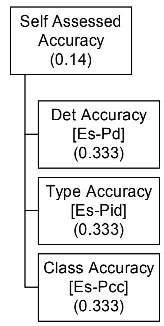


Figure 86. Sixth Group Screening, Self-Assessed Accuracy Portion of Value Hierarchy.

unity, providing value and utility of zero, as this evaluation measure was not part of the ATR CS testing. So testing this category as above is useless, as all combinations of weights and utility exponential values produce a utility of zero. Clearly the Self Assessed Accuracy measures are not significant.

Prioritization of Evaluation Measures. The ordered coefficients are contained in Table 40. They were determined by multiplying coefficients by the coefficient of parent groups to provide a global comparison. These provide a reasonable order in which single dimensional value functions should be converted to single dimensional utility functions in the hybrid model.

The value hierarchy reduced to those evaluation measures that were determined to be significant for $\alpha = 0.10$ during the screening process is depicted in Figure 87. Examining only these evaluation measures produces the subset of model coefficients listed in Table 41. The entries of Table 41 are identical to the top eleven entries of Table 40 except that Declaration Ability and Classification Robustness do not appear in Table 41. These evaluation measures were not found to be significant during the screening.

Table 40. Rank Ordered, by Absolute Value, Model Coefficients.

Evaluation Measure	**Coefficient**
Employment Concept	0.0738
Detection Robustness	0.0146
Probability of False Alarm	0.0123
Classification by Class	0.0122
Classification by Type	0.00980
Identification Robustness	0.00907
False Alarm Rate	0.00746
Declaration Ability	0.00211
Cost Use Time	0.000794
Classification Robustness	0.000729
Cost Redeployment Time	0.000334
Cost Use Expertise	0.000324
Cost Redeployment Risk	0.000322
Cost Redeployment Expertise	0.000238
Cost Use Money	0.000235
Cost Development Expertise	0.000183
Redeployment Money	0.000141
Cost Development Money	-5.48×10^{-5}
Cost Development Risk	-2.55×10^{-5}
Cost Development Time	2.38×10^{-5}
Es Pd	0
Es Pid	0
Es Pcc	0

They are in the eighth and tenth positions of Table 40. We conclude that employing the significance as a screening tool is reasonable.

Employment of the Hybrid Value-Utility Model. The hybrid utility algorithm was applied, using the priority sequence of variables presented in Table 41. Iteration zero employs the value model. Each succeeding iteration provides a hybrid model with the next highest priority evaluation measure value function replaced by the utility function. The results for the mission CID profile are presented in Table 42 and Table 43 contains

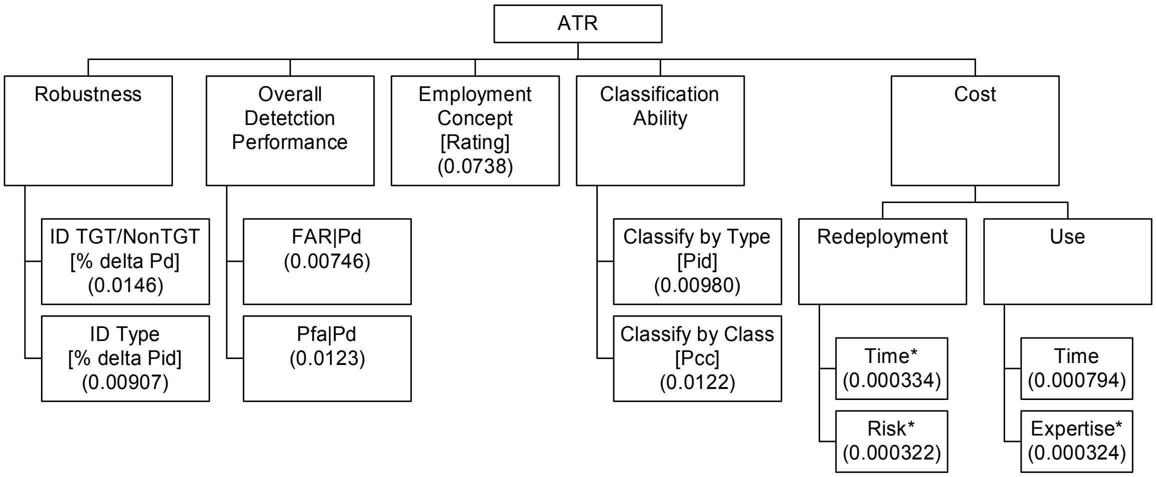


Figure 87. Value Hierarchy Reduced to Significant Taxa. The parenthetical numbers indicate the estimated coefficients. Taxa significant at alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

Table 41. Significant Rank Ordered Model Coefficients. Those significant at an alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

Evaluation Measure	**Coefficient**	**Priority**
Concept	0.0738	1
Detection Robustness	0.0146	2
Probability of False Alarm	0.0123	3
Classification by Class	0.0122	4
Classification by Type	0.00980	5
Identification Robustness	0.00907	6
False Alarm Rate	0.00746	7
Cost Use Time	0.000794	8
Cost Redeployment Time*	0.000334	9
Cost Use Expertise*	0.000324	10
Cost Redeployment Risk*	0.000322	11

the results for the ISR profile. The pure utility model results are also provided in the respective tables. Results for conversion of all variables, not only the significant ones, from value to utility are provided in Appendix E.

Table 42. CID Profile Value, Utility, and Hybrid Utility Results, Significant Evaluation Measures. Those significant at an alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

	Notes	ATR 33	ATR 55	ATR 89
Goal	Utility Model	0.572	0.507	0.518
Iteration				
0	Value Model	0.509	0.556	0.525
1	Employment Concept	0.464	0.511	0.480
2	Detection Robustness	0.455	0.471	0.442
3	Probability of False Alarm	0.529	0.500	0.512
4	Classification by Class	0.536	0.509	0.524
5	Classification by Type	0.538	0.511	0.528
6	Identification Robustness	0.529	0.505	0.509
7	False Alarm Rate	0.569	0.506	0.510
8	Cost Use Time	0.569	0.506	0.513
9	Cost Redeployment Time*	0.570	0.507	0.513
10	Cost Use Expertise*	0.570	0.507	0.513
11	Cost Redeployment Risk*	0.570	0.504	0.516

Table 43. ISR Profile Value, Utility, and Hybrid Utility Results, Significant Evaluation Measures. Those significant at an alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

	Notes	ATR 33	ATR 55	ATR 89
Goal	Utility Model	0.415	0.455	0.439
Iteration				
0	Value Model	0.497	0.531	0.500
1	Employment Concept	0.452	0.486	0.452
2	Detection Robustness	0.443	0.446	0.414
3	Probability of False Alarm	0.371	0.415	0.383
4	Classification by Class	0.392	0.430	0.403
5	Classification by Type	0.405	0.435	0.413
6	Identification Robustness	0.395	0.429	0.394
7	False Alarm Rate	0.412	0.455	0.432
8	Cost Use Time	0.412	0.455	0.434
9	Cost Redeployment Time*	0.413	0.455	0.434
10	Cost Use Expertise*	0.413	0.455	0.434
11	Cost Redeployment Risk*	0.413	0.452	0.434

The hybrid utility model results are shown graphically in Figure 88 for the ATR 33 alternative. Other alternatives behaved similarly and are omitted for clarity. All three alternatives converge towards the utility model after an initial period of instability. This is more readily observed in Figure 89 through Figure 92, where the differences between iterative hybrid model scores and the distance from the true utility model for the two mission profiles are illustrated. The decreasing differences between iterations and the decreasing distance from the utility model are obvious. The latter measure is a measure of the error of the hybrid model compared to the true utility model. After the initial period this error remains small.

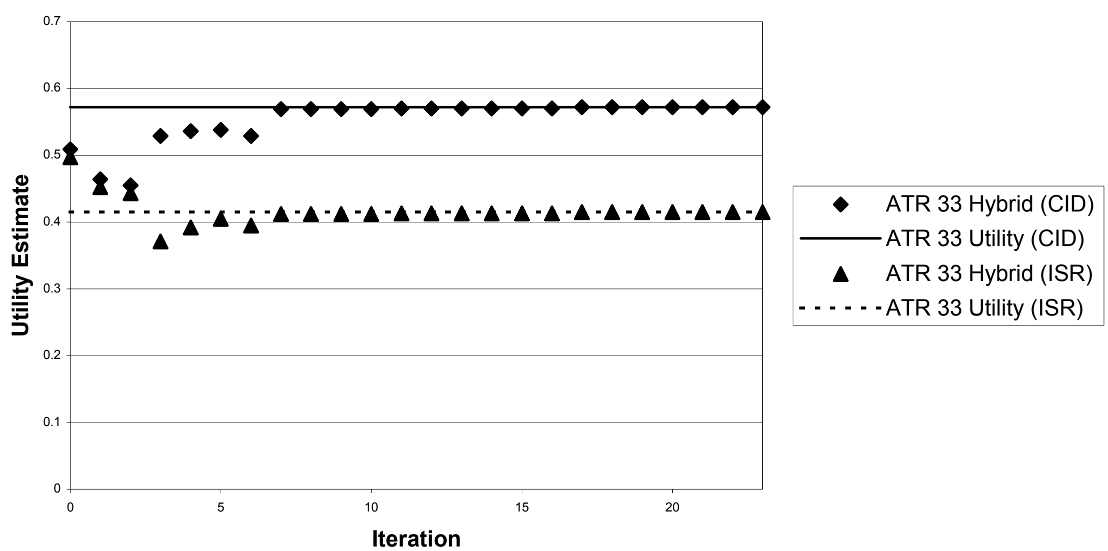


Figure 88. ATR Hybrid Utility Iterations for ATR 33.

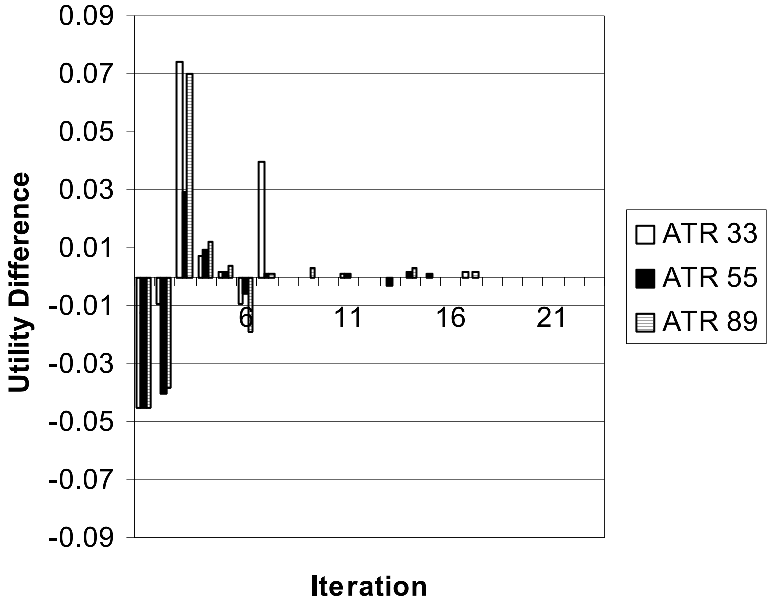


Figure 89. Hybrid Utility Model Difference Between Successive Iterations, CID Profile.

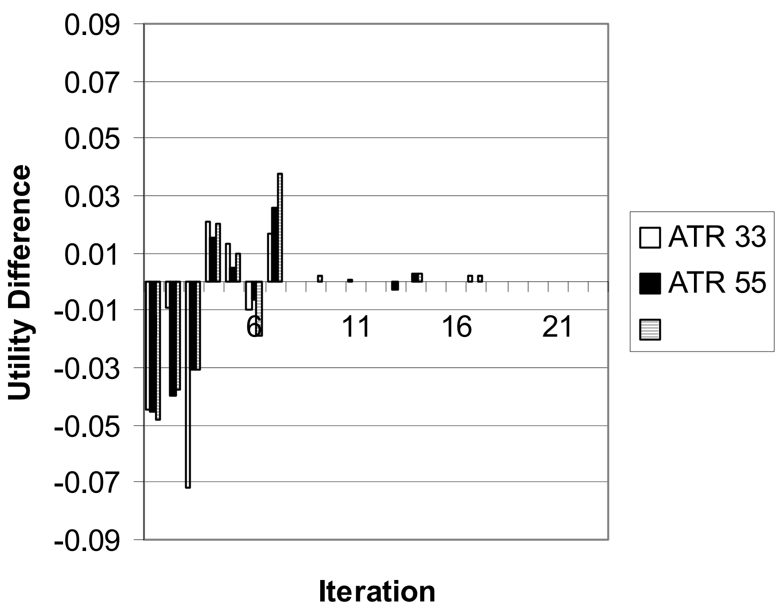


Figure 90. Hybrid Utility Model Difference Between Successive Iterations, ISR Profile.

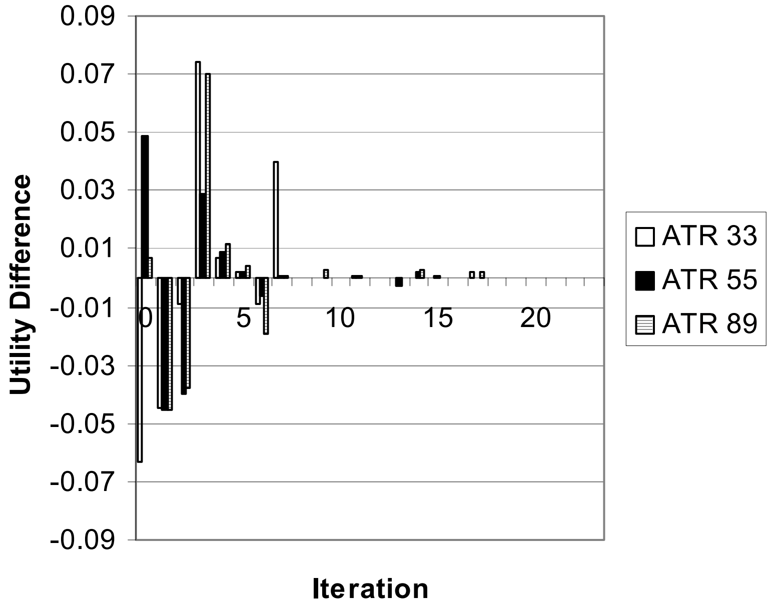


Figure 91. Hybrid Utility Model Difference Between Iteration and True Utility Model, CID Profile.

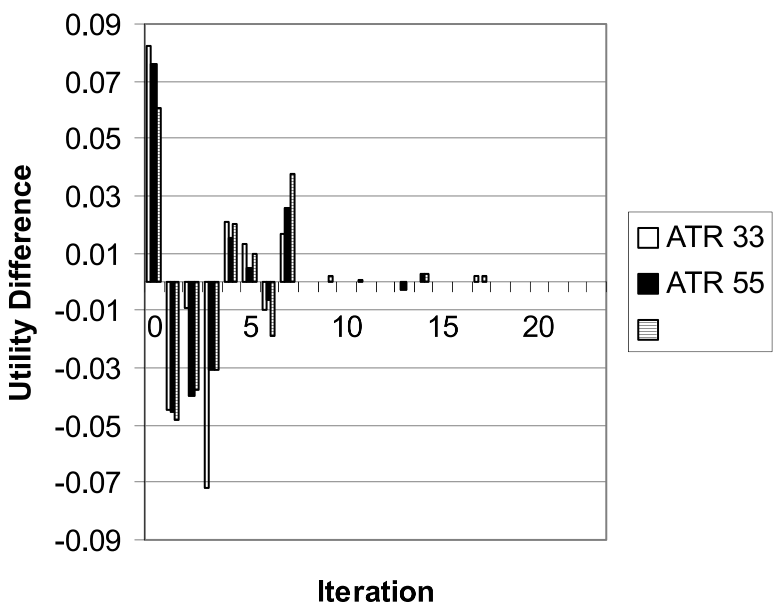


Figure 92. Hybrid Utility Model Difference Between Iteration and True Utility Model, ISR Profile.

To understand the behavior of the hybrid model as it is modified during each iteration, we examine the initial step for the ATR 33 alternative (see Table 42 and Figure 88). The complete value model had an overall value of 0.509 for this alternative. When the Employment Concept was converted to the single dimensional utility function, the hybrid utility became 0.464. The Employment Concept rating for the ATR 33 alternative (indeed, all alternatives) was poorly defined. The corresponding single dimensional value is 0.4. The single dimensional utility for the poorly defined rating is 0.1, so the difference between the preference functions is 0.3. When this difference is multiplied by the relative weight for the Employment Concept evaluation measure, 0.15, the result is 0.45, the difference between the results obtained in the first iteration compared to the starting value model.

It is interesting to note in Figure 88 that ATR 33 converges to the true utility model strictly from below for the CID profile, although the direction of movement was not identical for each iteration, while under the ISR case ATR 33 oscillated about the true utility value. In no case was the movement strictly towards the true utility, as would be expected if the decision maker was strictly relatively risk averse (seeking).

Once the significant evaluation measures, at $\alpha = 0.05$, were converted from value to utility functions the hybrid function provided estimates that differed at most 0.003 from the pure utility function result. The evaluation measures that became significant between $\alpha = 0.05$ and $\alpha = 0.1$ provided no improved estimate.

We conclude that for the example the evaluation measures determined to be significant through response surface methodology techniques provided an adequate set of variables to permit creation of an accurate hybrid model. The hybrid model may be used

to represent the utility model. The number of preference functions required to be elicited from the decision maker was reduced from 26 to eight for $\alpha = 0.05$.

Further, the rank ordering by regression coefficient was successful in providing successive hybrid estimates that in general approached the true utility value, each with generally reduced difference (δ_i) from the previous estimate.

Hybrid Value-Utility Summary

Research Summary. This portion of the research permits several conclusions. The first objective under this area of research was to establish when value and utility functions differed significantly. It was demonstrated that application of response surface methodology to the preference functions provides key information regarding the behavior of the utility model. This provides information that indicates for which evaluation measures value and utility differ significantly. This is a new contribution and is useful to identify those functions where discrimination must be made between value and utility. It would also be useful to identify where a linear preference function, employed in the absence of data, is adequate. When adequate, then no effort need be dedicated to determining the actual function.

The second objective was to develop an algorithm that provides an adequate hybrid value-utility model. The algorithm was successful in providing the hybrid model. The hybrid utility model, $\hat{U}$, provided an adequate representation of the true utility model, U. Elicitation of the hybrid utility model is more efficient with respect to

decision maker participation than employing the strict utility model. Under conditions of uncertainty and risk, the value model may provide differing results than the utility model, as was the case here, and so is not acceptable. This agrees with the results of Chapter IV, that value and utility functions often differ significantly.

Potential Future Work. Future research could examine improvements in the stopping criteria of the algorithm. Earlier stopping of the algorithm, within acceptable tolerances, provides returns on the efficiency with respect to the decision maker.

The method of incorporation of the alternative in the experimental design may be an area where future research would increase the efficiency of the hybrid model methodology. Treating the alternative as a categorical variable in the design greatly limits available designs. If the set of alternatives may be reduced during the examination of the preference functions potential significance, many more experimental designs are available. Clearly if a single alternative may be used for this step, the algorithm becomes much more efficient. Less grouping of variables would be required, meaning that fewer successive group screening steps would be required. Determination of sufficient conditions when this simplification is acceptable would be beneficial.

Rank statistical methods may provide an additional stopping criterion. If single dimensional substitutions are made in decreasing order of the strength of the subject's risk attitudes, then each iteration will have decreasing effects of the hybridization process. Each iteration *i* provides a set $R^{(i)}$ of the estimates of the utility for each alternative. The Mann-Whitney (or Wilcoxon) test provides a means of determining if the rankings of each iteration are drawn from the same population (Conover, 1971: 213 – 239). If consecutive rankings are not from significantly different populations, then the

model produced is not significantly different from its predecessor. It is possible that the algorithm may be terminated, although likely certain conditions may be necessary.

VI. Summary

General

The research had two major objectives, both pertaining to the relationship of value and utility preference functions in decision analysis. The first was to examine this relationship itself between value and utility functions. Possible relationships between the functions, each with underlying theoretical implications, were examined. Clarification of the relationships provides insight into risk attitudes as well as facilitates representation of empirical data with functional forms. The second was to improve the efficiency of decision analysis by determining when the distinctions between value and utility preference functions were significant. This chapter will summarize the research, highlight the original contributions, and identify possible future work. Detailed explanation of the results accompanies the presentation of the methodology, analysis, and results in Chapters IV and V.

Relationship of Value and Utility Preference Functions

Functional Relationship of Single Dimensional Value and Utility Functions. The analysis of the data for both the tactical scenario and the information operations study clearly show that the value and utility preference functions differ significantly and so are different constructs. For the tactical scenario, the linear model of $v_i(x_i) = u_i(x_i)$,

i.e., a linear transformation of value into utility, failed to achieve the best fit for any subject on any evaluation dimension. It achieved an acceptable fit, within the elicitation error, less than five percent of the time. This was a poorer performance than any other model. For the information operations analysis, the linear model also failed to achieve the best fit for any evaluation dimension. The linear model had an acceptable fit for only one-third of the evaluation measures, which was again, along with the logarithmic model, the poorest performance.

The sigmoid model, a sigmoid transformation of value into utility, performed well. It provided the best fit in over 90 percent of the cases as determined using WRMS (84 percent for RMS). The other functions were very poor performers, typically fitting best in only a few cases. The best of these other functions was the exponential, which is typically the relationship of choice of the decision analyst. The exponential and power functions achieved an acceptable fit (as opposed to best fit) about eleven percent of the time, logarithmic about seven, and linear less than in five percent of the cases. This supports the hypothesis that the value and utility functions are different constructs and different within a professional population of subjects. Further, it indicates that approximating a utility function with the corresponding value function is a poor approximation in the absence of other information.

Characteristics And Performance Of Various Common Decision Analysis Elicitation Methodologies. Initially it is important to consider if the distinction between value and utility are inconsequential. Several researchers have reported, as reviewed in Chapter II, that elicitation error often exceeds the differences between value and utility

functions. If so, then value and utility elicitation methods would be interchangeable in application.

No previous estimates of elicitation error were located, although Keller (1985b: 479) had proposed an ad hoc criterion of RMSE less than or equal to 0.05 for acceptance of a model fit to elicited data. This research includes an estimate of elicitation error between the certainty-equivalent and probability-equivalent methods of utility function elicitation. This provides the criterion of 0.107 for WRMSE (or 0.120 for RMSE if WRMSE is not used) for acceptable fit of utility functions. For the tactical scenario elicitations, which involved standard elicitation methodologies, the disparity between the value function and the mean utility function was 0.191. These differences are statistically significant. This indicates that value-based and utility-based approaches are not equivalent, and each should be used only in the proper context.

The two single dimensional utility elicitation procedures examined in the tactical scenario, the probability and certainty equivalent approaches, were found to produce strategically equivalent results. Employing either method alone would produce results that did not significantly differ, although the p critical value was 0.055. This suggests that either method may be employed, but eliciting both and using an average of the two may be advisable in critical applications. Subjects involved in the tactical scenario elicitations indicated that they were more comfortable providing answers to the probability equivalent approach rather than the certainty equivalent method. Single dimensional value and utility functions were clearly different constructs. This clearly supports employing value functions only under conditions of certainty, and using utility functions otherwise.

The multiattribute utility approach of Kirkwood (1997b: 155 – 166) was found to provide results that were not strategically equivalent. This is discussed further below.

Improving The Ability to Compare Between Preference Functions. Previous work employed the root mean square error (RMSE) as the method for comparing two preference functions. A weighted RMSE, WRMSE, which gives more weight to the differences further from the center of the independent variable midpoint, is more appealing. Differences of equal magnitude between two functions that occur at different distances from the center point occur under differing constraints. The functions are most constrained at the endpoints of the domain, so equal differences at different distances form the midpoint should be accorded different relative weighting. Such a WRMSE was developed in this work, and is provided in equation (58) in Chapter IV.

As utility functions were elicited employing two different, but compatible, methods, differences in these two elicited functions were assumed to capture the error of the elicitation process. Analysis of this error provided the criteria of 0.120 for RMSE and 0.107 for WRMSE for acceptable fit of utility functions. Differences between two functions below these criteria are not statistically significant different from the elicitation error (for these data), at the $\alpha = 0.05$ level. These criteria are a new contribution to the literature. It is recommended that the WRMSE be employed to detect significant differences between preference functions, with the criterion of 0.107.

Examining The Subject's Risk Attitudes. The curve shape of the $\hat{u}\left(v\left(x\right)\right)$ function (transformation function of value to utility) provides insight into a decision maker's risk attitudes, as does which mathematical model best fits the elicited data. In the tactical movement problem, the concave curve was exhibited most frequently (39 of

80 curves) and the curves were S-shaped in 32 of the cases. The sigmoid fit best for more than ninety percent of the evaluation measures. The exponential and logarithmic models fit for less than five percent of the cases. The sigmoid model (being more flexible) fit best in almost sixty percent of the cases; the exponential in more than forty. For the information operations analysis, the sigmoid function provided the best fit in seven cases and the exponential in the remaining five. Of the twelve continuous evaluation measures, the curve shapes of the assessed utility functions of the subjects were not strongly risk averse: two were linear or near linear, four were S-shaped, four were convex, and two were concave.

This lack of dominance of concave utility functions and the fit of the sigmoid function suggests that risk attitudes are more complex than the constant risk attitudes usually assumed. The usual assumption is constant risk aversion. If a constant risk attitude is to be assumed, constant risk aversion was the best model, which is supported by the high number of fits of the exponential function. However the assumption of constant risk aversion for military decision making is not supported by these data.

Consistent risk attitude was defined as the condition that the decision maker maintains risk attitudes on all evaluation measures that are consistent with respect to being generally risk averse, risk neutral, or risk seeking. If this condition is present, then the decision maker is risk averse (neutral) (seeking) on all evaluation measures if she is risk averse (neutral) (seeking) on any single evaluation measure. The subjects considering the tactical scenario did not demonstrate consistent risk attitudes. A new metric, Δ, was created and used to quantify the variability of the subjects' risk attitudes. This metric combines measures of local and global risk attitudes on an evaluation

measure, a new contribution. It appears that the subjects' risk attitudes varied widely between evaluation measures. Such variance contraindicates the employment of the multiattribute utility approach of Kirkwood (1997b), which assumes risk consistency, for military decision making.

Learning effects were studied. It was hypothesized that elicitation of single dimensional value functions would clarify the subjects' thought. So elicitation order of value-utility-utility should see lower elicitation error than utility-value-utility. No learning effects were observed.

Improving The Metric For Assessing Risk Aversion. This research improved the ability to examine subjects' risk attitudes. The measures $R(f)$ and $V(f)$ were developed and provide more discrimination that the previous metrics – curve shape and preference area. These metrics were also employed in the search for consistent risk attitudes for subjects, measured by Δ, discussed above.

Hybrid Value-Utility Decision Analysis Models

The second major portion of the research dealt with the sensitivity of the decision analysis model to the preference function, and how to take advantage of lack of sensitivity. The work of Chapter V is summarized here.

Establishing When Value-Utility Differences Are Significant. Response surface methodology was successfully applied to the problem of determining when the model is sensitive to the preference functions. Response surface methodology was used to examine the margins of an envelope of reasonable curves for $\hat{u}_i\left(v_i\left(x_i\right)\right)$. When the

function failed to be significant, then the true function was assumed to be not significant for evaluation measure *i*.

Developing an Algorithm for Employment of a Hybrid Value-Utility Model.

An algorithm was created that was successful in providing the hybrid model. The hybrid multiattribute utility model, $\hat{U}$, provided at adequate representation of the true multiattribute utility model, U. Elicitation of the hybrid utility model is more efficient with respect to decision maker participation than employing the strict utility model, especially in cases where there are a large number of evaluation measures or cases where several of the measures require an inordinate amount of time to elicit additional utility information. Such efficiency promotes use of decision analysis and encourages sound decision making.

Original Contributions

The results discussed above include original contributions to the literature. This study extended value-utility studies into multidimensional cases employing military professionals as subjects. An improved measure of differences between preference functions, WRMSE, was created. Acceptance criteria for adequate model fit, based on elicitation error, were established for WRMSE and RMSE. The concept of a hybrid value-utility decision analysis model was first advanced. The sensitivity of decision analysis models to perturbation of the preference functions was examined. This involved the first use of RSM on the preference functions. This information was used to establish an efficient algorithm for creation of hybrid value-utility models.

Future Research

As with any work, additional research topics emerge. The examination of value and utility function relationships presented in Chapter IV could be extended. Simply enlarging the sample size would improve the clarity of the elicitation error information. Repeating elicitations but limiting the domain for the utility function elicitation to the region where $v_i\left(x_i\right) = a, a \in \left(0,1\right)$, would provide stronger evidence for the usefulness of the sigmoid function if it continued to fit well. Repeating the work with military professionals but employing a different scenario would strengthen insight that the preference function results are not scenario-specific.

Learning effects discovered during the course of this research were significant, but counterintuitive. Examination of this phenomenon may provide useful insights into risk attitudes. Other weighting functions for the weighted root mean square calculations may offer improved metrics. Application of WRMSE to discrete functions was not addressed in this research, and so remains an open question.

Examination of the risk attitude metrics developed in this work for non-military decision making may show improved opportunity for use. As risk attitude consistency is usually assumed, it may be present in other groups of decision makers. If so, the metric Δ may prove useful. If some professional group is found to possess consistent risk attitudes, then risk attitude information for a subset of evaluation dimensions and Δ may be used to estimate risk attitudes on other dimensions. Such an approach could provide improved efficiency for problems with many evaluation dimensions.

Improved stopping criteria for the hybridization algorithm would improve efficiency. Rank order statistics may provide such a benefit, particularly if conclusions about strict monotonicity of preference functions could be elicited from the decision maker.

Conclusion

This work is the first to enter the literature regarding comparison of value and utility functions for decision analysis examining multidimensional data from military professionals as subjects. This research supports the hypothesis that single dimensional value and utility functions are not equivalent. This conclusion was confirmed for two separate military decision making situations. No single functional relationship defined the relationship between value and utility functions. Certainty equivalent and probability equivalent elicitation methodologies provided equivalent results. Value and utility functions differed significantly more than can be explained by elicitation error. Aggregating single dimensional utilities into a multiattribute model and the Kirkwood (1997b) multiattribute utility produce results that are not strategically equivalent.

The research demonstrated that response surface methodology may be applied to the preference function. The potential significance of a preference function may be employed to construct a hybrid value-utility model that facilitates more efficient elicitation of information from the decision maker. This promotes use of decision analysis techniques and therefore improves decision making for large multidimensional problems.

While value and utility are differing constructs, prudent exploitation of their differences and similarities permits more efficient use of the decision maker's time. This work emphasizes that the distinction between value and utility must not be ignored. But the lack of strategic differences for specific cases may be identified and exploited.

Appendix A. Glossary of Technical Terms

ALLAIS PARADOX – Allais constructed two sequential lottery choices, that most subjects made alternative selections in each that are mutually incompatible without axiomatic violations of expected utility. Consider four lotteries. The notation indicates the payoff then, parenthetically, the associated probability.

$$L_1 : [\$30{,}000\,(0.33);\ \$25{,}000\,(0.66);\ \$0\,(0.01)]$$

$$L_2 : [\$25{,}000\,(1)]$$

$$L_3 : [\$30{,}000\,(0.33);\ \$0\,(0.67)]$$

$$L_4 : [\$25{,}000\,(0.34);\ \$0\,(0.66)]$$

Initially the subject must choose between L_1 and L_2. Most subjects choose L_2. Then the same subject who chose L_2 generally chooses L_3 when presented with a choice between L_3 and L_4. This combination of choices violates expected utility. Counter arguments included bounded rationality, FRAMING, and DYNAMIC CONSISTENCY. (Biswas, 1997: 5 – 6)

ALTERNATIVE – One of several courses of action that may be chosen (Decision Analysis Society, November 3, 2000).

BETWEENESS AXIOM. – "If someone is indifferent between two lotteries L_1 and L_2, then she is indifferent between either of these lotteries and the compound lottery $[L_1(\alpha), L_2(1-\alpha)]$," where α is the associated probability of lottery L_1. Normally this axiom is employed in conjunction with the WEAK INDEPENDENCE AXIOM. (Biswas, 1997: 12) See INDEPENDENCE AXIOM.

CASH EQUIVALENT – See CERTAINTY EQUIVALENT.

CERTAIN EQUIVALENT – See CERTAINTY EQUIVALENT.

CERTAINTY – Condition under which decision are made with known deterministic alternatives.

CERTAINTY EQUIVALENT – The quantity at which a decision maker is indifferent between two alternatives, one being that amount under certainty and the other an uncertain situation where the possibility exists of either a greater of lesser payoff, each with a specified probability. When the decision maker is risk averse, it is less than the expected monetary value. (Clemen and Reilly, 2001: 535) Also called CASH EQUIVALENT by Pratt (1964b: 122) and CERTAIN EQUIVALENT by Skinner (1999: 57). See RISK PREMIUM.

CERTAINTY-EQUIVALENT (CE) ASSESSMENT TECHNIQUE – Utility function elicitation employing a two-result lottery with equal probability of each outcome and an alternative with a certain outcome that is variable. Adjustments are made to the certain alternative until the subject is ambivalent between the uncertain lottery and the certain alternative. (Clemen, 1996: 474-475) This method is referred to as the VARIABLE CERTAINTY EQUIVALENT METHOD by von Winterfeldt and Edwards (1986: 249).

CLARITY OF ACTION – condition of the decision maker knowing the appropriate action although the outcome is uncertain. (Spradlin, 1997).

COEFFICIENT OF VALUE SATIATION – See VALUE SATIATION.

CONSISTENT RISK ATTITUDES – The condition of the decision maker maintaining risk attitudes on all evaluation measures that are consistent with respect to being risk averse, risk neutral, or risk seeking. If this condition is present, then the decision maker is risk averse (neutral) (seeking) on all evaluation measures if she is risk averse (neutral) (seeking) on any single evaluation measure.

CONSTANT RISK ATTITUDES – The condition of the decision maker maintaining a risk attitude throughout the domain of an evaluation measure with respect to being risk averse, risk neutral, or risk seeking. If this condition is present, then the decision maker is risk averse (neutral) (seeking) over the entire domain of that evaluation measure if he is risk averse (neutral) (seeking) at any point in the domain for that evaluation measure. (Bell and Raiffa, 1988a)

CONSTANT RELATIVE RISK ATTITUDES – The condition of the decision maker maintaining a risk attitude throughout the value domain ($v(x)$) of an evaluation measure with respect to being risk averse, risk neutral, or risk seeking. If this condition is present, then the decision maker is risk averse (neutral) (seeking) over the entire domain of that evaluation measure if he is risk averse (neutral) (seeking) at any point in the domain for that evaluation measure. (Bell and Raiffa, 1988a)

DECISION – "A conscious, irrevocable allocation of resources with the purpose of achieving a desired objective" (Skinner, 1999: 11).

DECISION ANALYSIS – 1. A structured approach in analyzing how alternatives for a decision would produce a desired result (Decision Analysis Society, November 3, 2000). 2. Analysis of a decision problem strictly by employing a value or utility function approach (Keeney, 1982: 813).

DECISION ANALYSIS CYCLE – "A systematic approach to solving problems: STRUCTURING a problem to capture the essentials, EVALUATION to gain insight and AGREEMENT with the world to make something happen" (Skinner, 1999: 356).

DECISION MAKER – “Person or team with the responsibility and authority to allocate resources and implement the decision” (Skinner, 1999: 356).

DECISION MAKING UNDER RISK – Pertains to choice among lotteries that assign objective probabilities to outcomes (Karni, 1990).

DECISION MAKING UNDER UNCERTAINTY – Pertains to choice among alternative acts that assign outcomes to states of nature whose likelihood of being realized is a matter of subjective belief (Karni, 1990).

DECISION NODE – “A point in a decision tree where a decision must be made” (Skinner, 1999: 357).

DECISION POLICY – A rule for the selection of alternatives. Decision policy designates which alternative to take at each and every decision node in a decision tree. Also referred to as a DECISION STRATEGY or STRATEGY. (Skinner, 1999: 357)

DECISION THEORY – “The mathematical theory of decision making under uncertainty” (Skinner, 1996: 357).

DECISION TREE – A sequential graphical representation of decisions and uncertain events that provide all possible outcomes to a decision situation. (Skinner, 1999: 357)

DYNAMIC CONSISTENCY – Maintenance of alternative choices when a subject is presented a decision with the decision in differing orders. For example, if a decision made with consideration of later probabilistic events, a decision maker is said to be dynamically consistent if she would maintain the same decision choice if the probabilistic events were known, i.e., occurring before the decision. (Biswas, 1997: 15)

ECONOMIC VALUE – A value measured in units of currency (Spradlin, 1997).

ELASTICITY – The relative change in the response variable to a relative change in a predictor variable,

$$E_i = \frac{\Delta y / y}{\Delta x_i / x_i}$$

where y is the response variable and x_i an independent variable (Joish and Armstrong, 2000: 537).

ELLSBERG PARADOX – Given an opaque container in which are 30 marbles, ten of red and the balance being either blue or yellow, and then offered two sequential bets: (1) Call red or blue, if correct win $5,000; and (2) Call red or blue, if wrong, collect $5,000. Ellsberg found that most subjects choose red for both bets. This violates expected utility theory because red as the choice in bet (1) evinces

$$p_r \cdot u(5000) + (1 - p_r) \cdot u(0) > p_b \cdot u(5000) + (1 - \mathrm{p_b}) \cdot u(0)$$

where the subscripts p_r and p_b indicate the respective probabilities of selecting a marble of that color. The expected utility relationship of bet (2) is captured by

$$p_r \cdot u(0) + (1 - p_r) \cdot u(5000) > p_b \cdot u(0) + (1 - p_b) \cdot u(5000)$$

which may be manipulated to form

$$p_r \cdot u(5000) + (1 - p_r) \cdot u(0) < p_b \cdot u(0) + (1 - p_b) \cdot u(0).$$

This clearly violates the equation resultant from bet (1), and so violates expected utility theory. Ellsberg advanced the argument that decision makers are likely to place more confidence in objective rather than subjective probabilities. The red probabilities are objective while those for blue are subjective, leading a decision maker so inclined to select red. (Ellsberg, 1961: 653 – 655)

ENCODING UNCERTAINTY – "The process by which individual judgments about an uncertain variable are characterized by a probability distribution" (Skinner, 1999: 357).

EVALUATION CONSIDERATION – (1) "Any matter that is significant enough to be taken into account while evaluating alternatives. Other terms sometimes used for evaluation considerations are EVALUATION CONCERN or AREA OF CONCERN." (Kirkwood, 1997: 11-12) (2) "A factor to compare alternatives (e.g., annual income)." (Kirkwood, 1997, as quoted by Parnell, undated)

EVALUATION MEASURE – "Measuring scale for the degree of attainment of an objective," e.g., annual salary in dollars. Also called measure of effectiveness, measure of merit, or metric. (Kirkwood, 1997: 12)

EXPECTED MONETARY VALUE – The weighted mean of outcomes, expressed in currency units. Weighting is provided by multiplying the outcome by the corresponding probability. (Skinner, 1999: 357).

EXPECTED UTILITY – The theory as developed by von Neumann and Morgenstern (1947) that the alternative may be rank ordered by their expected utility. The expected utility is determined by multiplying possible outcomes by their probabilities and summing. (Keller, 1992: 4) Also referred to as UTILITY THEORY.

EXPECTED VALUE – 1. "The mean of the values one would expect to obtain upon sampling from the distribution a large number of times" (Decision Analysis Society, November 3, 2000). Also known by the older term of MATHEMATICAL EXPECTATION. 2. A variation of EXPECTED UTILITY where value functions replace utility functions.

FOURFOLD PATTERN OF RISK ATTITUDES – Tversky and Kahneman (1992: 306 – 307) found that subject's risk behavior followed a pattern where risk seeking behavior was demonstrated for low probability potential gains and high probability potential

losses. Risk averse behavior was presented in the converse cases. This is summarized in Table 44. They state (1995: 1256) that numerous studies confirm this construct.

Table 44. Certainty Equivalents Under Extremes of Probability and Prospects. The Uncertain Alternative was for a Gain of 100 or a Loss of 100, with Probability of Either 0.05 or 0.95. (Tversky and Kahneman, 1992: 307)

	Gain	Loss
Low Probability	$u(14) = 0.05$, risk seeking	$u(-8) = -0.05$, risk averse
High Probability	$u(95) = 0.78$, risk averse	$u(-84) = -0.95$, risk seeking

FRAMING – The manner of presentation of questions employed in elicitation in decision analysis. Framing has been shown to be a significant factor affecting the answers received from the decision maker. (Tversky and Kahneman, 1986)

GOAL – (1) "Threshold of achievement with respect to an evaluation consideration," (Kirkwood, 1997: 12). (2) "Specific degree of satisfaction of a given objective" (Decision Analysis Society, November 3, 2000).

INDEPENDENCE AXIOM – "Consider three lotteries, L_1, L_2, and L_3. Suppose that the relation of the indifference (or strict preference) holds between L_1 and L_2. The relation of indifference (or strict preference) must hold between the compound lotteries $[L_1(\alpha), L_3(1-\alpha)]$ and $[L_2(\alpha), L_3(1-\alpha)]$" where α is the associated probability of lotteries L_1 and L_2. (Biswas, 1997: 11) See BETWEENESS AXIOM.

INDIFFERENCE – Indifference between two objects indicates that the decision maker desires each equally. The objects are considered to have equal value or utility. (Skinner, 1999: 357)

INDIFFERENCE CURVE – The plot of continuous points in n attributes where the utility (or value) is constant (Clemen, 1996: 540 – 541). Also known as isopreference curves. The concept of indifference curves was conceived by the economist Edgeworth (Stigler, 1950: 325).

INDIRECT VALUES – "Things that the decision maker values that are not likely to show up on accounting statements." (Spradlin, 1997).

INFLUENCE DIAGRAM – A graphical representation of a decision situation. Decisions are represented by rectangles, ovals represent stochastic events, diamonds represent the

final outcome, and arcs indicate information status and conditional relationships. (Clemen and Reilly, 2001: 52).

LAYER – The levels in the value tree. Also called a **tier**. (Kirkwood, 1997b, as quoted by Parnell, undated)

LEVEL – "Specific numerical rating for a particular alternative with respect to a specified evaluation measure," e.g., $55K; also called a SCORE (Kirkwood, 1997b: 12, 55).

LOCAL RISK AVERSION – Pratt (1964) introduced a measure of local risk aversion, $r(x) = -u''(x)/u'(x)$. This was also independently developed by Arrow.

MEASURABLE VALUE FUNCTION – "A value function that may be used to order the differences in the strength of preference between pairs of alternatives" (Dyer and Sarin, 1979: 810).

OBJECTIVE – (1) "The desired level of performance against a value" (Skinner, 1999: 358). (2) "Preferred direction of attainment with respect to an evaluation consideration" (e.g., higher annual income). See STRATEGIC OBJECTIVE. (Kirkwood, 1997: 12) (3) "What the decision maker hopes to achieve by allocating the resources" (Spradlin, 1997).

OPTION – "An alternative that permits a future decision following revelation of information. All options are alternatives, but not all alternatives are options." (Spradlin, 1997).

ORDINAL UTILITY FUNCTION – See VALUE FUNCTION.

OUTCOME – "The result of the decision situation measured on the scale of the decision maker's values" (Spradlin, 1997).

OUTCOME BRANCH – "A branch emanating from a probability node representing the possible outcome and its probability of occurrence" (Skinner, 1999: 358).

PORTFOLIO PROBLEM – "The various decisions faced in the strategy are of a similar nature, and the decision maker does not have sufficient resources for funding all combinations of alternatives." (Spradlin, 1997).

PREFERENCE – "The decision maker's attitude toward the value, timing, and uncertainty of outcomes" (Skinner, 1999: 358).

PREFERENCE AREA – (1) The area under the utility curve, plotted in a unit square, that serves as a measure of the decision maker's risk attitude. Areas below 0.45 indicate risk aversion, above 0.55 indicate risk seeking, and between these values indicates risk

neutrality. (Kimbrough and Weber, 1994: 627) (2) More generally, the same definition applied to any preference function.

PREFERENCE FUNCTION – Either a value or a utility function.

PROBABILITY – “A number between zero and one (inclusively) representing the degree of belief a person attaches to the occurrence of an event” (Skinner, 1999: 358).

PROBABILITY-EQUIVALENT (PE) ASSESSMENT TECHNIQUE – Utility function elicitation employing a two-result lottery where the probability p of the desired outcome is adjusted, the undesired outcome occurs with probability (1-p), and the alternative with the certain outcome is fixed. Adjustments are made to p until the subject is ambivalent between the uncertain lottery and the certain alternative. (Clemen, 1996: 475) This method is called the variable probability method by von Winterfeldt and Edwards (von Winterfeldt, 1986: 243), who also say the method is referred to as the basic reference lottery ticket (BRLT) method.

PROBABILITY NODE – “A point in a decision tree where an uncertainty will be resolved; often called chance node” (Skinner, 1999: 358).

RATING – $v(x)$ where v is a value function; also called a “value.” (Kirkwood, 1997b: 55).

RESPONSE SURFACE METHODOLOGY (RSM) – A collection of knowledge and techniques from statistical experimental design, regression modeling, and optimization (Myers and Montgomery, 1995: xiii).

RISK – (1) the chance of loss; (2) the possibility of loss; (3) uncertainty; (4) the dispersion of actual from expected results; (5) the probability of any outcome different from the one expected; (6) the probability of an event times the probable cost (or loss) if the event occurs (Dargahi-Noubary & Growney, 1998); (7) the condition under which alternatives involve known probabilities.

RISK AVERSION – See LOCAL RISK AVERSION.

RISK AVERSION COEFFICIENT – When an exponential utility function is employed “$u(x) = a - be^{-x/R}$ where x is the value (such as dollars), R is the risk tolerance and a and b are parameters set by the choice of two points in the utility curve…One sometimes sees reference to the RISK AVERSION COEFFICIENT, which is defined as $1/R$” (McNamee, 1987: 94). See also LOCAL RISK AVERSION and PREFERENCE AREA.

RISK PREMIUM – The expected monetary value of an uncertain alternative less the CERTAINTY EQUIVALENT of the decision maker. It is a measure of the decision maker’s risk aversion. (Clemen and Reilly, 2001: 534)

RISK TOLERANCE – "Describes the decision maker's attitude toward risk" (Spradlin, 1997).

RISK-AVERSE DECISION MAKER – "One who values alternatives at less than their expected values" (Spradlin, 1997).

RISK-NEUTRAL DECISION MAKER – "One who is willing to play the long-run odds when making decisions and will evaluate alternatives according to their expected values" (Spradlin, 1997).

RISK-SEEKING DECISION MAKER – One who values an alternative more than the expected value.

REQUISITE DECISION MODEL – "A model whose form and content are sufficient to solve a particular problem." (Phillips, 1984: 29)

SCORE – See LEVEL**.**

SCORING FUNCTION – "A single dimensional value function that assigns value to an evaluation measure level" (Kirkwood, 1997b, as quoted by Parnell, undated).

SENSITIVITY ANALYSIS – (1) "Examining the impact of reasonable changes in base-case assumptions (Eschenbach, 1992: 41). (2) "Measuring the impact of each uncertainty on the value of an alternative to determine which are critical, i.e., would change the order of preference for the alternatives" (Skinner, 1999: 358).

SENSITIVITY CHART – See TORNADO DIAGRAM (Skinner, 1999: 358).

SIMPLE DECISION – "One in which there is only one decision to be made, even though there might be many alternatives." (Spradlin, 1997).

SPIDERPLOT – A Cartesian plot of response the dependent variable to percentage change in independent variables where each independent variable is represented by a curve. The curves intersect at the case point $\left(100\%, E\left[u\left(x\right)\right]\right)$ creating a spider web-like appearance. The plot shows sensitivity of the decision problem.

STRATEGIC EQUIVALENCE – (1) Condition under which preference functions provide identical rank orderings of a set of alternatives (Kirkwood, 1997b: 229 and 245). (2) Condition under which methodologies produce identical rank orderings of alternatives.

STRATEGIC OBJECTIVE – "The ultimate objective for the decision" (Kirkwood, 1997b, as quoted by Parnell, undated).

STRATEGY – A set of alternatives for sequential decisions in a decision situation where the alternatives follow some policy or principle. For example, one strategy may involve only conservative alternatives while another competing strategy employs aggressive choices. Also known s a POLICY. (Skinner, 1999: 358).

STRATEGY GENERATION TABLE – Employed to create or display strategies. The first column contains titles for generated strategies. Decisions are assigned to each column. The alternatives populate the column. A strategy is indicated by linking the name to each alternative chosen under that strategy. Strategy names are chosen to highlight differences among the strategies. (Kirkwood, 1997b: 47 – 48)

STRATEGY TABLE – See Strategy Generation Table (Skinner, 1999: 358).

SUBJECTIVE EXPECTED UTILITY – The EXPECTED UTILITY approach where objective probabilities are replaced with subjective probabilities. The idea was introduced by Ramsey (1954). (Keller, 1992: 4)

SUBJECTIVE PROBABILITY – "Numerical specification of an individual's degree of belief that an uncertain event will occur (Kirkwood, 1997: 109).

TORNADO DIAGRAM – A graphical representation of the decision analysis model optimal solution response to variation in model parameters. The diagram is a series of horizontal bars, each representing the response variation. By convention the bars are ordered in descending impact, creating an inverted triangle reminiscent of an atmospheric tornado. Also known as a SENSITIVITY CHART.

TRADE-OFFS – "Judgments about how much a decision maker is willing to sacrifice on one value in order to receive more of another" (Decision Analysis Society, November 3, 2000).

UNCERTAINTY – (1) "Any event for which the outcome is not known at the time a decision is made" (Skinner, 1999: 359). (2) "Uncertainty means unknown probabilities," (Bernstein, 1996: 133). This suggests a distinction beyond the division between deterministic and stochastic, or that a stochastic situation may either be known, and still certain, and unknown (uncertain). (3) "Uncontrollable elements" (Spradlin, 1997).

UTILITY – "A mathematical function (curve) that represents the decision maker's risk attitude" (Skinner, 1999: 359).

UTILITY FUNCTION – (1) "… a real-valued function defined on a set and inducing or corresponding to a preference order by having $u(x)<=u(y)$ exactly when y is preferred to x" (Borowski, 1991: 620). [Note that this is a definition from a non-DA source. The term VALUE FUNCTION does not appear in this source.] (2) "… encodes a decision maker's attitude toward risk taking in mathematical form by relating the decision maker's satisfaction with the outcome (or "utility" associated with the outcome) to the monetary

value of the outcome itself.” (Decision Analysis Society, November 3, 2000). (3) “A preference function under uncertainty” (Dyer and Sarin, 1979: 810).

UTILITY THEORY – See EXPECTED UTILITY.

TIER – See LAYER.

VALUE – (1) “An outcome measure, e.g., NPV” (Skinner, 1996). (2) “$v(x)$ where v is a value function; also called a ‘rating’” (Kirkwood, 1997b: 55).

VALUE FUNCTION – (1) A function $v(x)$ is a value function if it is true that $v(x') > v(x'')$ iff $x' \succ x''$ where x' and x'' are specified but arbitrary levels of x (Kirkwood, 1997: 229). (2) Used under conditions of certainty to rank-order alternatives; also called ORDINAL UTILITY FUNCTIONS (Clemen, 1996: 552). (3) “A preference function under certainty” (Dyer and Sarin, 1979: 810).

VALUE HIERARCHY – (1) “A value structure with a hierarchical structure”; also called a VALUE TREE. (Kirkwood, 1997: 12). (2) “Pictorial representation of the structure of the evaluation considerations” (Kirkwood, 1997b, as quoted by Parnell, undated).

VALUE OF INFORMATION – “The maximum price one should pay for knowing the actual value of uncertainty prior to making the decision” (Skinner, 1999: 359).

VALUE MODEL – “A mathematical model of the value structure that includes scoring functions and weights” (Kirkwood, 1997b, as quoted by Parnell, undated).

VALUE SATIATION – A construct introduced by Dyer and Sarin (1982: 877) for measurable value functions that is the analog of Pratt’s local risk aversion for utility functions. The coefficient of value satiation, defined $m(x) = -v''(x)/v'(x)$, is a local measure of strength of preference at asset level x.

VALUE STRUCTURE – “The entire set of evaluation considerations, objectives, and evaluation measures for a particular decision analysis.” (Kirkwood, 1997b: 12).

VALUE TREE – See VALUE HIERARCHY.

VALUES – Things that are important to you in a general sense (Clemen, 1996: 19).

VARIABLE CERTAINTY EQUIVALENT METHOD – See CERTAINTY-EQUIVALENT ASSESSMENT TECHNIQUE.

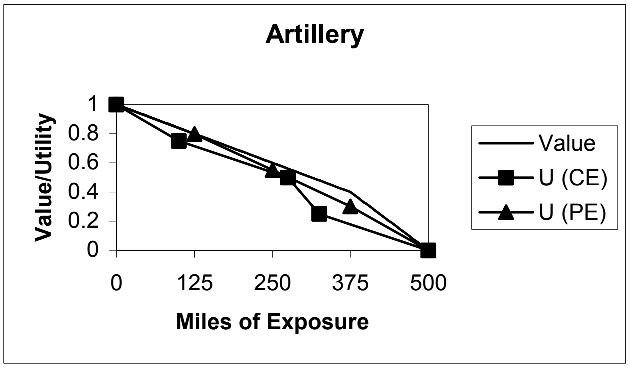
VARIABLE PROBABILITY METHOD – See PROBABILITY-EQUIVALENT ASSESSMENT TECHNIQUE.

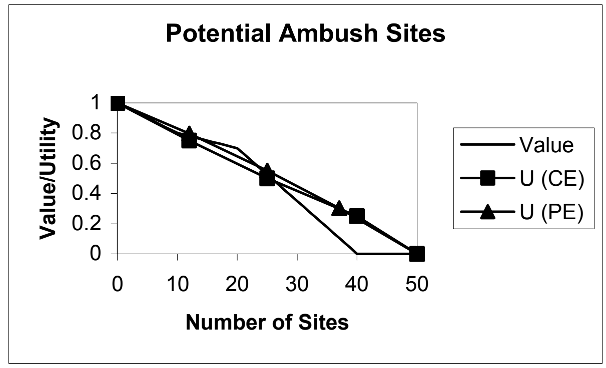
WEAK INDEPENDENCE AXIOM – “Consider three lotteries, L_1, L_2, and L_3 and an individual who is indifferent between L_1 and L_2. Given an y compound lottery $[L_1(p), L_3(1-p)]$ there must exist a q such that the individual is indifferent between this compound lottery and $[L_2(q), L_3(1-q)]$,” where p and q are the associated probabilities of lotteries L_1 and L_2. Normally this axiom is employed in conjunction with the BETWEENESS AXIOM to guarantee non-intersection of indifference curves. (Biswas, 1997: 13)

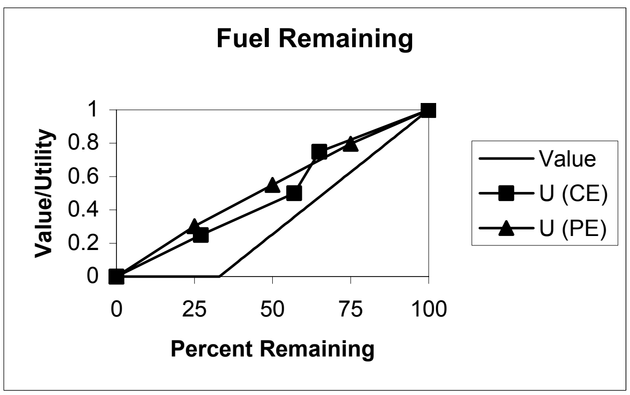
WEIGHTS – “Our relative preference for evaluation considerations and evaluation measures. Weights depend on the domain of the scoring functions.” (Kirkwood, 1997b, as quoted by Parnell, undated)

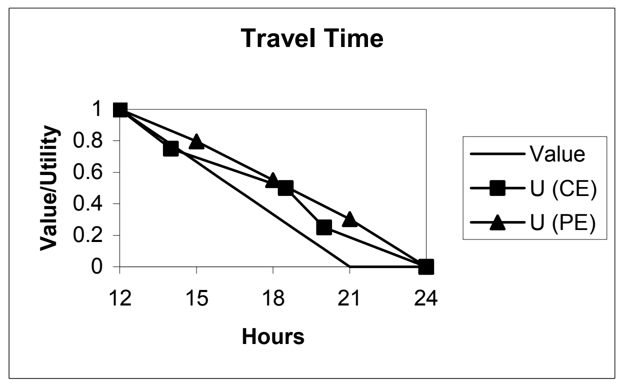
Appendix B. Tactical Questionnaire Detailed Results

Subject FJ1









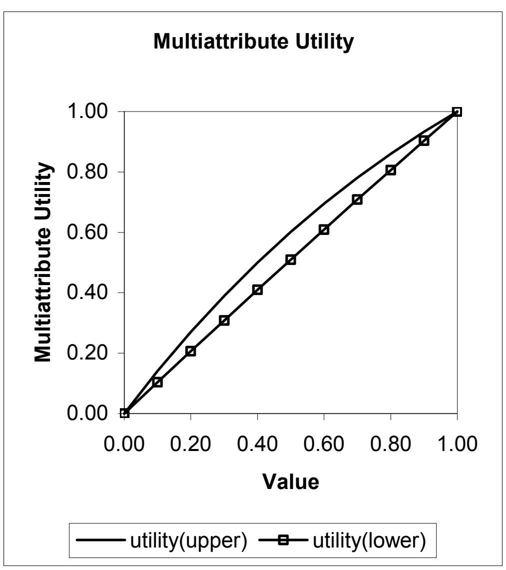


Figure 93. Subject FJ1's Preference Functions.

Subject FJ2

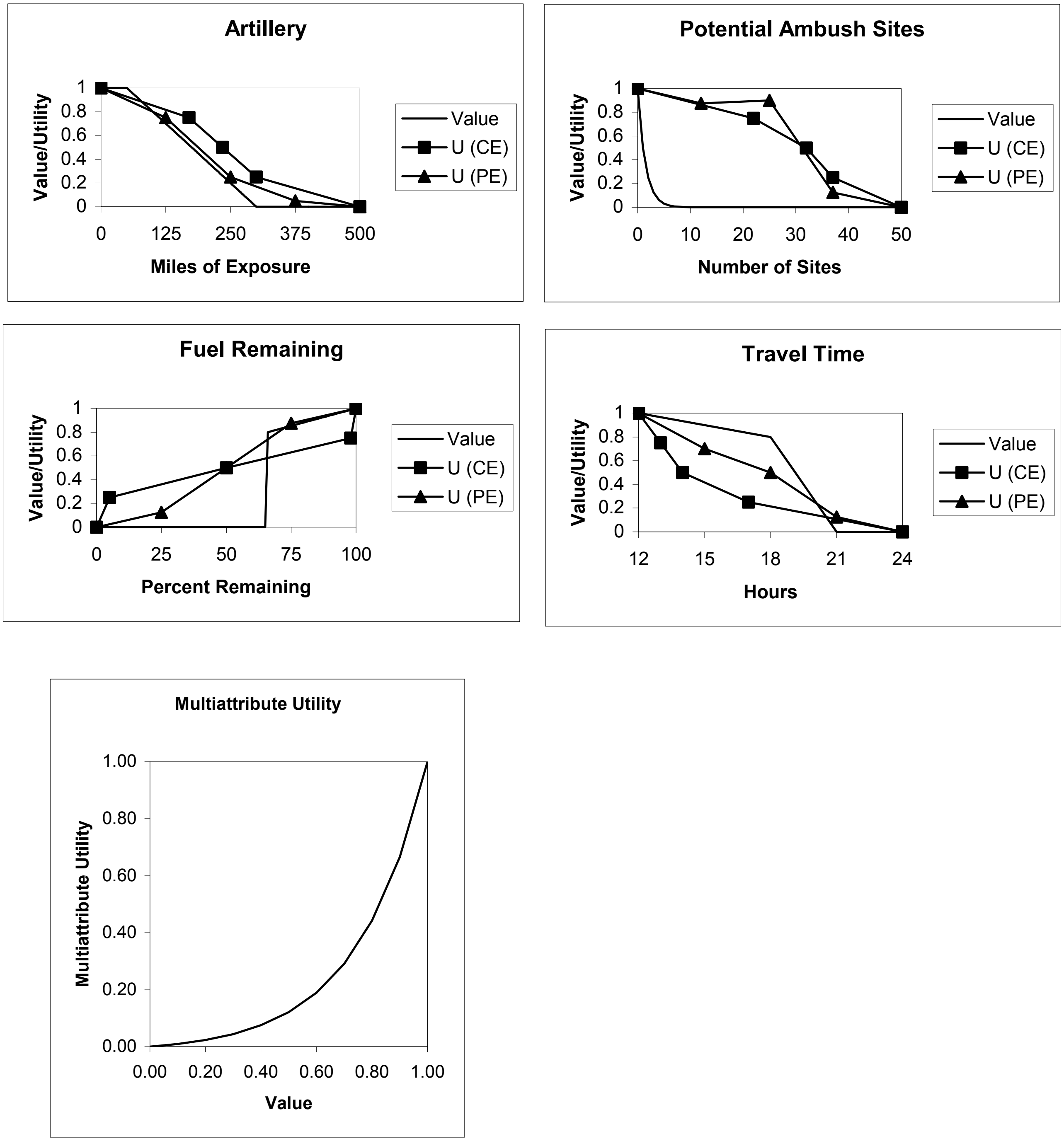


Figure 94. Subject FJ2's Preference Functions.

Subject FJ3

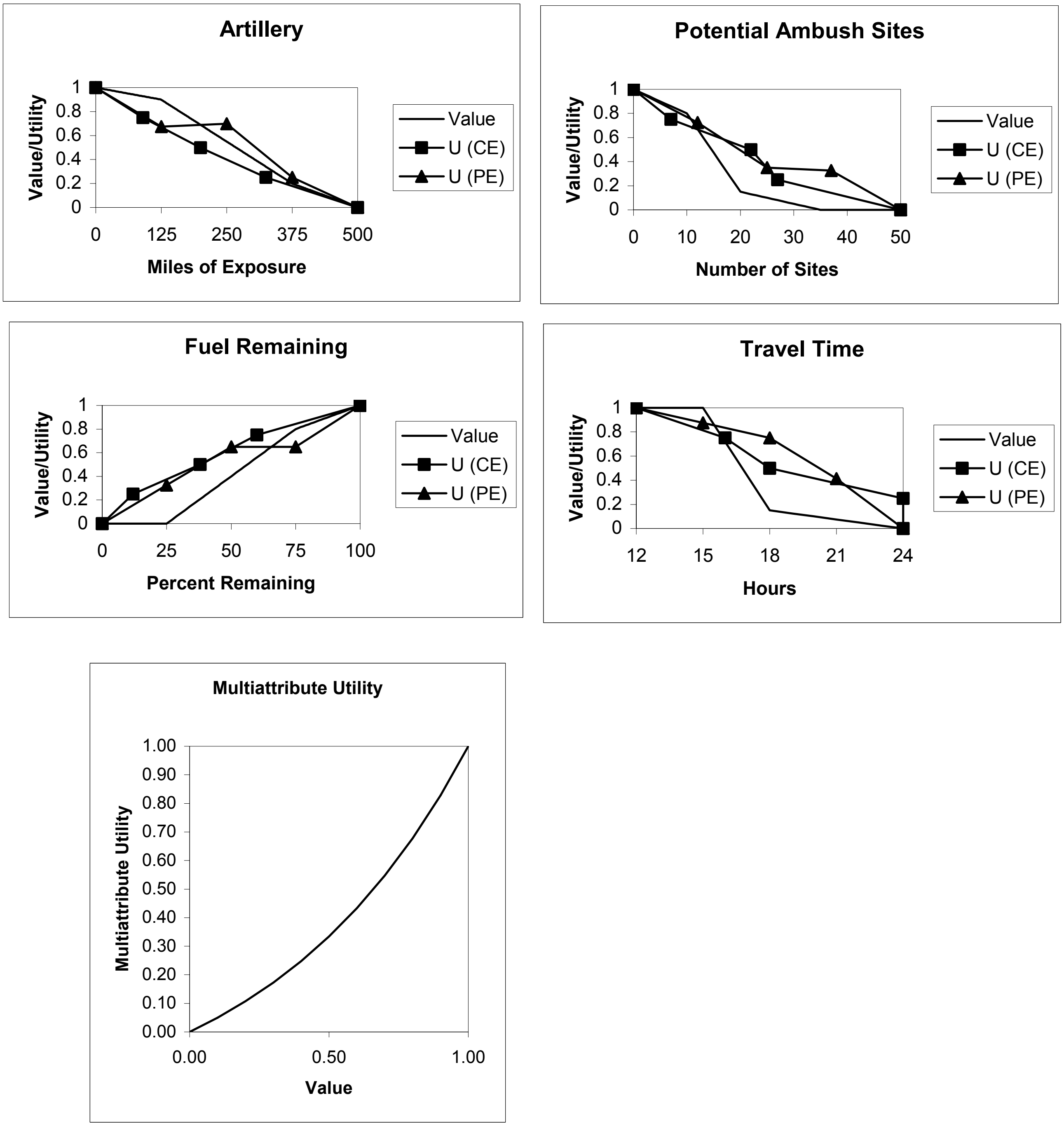


Figure 95. Subject FJ3's Preference Functions.

Subject FJ4

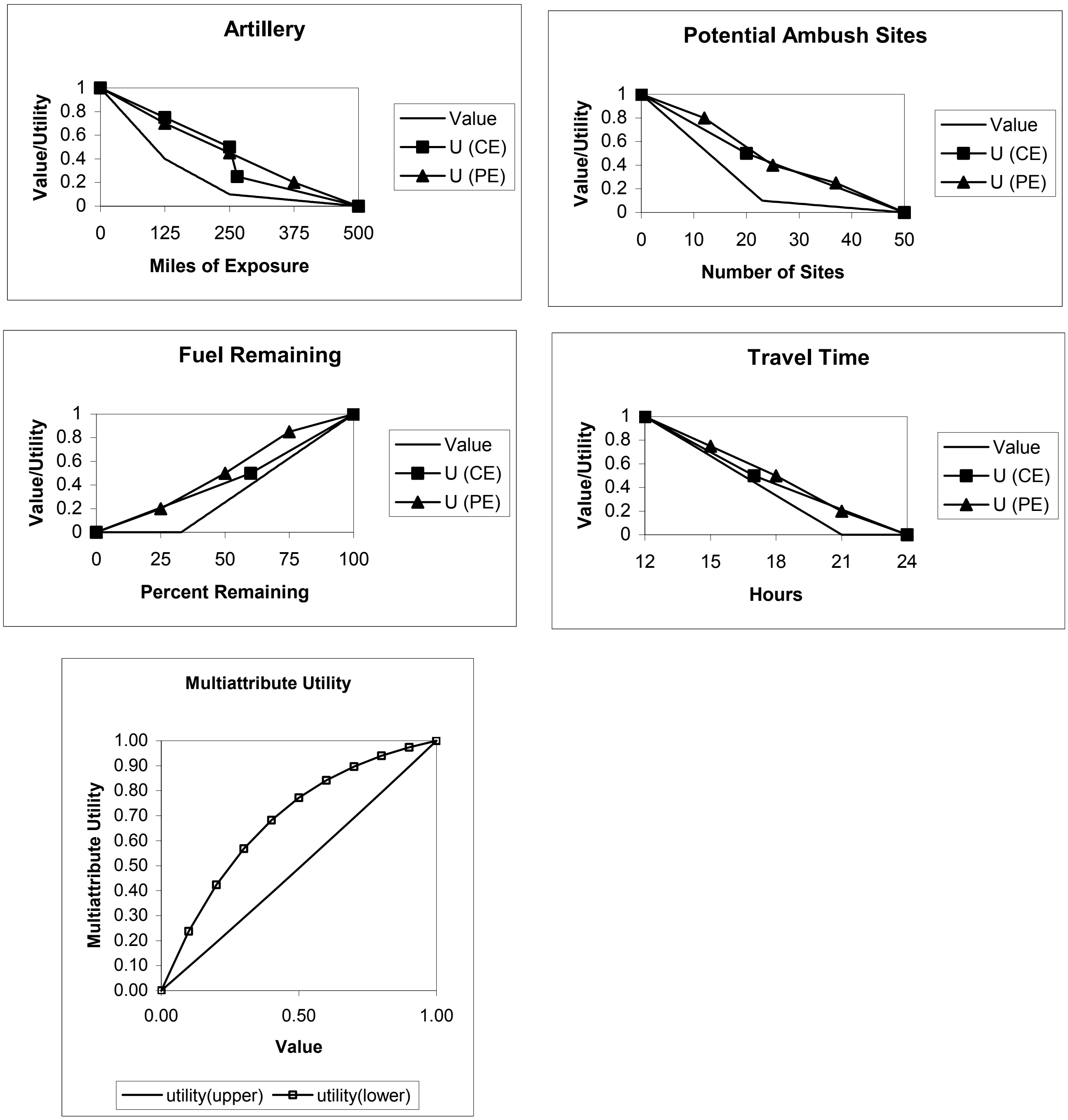


Figure 96. Subject FJ4's Preference Functions.

Subject FJ5

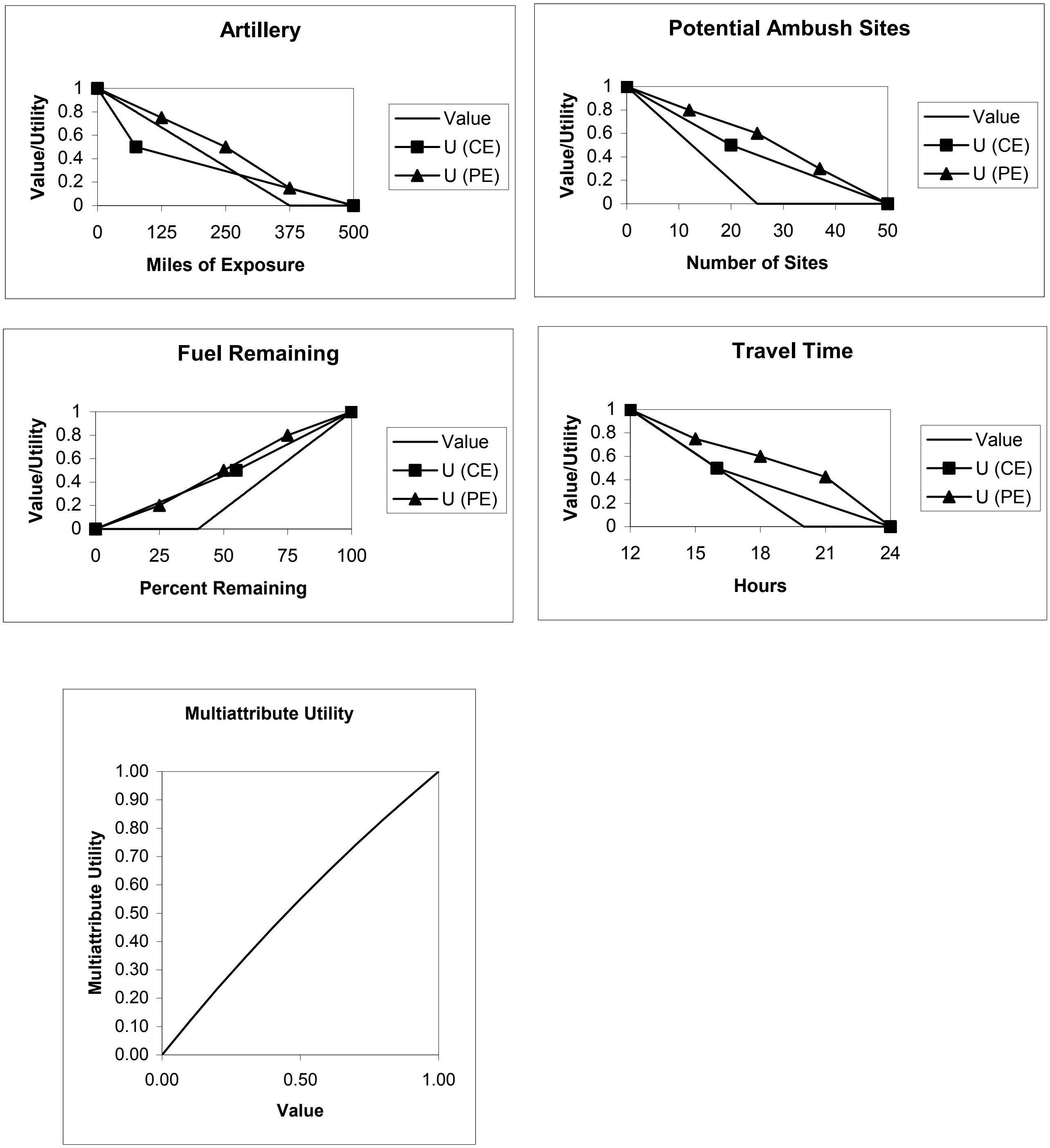


Figure 97. Subject FJ5's Preference Functions.

Subject FJ6

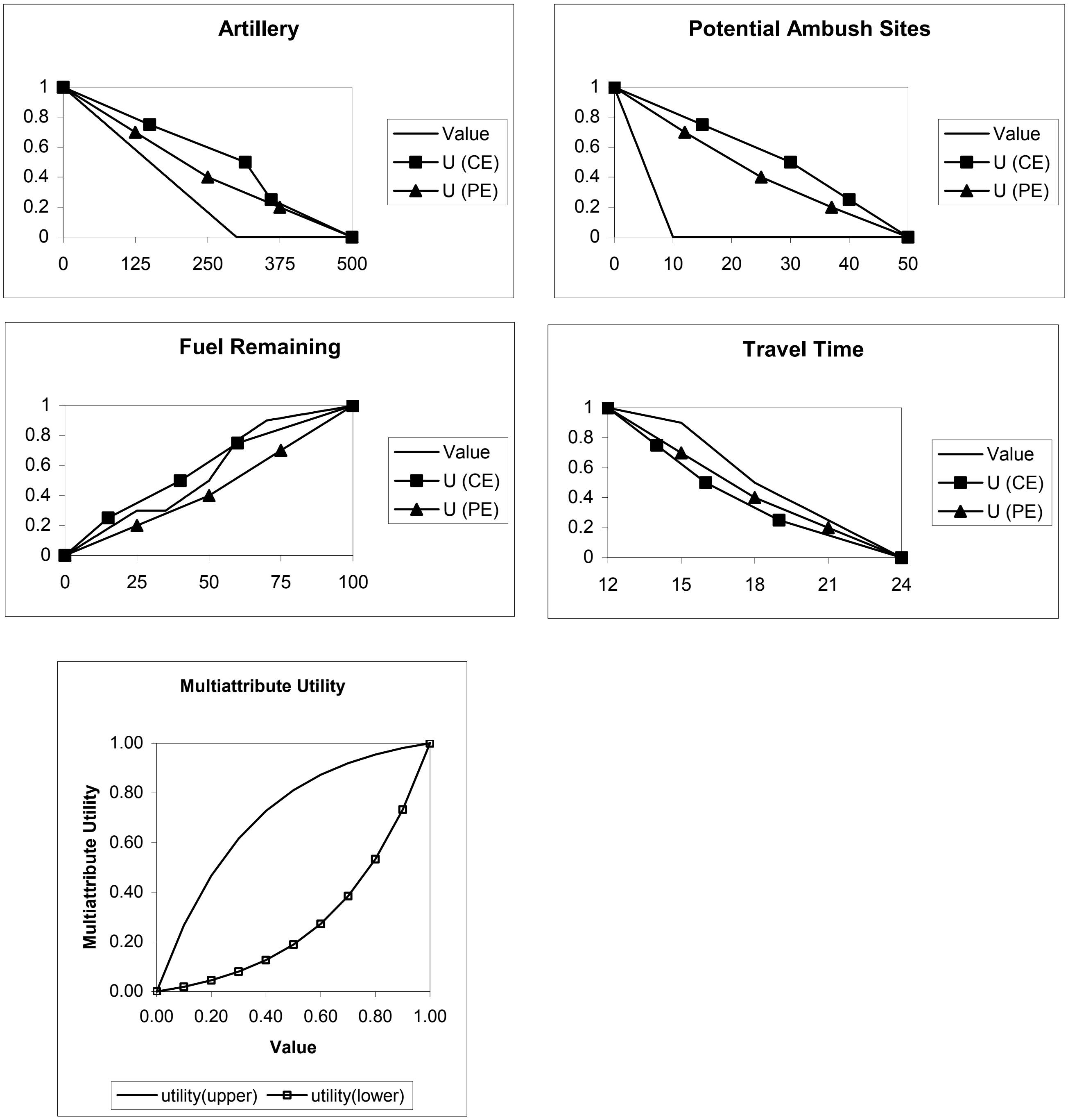


Figure 98. Subject FJ6's Preference Functions.

Subject FJ7

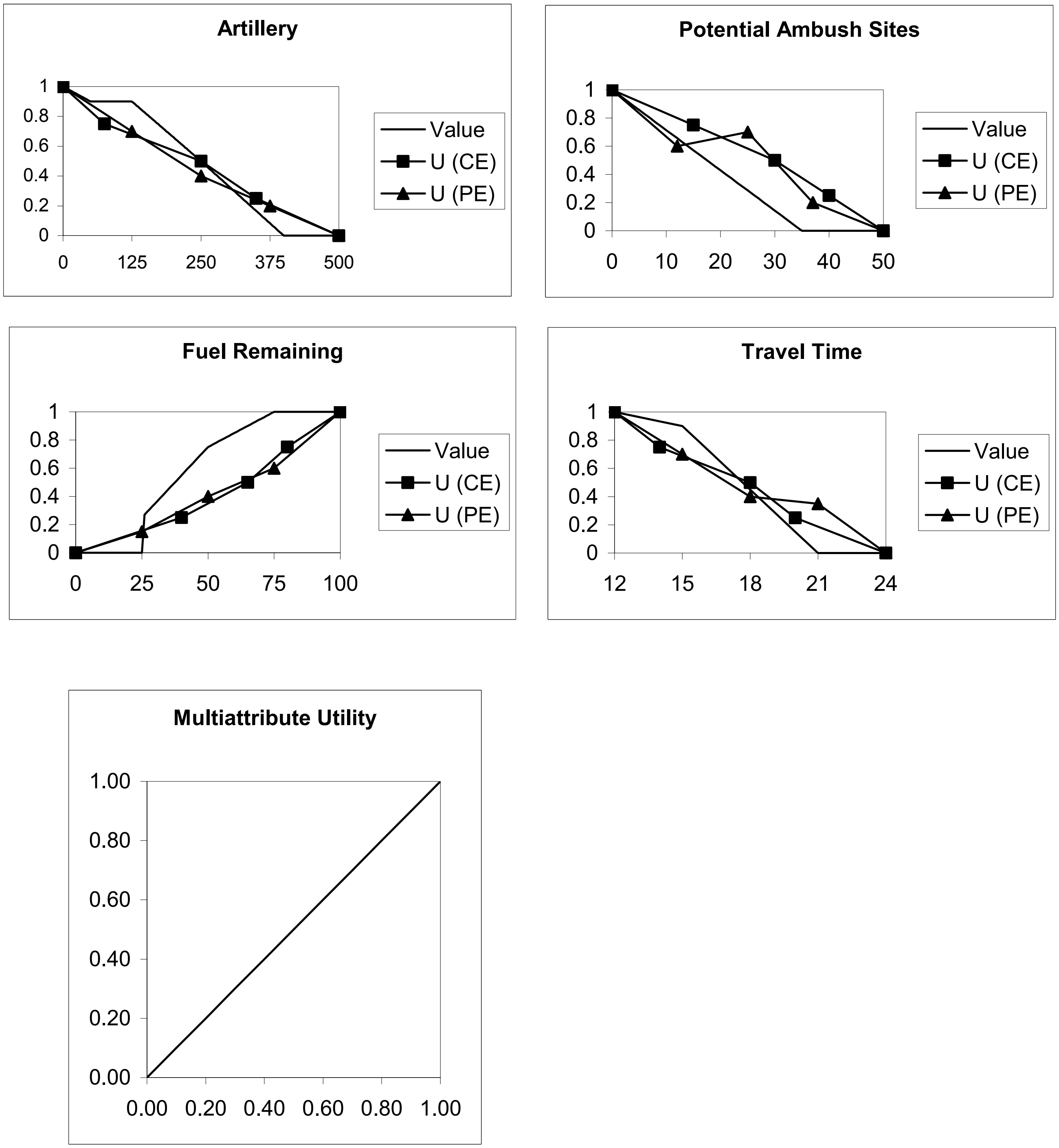


Figure 99. Subject FJ7's Preference Functions.

Subject FJ8

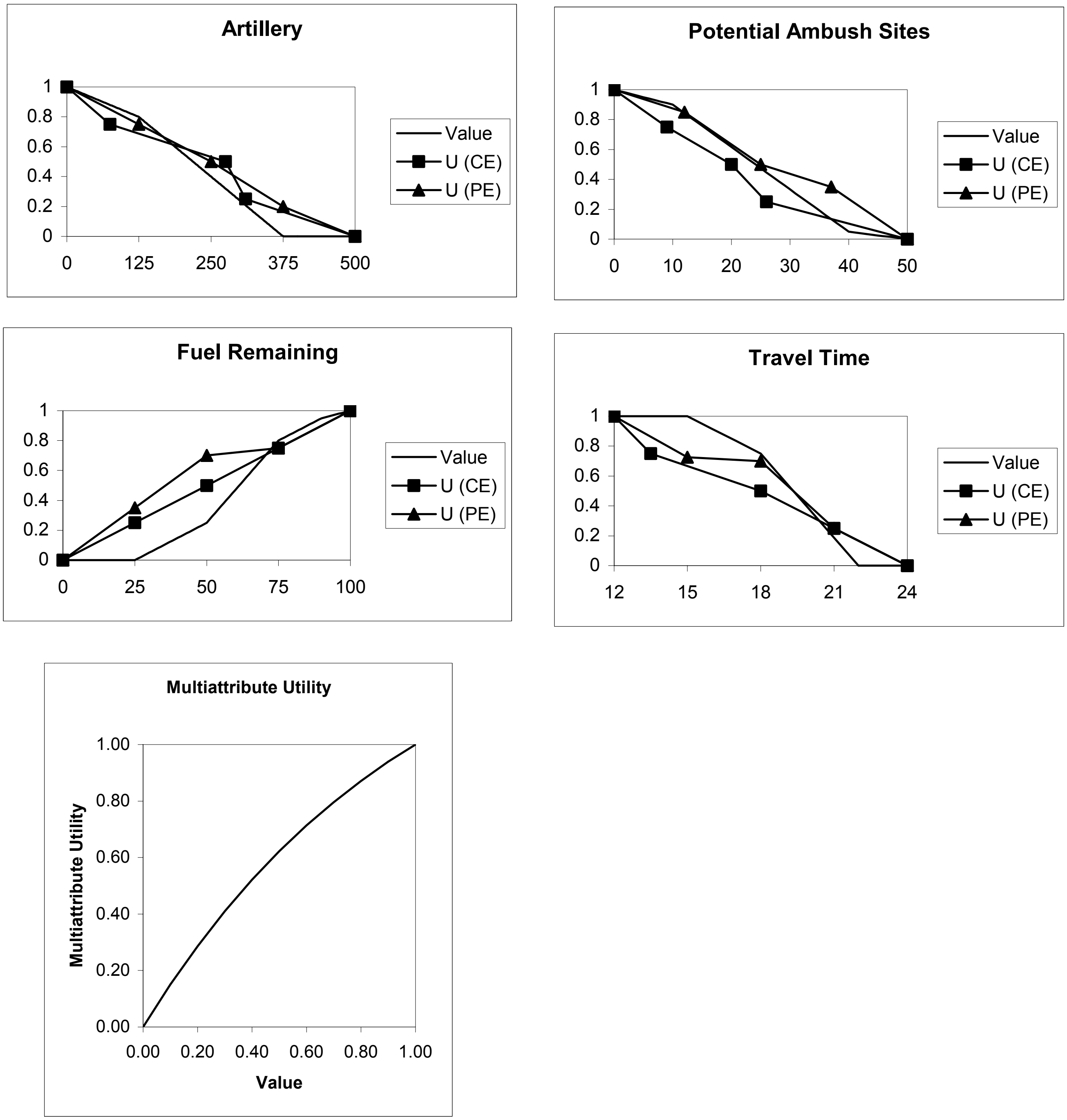


Figure 100. Subject FJ8's Preference Functions.

Subject FJ9

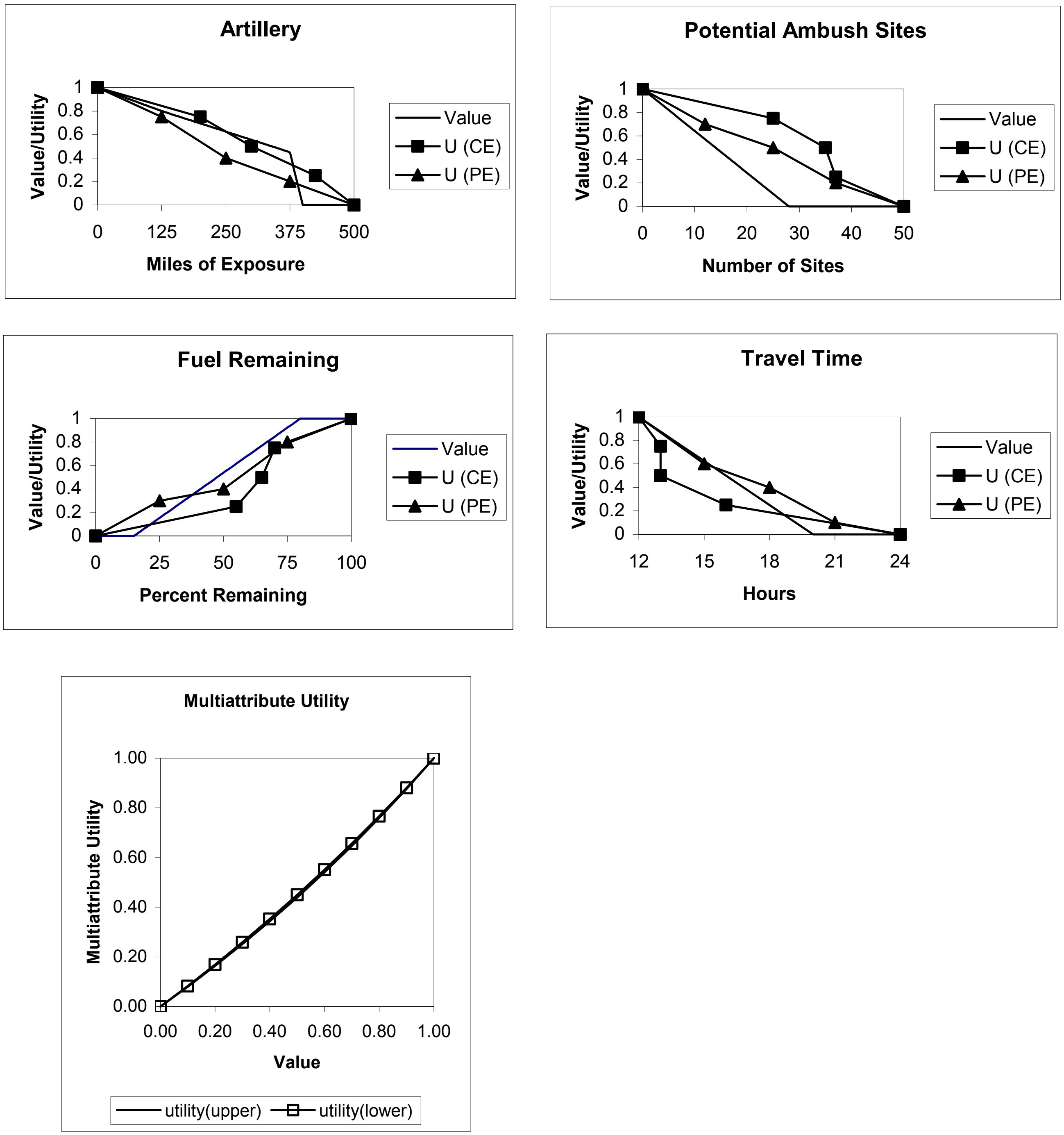


Figure 101. Subject FJ9's Preference Functions.

Subject FJ10

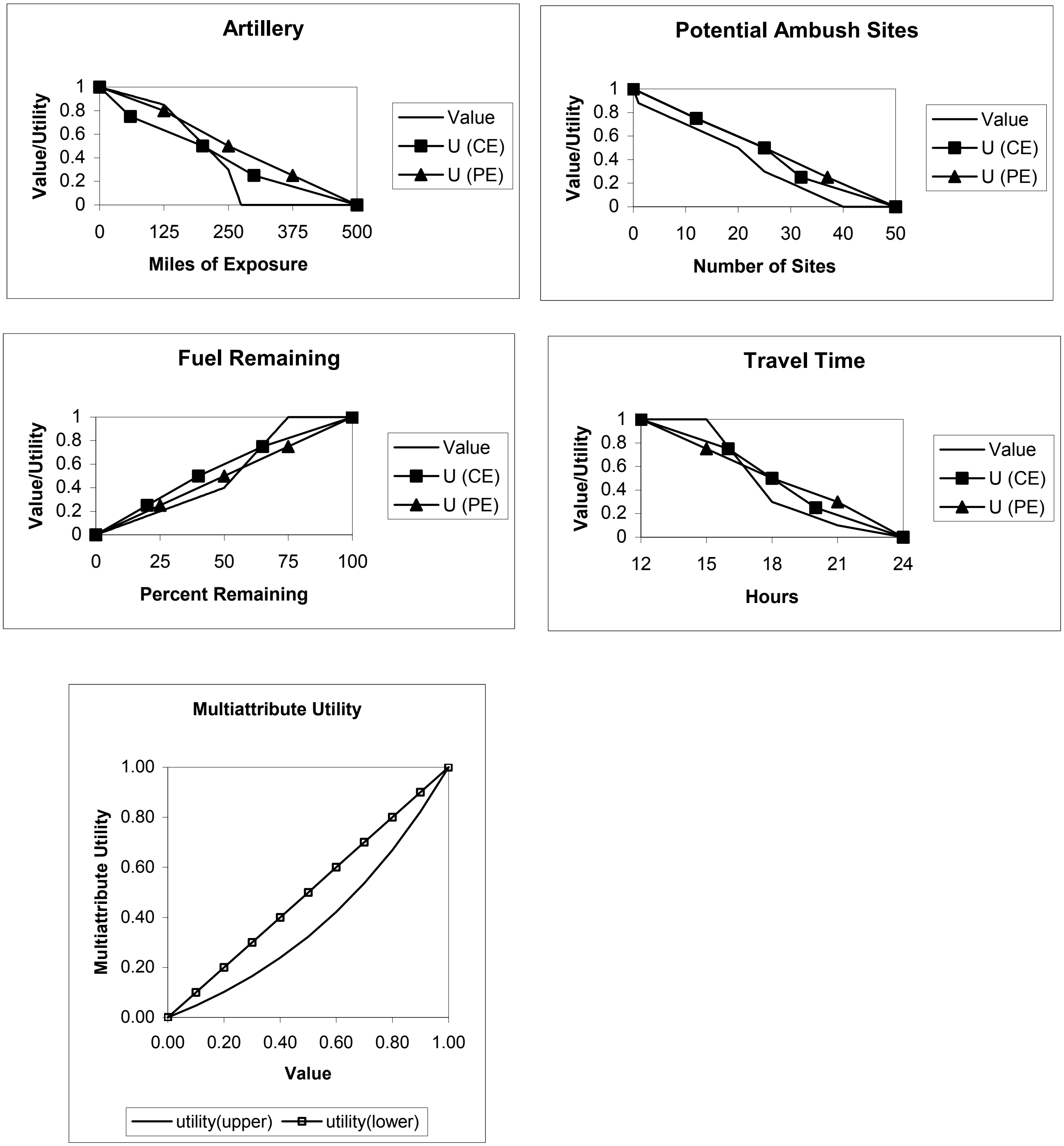


Figure 102. Subject FJ10's Preference Functions.

Subject FJ11

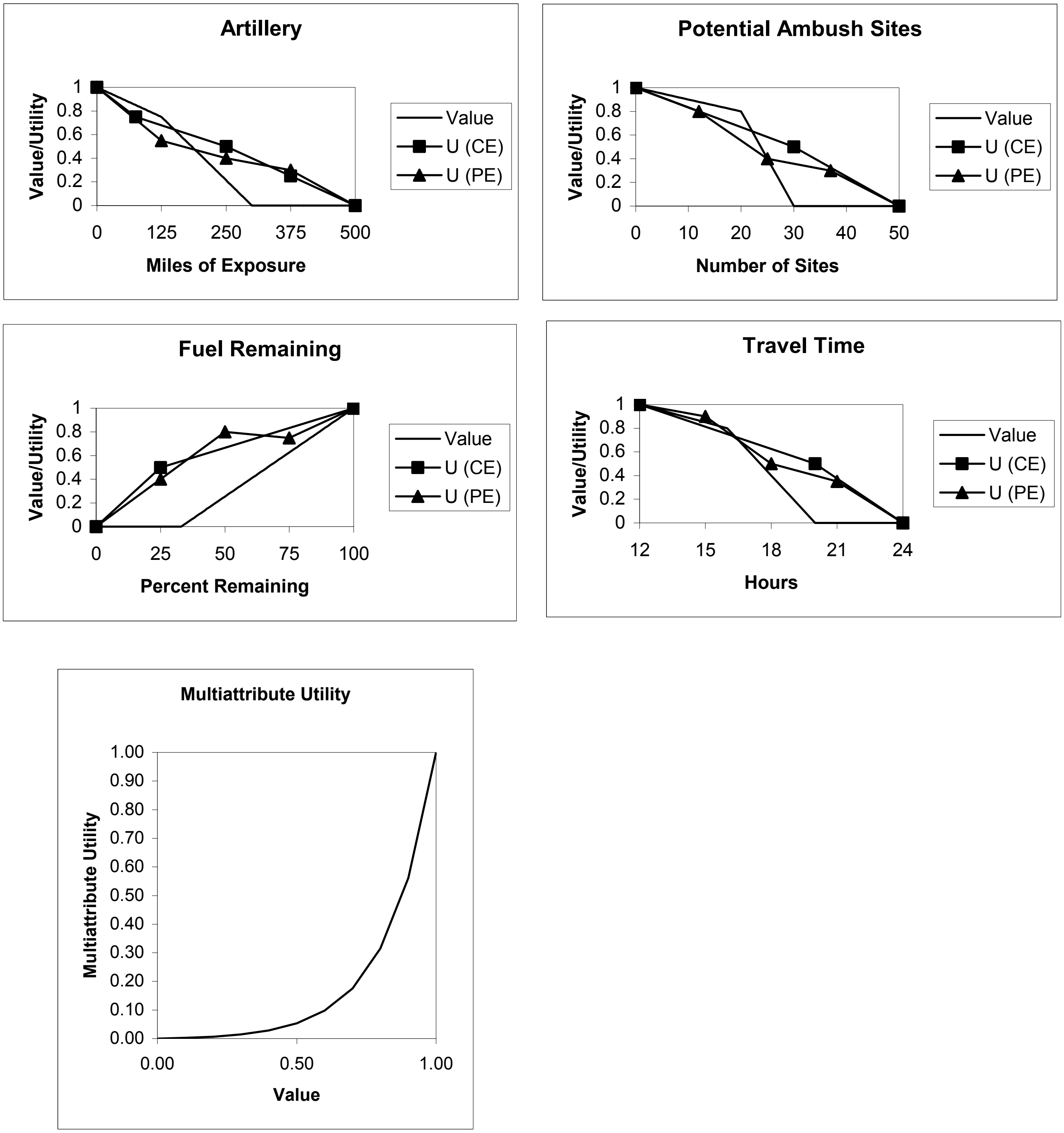
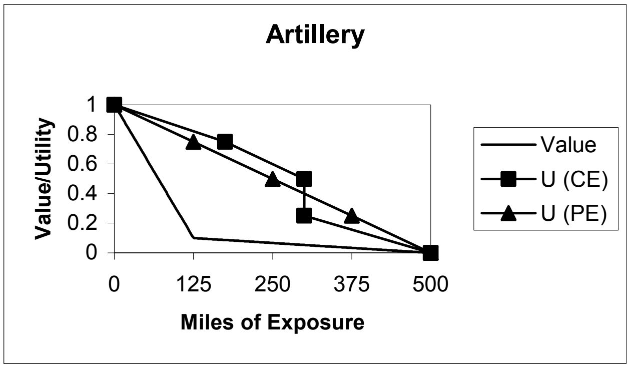
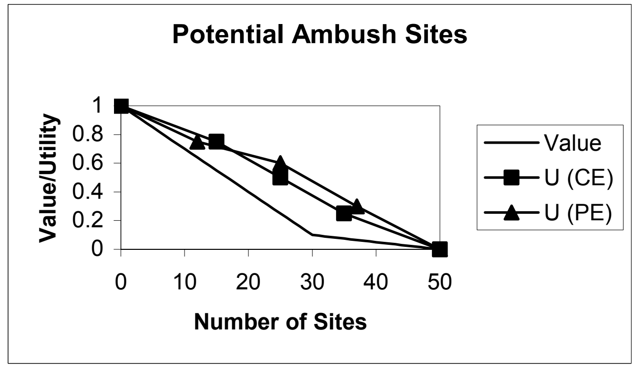
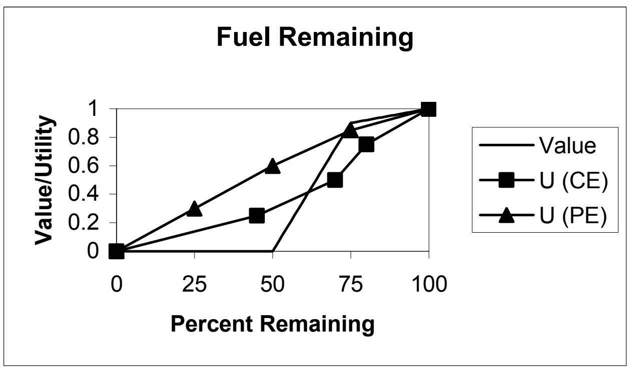


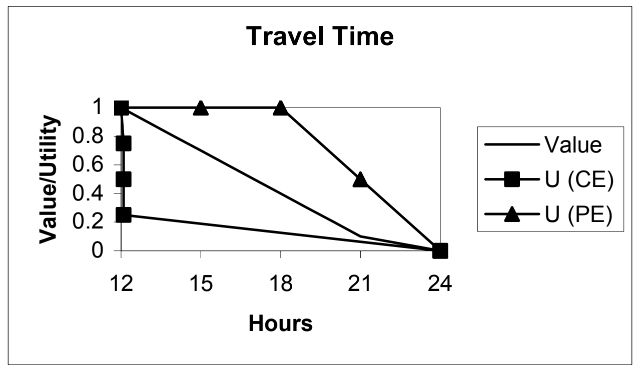
Figure 103. Subject FJ11's Preference Functions.

Subject FJ12









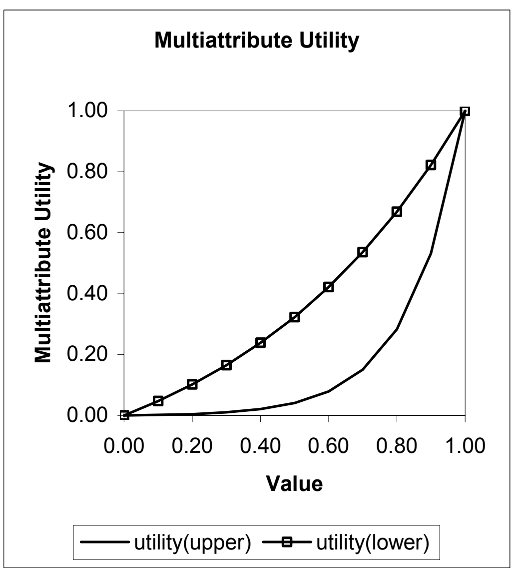


Figure 104. Subject FJ12's Preference Functions.

Subject F13

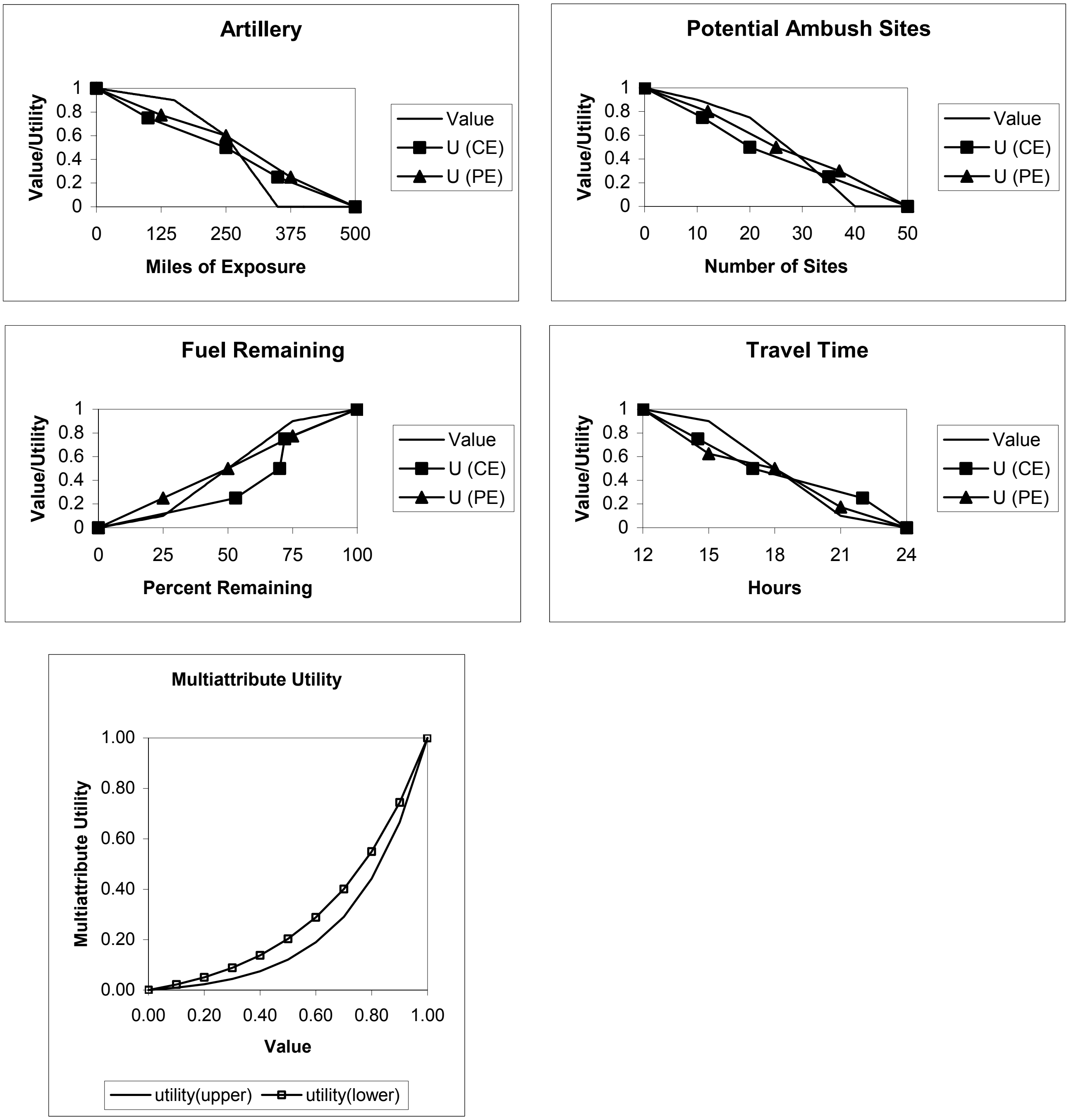


Figure 105. Subject FJ13's Preference Functions.

Subject FJ14

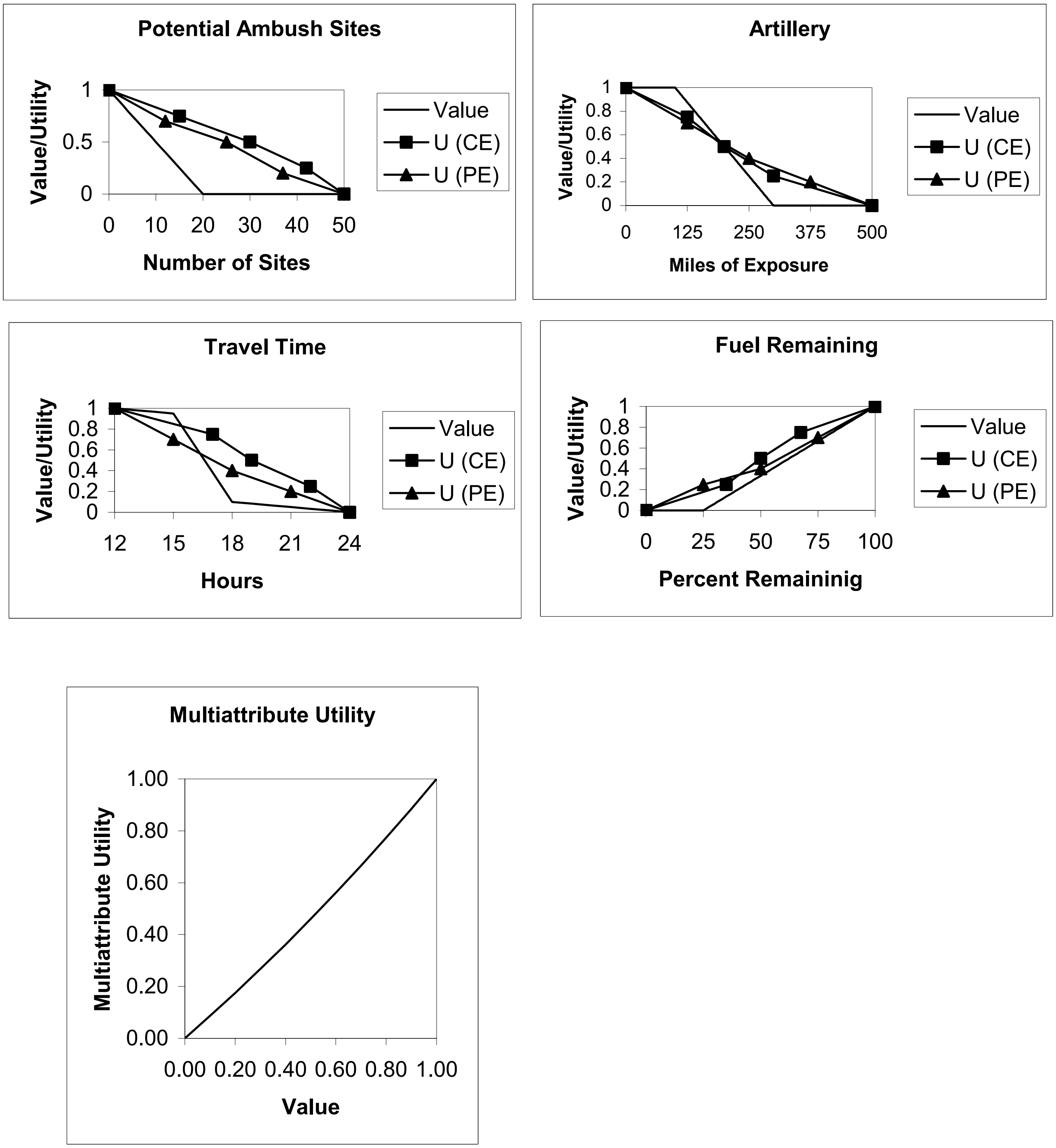


Figure 106. Subject FJ14's Preference Functions.

Subject FJ15

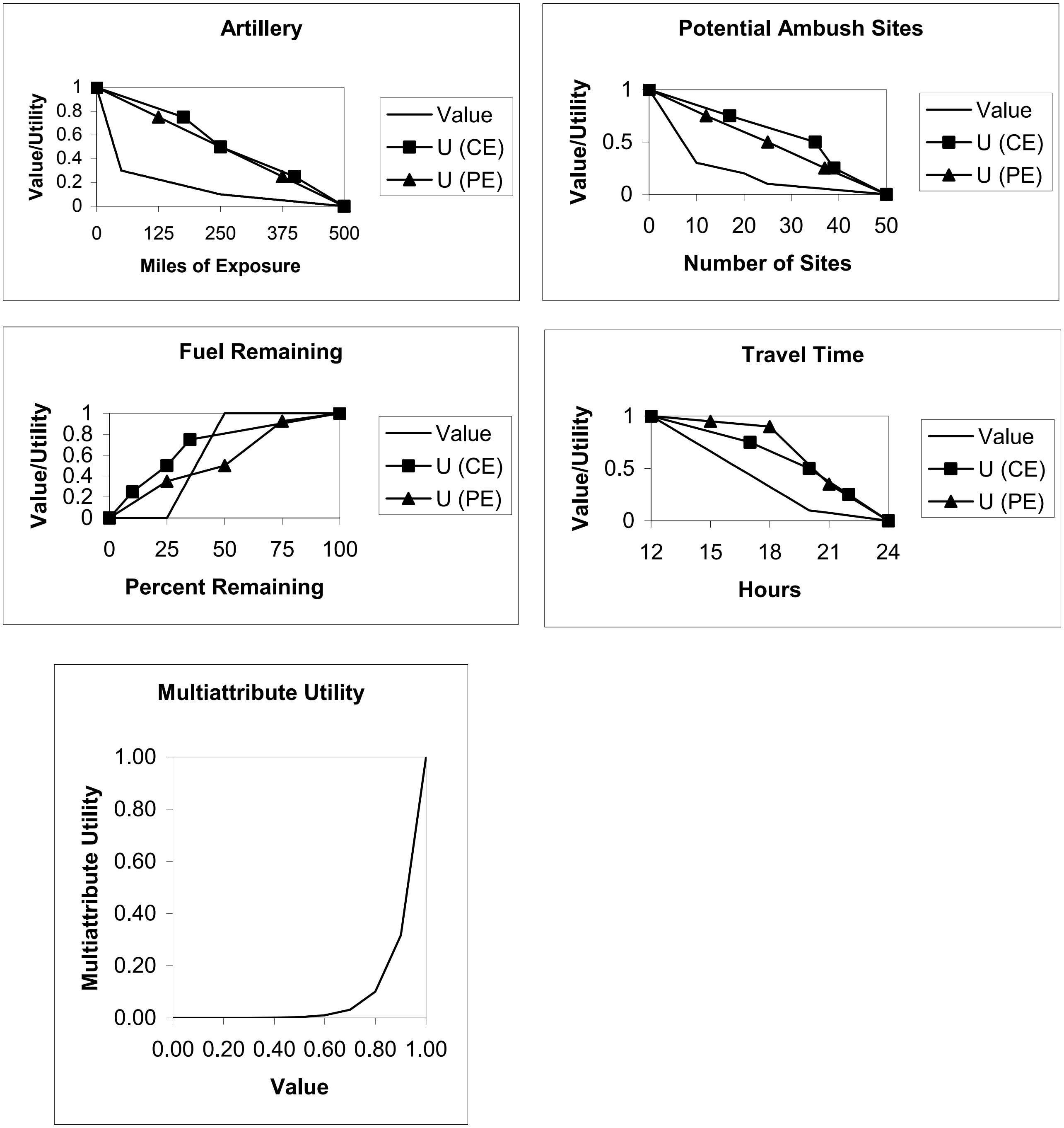
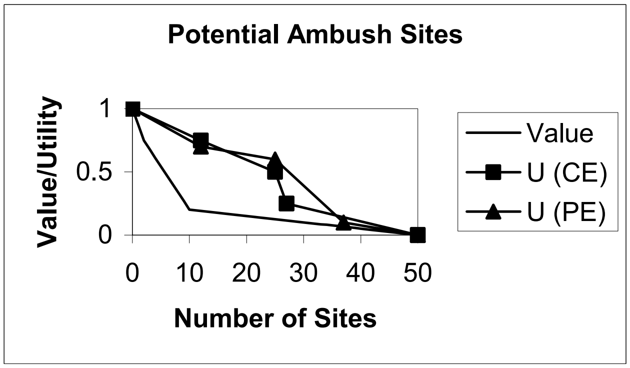
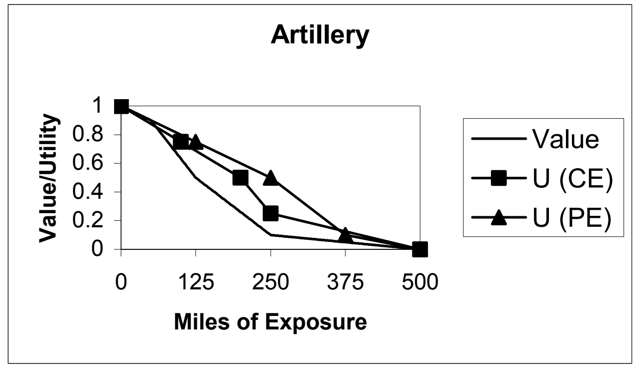
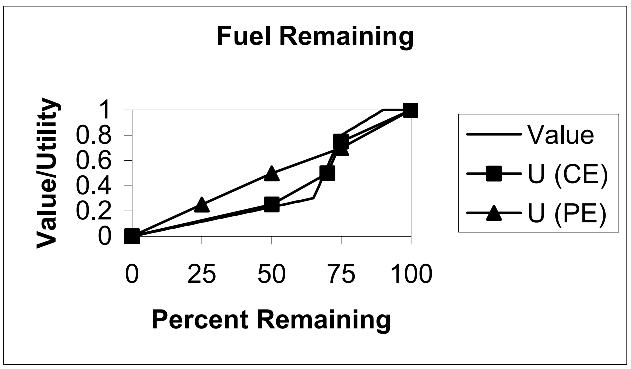


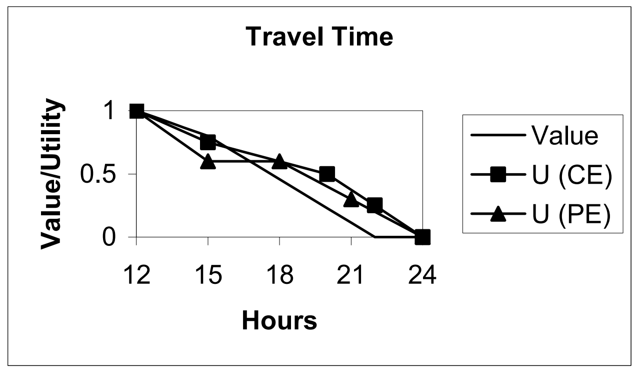
Figure 107. Subject FJ15's Preference Functions.

Subject FJ16









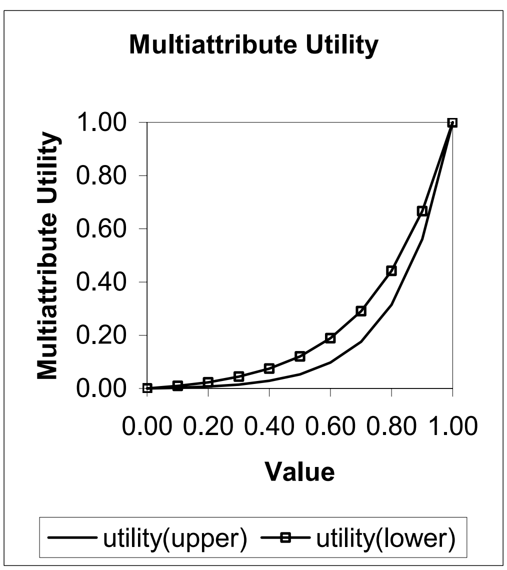
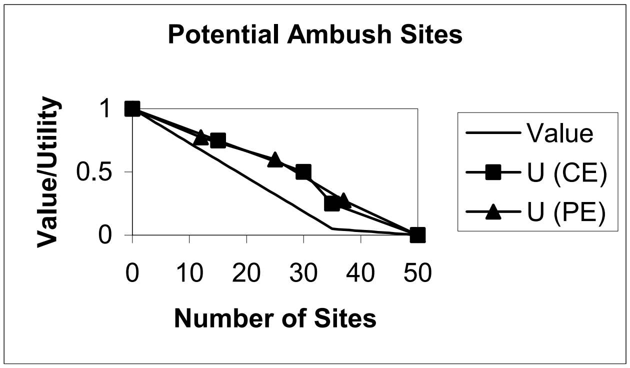
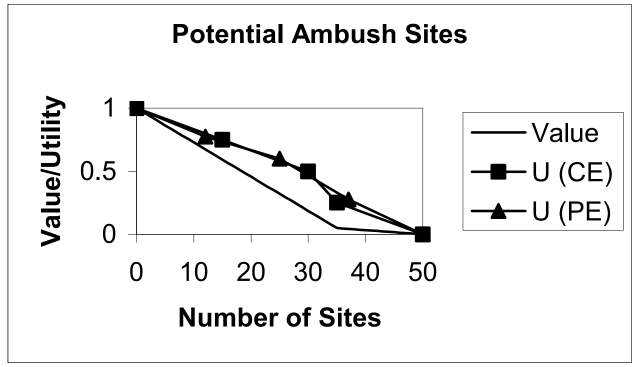
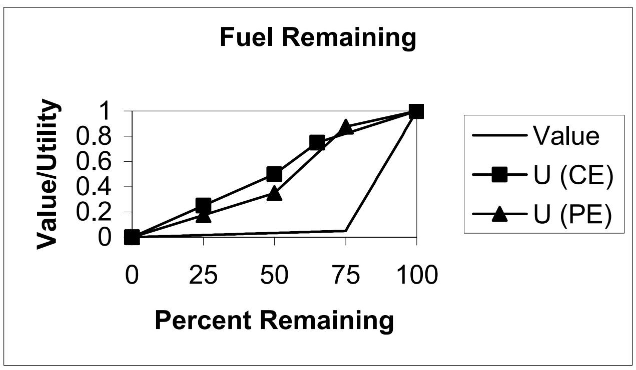


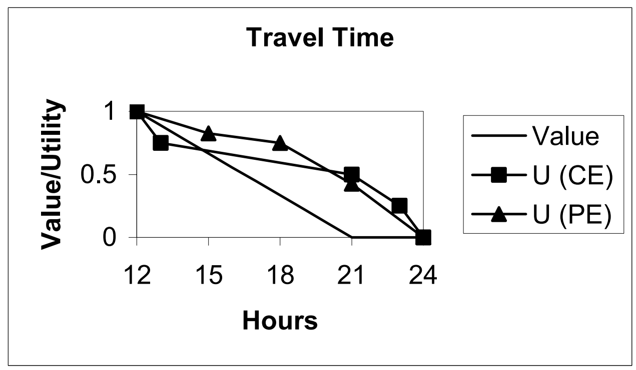
Figure 108. Subject FJ16's Preference Functions.

Subject FJ17









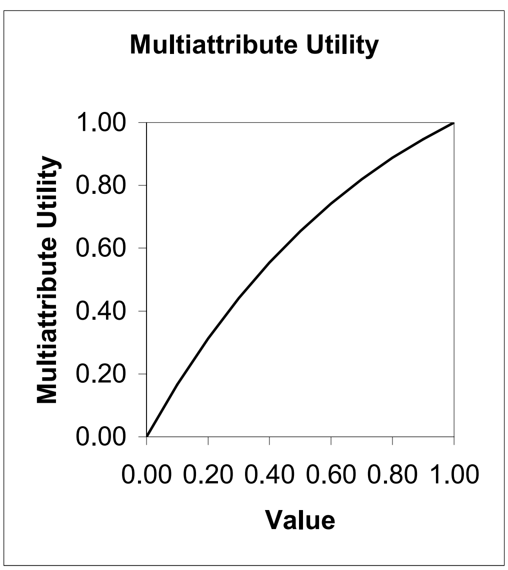
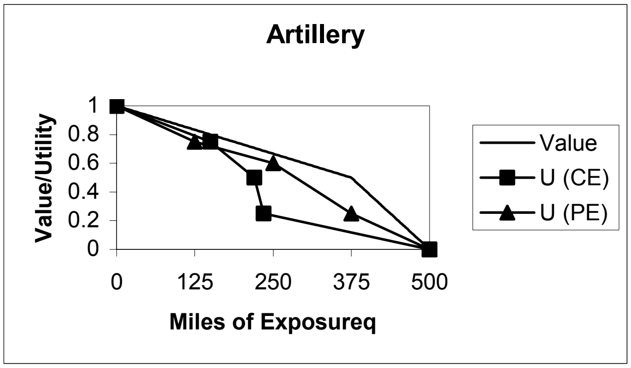
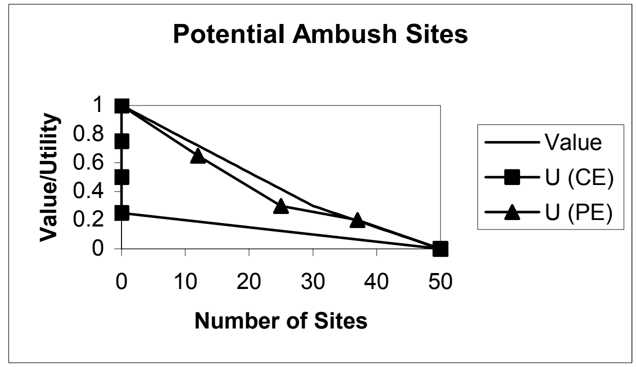
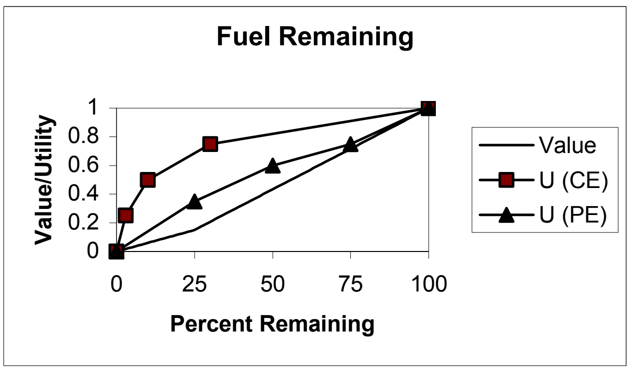


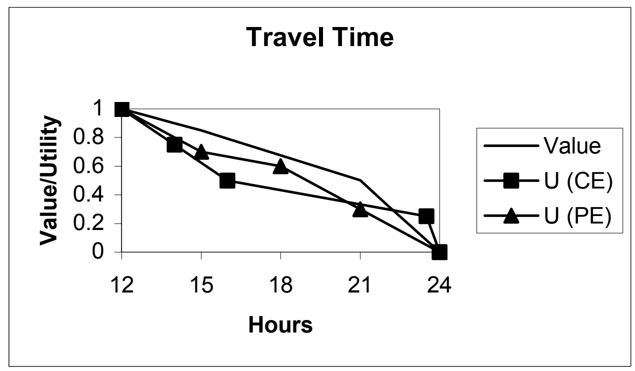
Figure 109. Subject FJ17's Preference Functions.

Subject FJ18









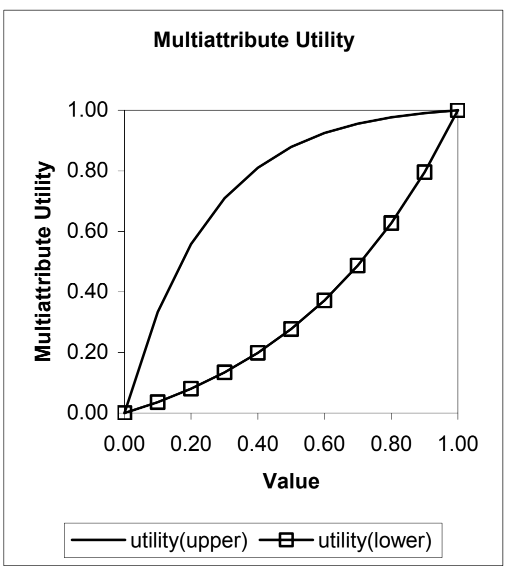
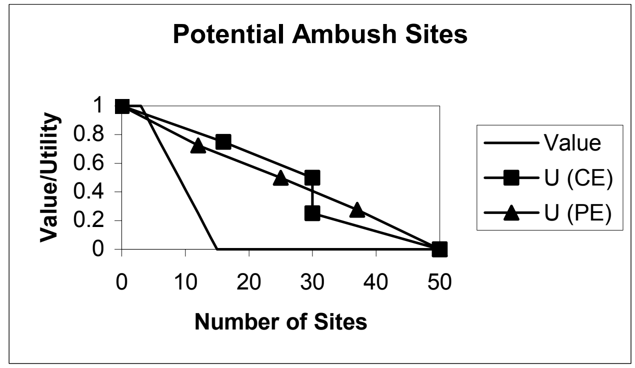
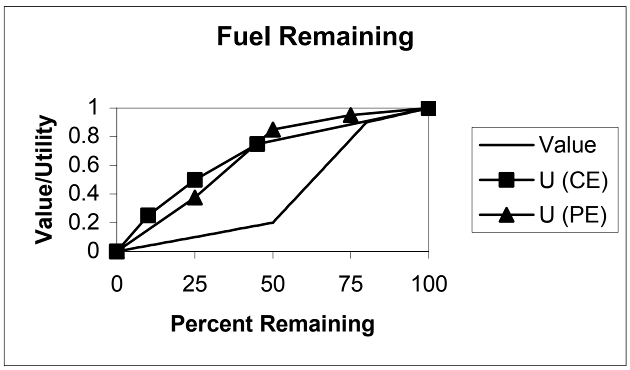
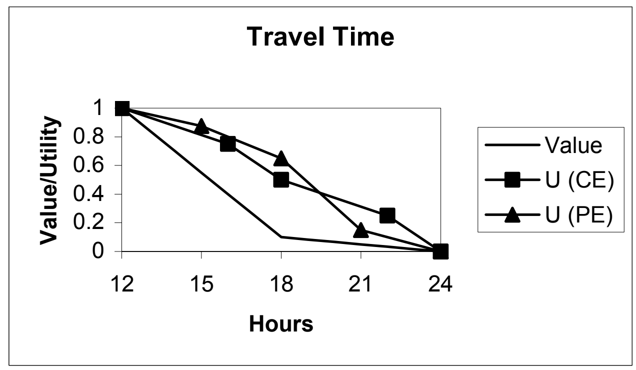


Figure 110. Subject FJ18's Preference Functions.

Subject FJ19







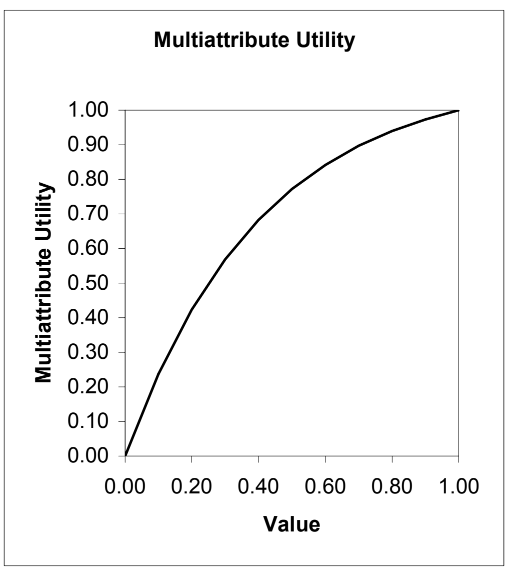


Figure 111. Subject FJ19’s Preference Functions.

Subject FJ20

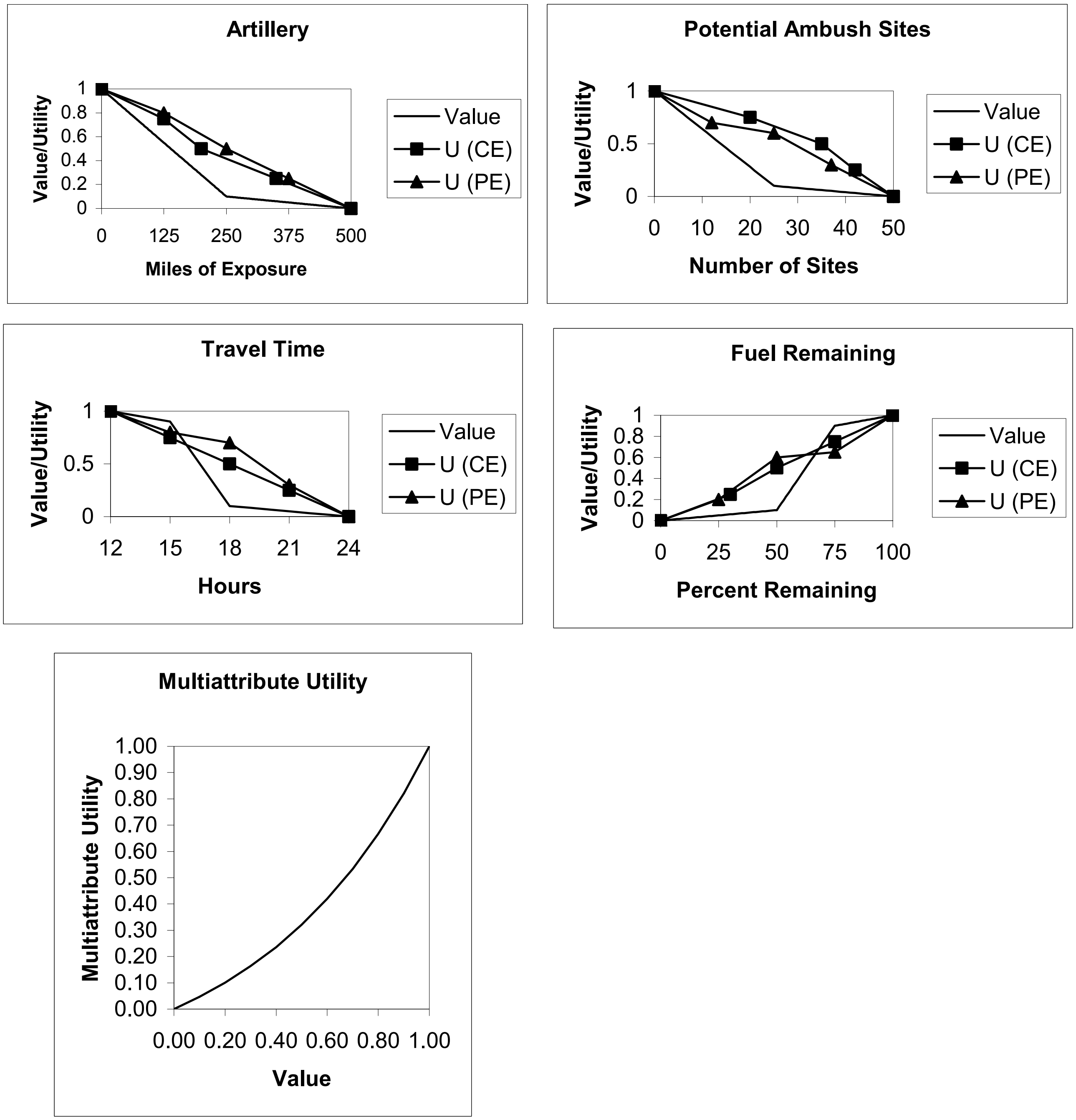


Figure 112. Subject FJ20's Preference Functions.

Table 45. Subjects' Attribute Weightings and Utility Data.

ID	Weights		CE Utility				PE Utility				Multattribute Utility	
	Att	w_i	Att	0.25	0.5	0.75	Att	Q_1	Q_2	Q_3	Att	x_i
FJ1	FA	0.27	FA	325	275	100	FA	0.30	0.55	0.80	FA	
	Amb	0.40	Amb	40	25	12	Amb	0.30	0.55	0.80	Amb	
	Fuel	0.22	Fuel	27	57	65	Fuel	0.80	0.55	0.30	Fuel	
	Time	0.11	Time	20	18.5	14	Time	0.30	0.55	0.80	Time	
FJ2	FA	0.17	FA	300	235	170	FA	0.03	0.25	0.63	FA	0
	Amb	0.33	Amb	37	32	22	Amb	0.23	0.90	0.98	Amb	0
	Fuel	0.33	Fuel	5	50	98	Fuel	0.88	0.50	0.13	Fuel	100
	Time	0.17	Time	17	14	13	Time	0.13	0.50	0.70	Time	24
FJ3	FA	0.45	FA	325	200	90	FA	0.35	0.70	0.81	FA	0
	Amb	0.30	Amb	27	22	7	Amb	0.23	0.35	0.64	Amb	50
	Fuel	0.21	Fuel	12	38	60	Fuel	0.76	0.65	0.42	Fuel	100
	Time	0.04	Time	24	18	16	Time	0.62	0.75	0.94	Time	24
FJ4	FA	0.25	FA	265	250	125	FA	0.18	0.45	0.67	FA	
	Amb	0.31	Amb		20		Amb	0.20	0.40	0.76	Amb	
	Fuel	0.27	Fuel		60		Fuel	0.85	0.50	0.20	Fuel	
	Time	0.18	Time		17		Time	0.20	0.50	0.75	Time	
FJ5	FA	0.05	FA		75		FA	0.15	0.50	0.75	FA	0
	Amb	0.45	Amb		20		Amb	0.36	0.60	0.84	Amb	50
	Fuel	0,17	Fuel		55		Fuel	0.80	0.50	0.20	Fuel	0
	Time	0.33	Time		16		Time	0.51	0.60	0.80	Time	24
FJ6	FA	0.22	FA	360	315	150	FA	0.16	0.40	0.64	FA	
	Amb	0.44	Amb	40	30	15	Amb	0.16	0.40	0.64	Amb	
	Fuel	0.11	Fuel	15	40	60	Fuel	0.64	0.40	0.16	Fuel	
	Time	0.22	Time	19	16	14	Time	0.16	0.40	0.64	Time	
FJ7	FA	0.17	FA	350	250	75	FA	0.16	0.40	0.64	FA	500
	Amb	0.33	Amb	40	30	15	Amb	0.26	0.70	0.76	Amb	0
	Fuel	0.17	Fuel	40	65	80	Fuel	0.52	0.40	0.12	Fuel	100
	Time	0.33	Time	20	18	14	Time	0.28	0.40	0.64	Time	24
FJ8	FA	0.63	FA	310	275	75	FA	0.20	0.50	0.75	FA	500
	Amb	0.13	Amb	26	20	9	Amb	0.35	0.50	0.85	Amb	0
	Fuel	0.20	Fuel	25	50	75	Fuel	0.75	0.70	0.35	Fuel	100
	Time	0.05	Time	21	18	13.5	Time	0.25	0.70	0.73	Time	12
FJ9	FA	0.42	FA	425	300	200	FA	0.16	0.40	0.70	FA	
	Amb	0.21	Amb	37	35	25	Amb	0.20	0.50	0.70	Amb	
	Fuel	0.12	Fuel	55	65	70	Fuel	0.76	0.40	0.24	Fuel	
	Time	0.24	Time	16	13	13	Time	0.08	0.40	0.52	Time	
FJ10	FA	0.25	FA	300	200	60	FA	0.25	0.50	0.80	FA	
	Amb	0.50	Amb	32	25	12	Amb	0.25	0.50	0.75	Amb	
	Fuel	0.17	Fuel	20	40	65	Fuel	0.75	0.50	0.25	Fuel	
	Time	0.08	Time	20	18	16	Time	0.30	0.50	0.75	Time	

Table 45. Subjects' Attribute Weightings and Utility Data, continued.

ID	Weights		CE Utility				PE Utility				Multattribute Utility	
	Att	w_i	Att	0.25	0.5	0.75	Att	Q_1	Q_2	Q_3	Att	x_i
FJ11	FA	0.53	FA	325	250	75	FA	0.24	0.40	0.46	FA	0
	Amb	0.27	Amb		30		Amb	0.24	0.40	0.76	Amb	0
	Fuel	0.12	Fuel		25		Fuel	0.90	0.80	0.64	Fuel	0
	Time	0.08	Time		20		Time	0.35	0.50	0.90	Time	12
FJ12	FA	0.55	FA	300	300	175	FA	0.25	0.50	0.75	FA	
	Amb	0.11	Amb	35	25	15	Amb	0.36	0.60	0.80	Amb	
	Fuel	0.30	Fuel	45	70	80	Fuel	0.88	0.60	0.36	Fuel	
	Time	0.03	Time	12.15	12.1	12.05	Time	1	1	1	Time	
FJ13	FA	0.61	FA	350	250	100	FA	0.30	0.60	0.82	FA	
	Amb	0.06	Amb	35	20	11	Amb	0.30	0.50	0.80	Amb	
	Fuel	0.17	Fuel	53	70	72	Fuel	0.78	0.50	0.25	Fuel	
	Time	0.17	Time	22	17	14.5	Time	0.18	0.50	0.63	Time	
FJ14	FA	0.45	FA	300	200	125	FA	0.16	0.40	0.64	FA	0
	Amb	0.38	Amb	42	30	15	Amb	0.20	0.50	0.70	Amb	50
	Fuel	0.08	Fuel	35	50	67.5	Fuel	0.64	0.40	0.20	Fuel	0
	Time	0.08	Time	22	19	17	Time	0.16	0.40	0.64	Time	12
FJ15	FA	0.42	FA	400	250	175	FA	0.25	0.50	0.75	FA	0
	Amb	0.42	Amb	39	35	17	Amb	0.25	0.50	0.75	Amb	0
	Fuel	0.11	Fuel	10	25	35	Fuel	0.93	0.50	0.35	Fuel	0
	Time	0.06	Time	22	20	17	Time	0.63	0.90	0.99	Time	24
FJ16	FA	0.10	FA	250	200	100	FA	0.10	0.50	0.75	FA	
	Amb	0.81	Amb	27	25	12	Amb	0.12	0.60	0.76	Amb	
	Fuel	0.07	Fuel	50	70	75	Fuel	0.70	0.50	0.25	Fuel	
	Time	0.02	Time	22	20	15	Time	0.36	0.60	0.68	Time	
FJ17	FA	0.19	FA	340	300	150	FA	0.36	0.55	0.82	FA	0
	Amb	0.56	Amb	35	30	15	Amb	0.33	0.60	0.82	Amb	50
	Fuel	0.17	Fuel	25	50	65	Fuel	0.84	0.35	0.12	Fuel	100
	Time	0.08	Time	23	21	13	Time	0.64	0.75	0.91	Time	24
FJ18	FA	0.08	FA	235	220	150	FA	0.30	0.60	0.80	FA	
	Amb	0.08	Amb	0	0	0	Amb	0.12	0.30	0.51	Amb	
	Fuel	0.21	Fuel	3	10	30	Fuel	0.80	0.60	0.42	Fuel	
	Time	0.62	Time	23.5	16	14	Time	0.36	0.60	0.76	Time	
FJ19	FA	0.38	FA	325	200	75	FA	0.33	0.65	0.81	FA	500
	Amb	0.38	Amb	30	30	16	Amb	0.28	0.50	0.73	Amb	50
	Fuel	0.17	Fuel	10	25	45	Fuel	0.99	0.85	0.64	Fuel	100
	Time	0.08	Time	22	18	16	Time	0.20	0.65	0.91	Time	12
FJ20	FA	0.17	FA	350	200	125	FA	0.25	0.50	0.80	FA	0
	Amb	0.33	Amb	42	35	20	Amb	0.36	0.60	0.76	Amb	0
	Fuel	0.33	Fuel	30	50	75	Fuel	0.72	0.60	0.24	Fuel	0
	Time	0.17	Time	21	18	15	Time	0.42	0.70	0..88	Time	12

Table 46. Subjects' Value Function Data.

ID		FA				Ambush				Fuel				Time			
FJ1	x	0	125	375	500	0	10	20	40	0	33	100		0	21	24	
	v	1	.8	.4	0	1	.8	.7	0	0	0	1		1	0	0	
FJ2	x	0	50	300	500	1	2	3	20	0	66	75	100	12	18	21	24
	v	1	1	0	0	.5	.4	.3	0	0	0	.8	1	1	.8	0	0
FJ3	x	0	125	375	500	0	10	20	35	0	25	75	100	12	15	18	24
	v	1	.9	.2	0	1	.8	.15	0	0	0	.8	1	1	1	.15	0
FJ4	x	0	125	250	500	0	23	50		0	33	100		12	21	24	
	v	1	.4	.1	0	1	.1	0		0	0	1		1	0	0	
FJ5	x	0	375	500		0	25	50		0	40	100		12	20	24	
	v	1	0	0		1	0	0		0	0	1		1	0	0	
FJ6	x	0	300	500		0	10	50		0	25	35	50	12	15	18	21
	v	1	0	0		1	0	0		0	.3	.3	.5	1	.9	.5	.25
	x									60	70	100		24			
	v									.77	.9	1		0			
FJ7	x	50	125	250	400	0	35	50		25	50	75	100	12	15	21	24
	v	.9	.9	.5	0	1	0	0		.25	.75	1	1	1	.9	0	0
FJ8	x	125	250	375	500	0	10	40	50	25	50	75	90	15	18	22	24
	v	.8	.4	0	0	1	.9	.05	0	0	.25	.8	.95	1	.75	0	0
FJ9	x	0	125	375	400	0	28	50		0	15	80	100	12	20	24	
	v	1	.8	.45	0	1	0	0		0	0	1	1	1	0	0	
FJ10	x	0	125	250	275	0	10	20	25	0	25	50	75	12	15	18	21
	v	1	.85	.3	0	1	.7	.5	.3	0	.2	.4	1	1	1	.3	.1
	x	500				30	40	50		100				24			
	v	1				.2	0	0		1				0			
FJ11	x	0	125	300	500	0	20	30	50	0	33	100		12	16	20	24
	v	1	.75	0	0	1	.8	0	0	0	0	1		1	.8	0	0
FJ12	x	0	125	500		0	30	50		0	50	75	100	12	21	24	
	v	1	.1	0		1	.1	0		0	0	.8	1	1	.1	0	
FJ13	x	150	250	350	500	10	20	30	40	25	50	75	100	12	15	21	24
	v	.9	.6	0	0	.9	.75	.4	0	.1	.5	.9	1	1	.9	.1	0
FJ14	x	0	100	300	500	0	20	50		0	25	100		12	15	18	24
	v	1	1	0	0	1	0	0		0	0	1		1	.95	.1	0
FJ15	x	0	50	250	500	10	20	25	50	0	25	50	100	12	20	24	
	v	1	.3	.1	0	.3	.2	.1	0	0	0	1	1	1	.1	0	
FJ16	x	50	125	250	500	0	2	10	50	65	75	90	100	12	15	22	24
	v	.9	.5	.1	0	1	.75	.2	0	.3	.8	1	1	1	.8	0	0
FJ17	x	0	300	500		0	35	50		0	75	100		12	21	24	
	v	1	.05	0		1	.05	0		0	.05	1		1	0	0	
FJ18	x	0	375	500		0	30	50		0	25	100		12	15	21	24
	v	1	.5	0		1	.3	0		0	.15	1		1	.85	.5	0
FJ19	x	0	125	500		0	3	15	50	0	50	80	100	12	18	24	
	v	1	.9	0		1	1	0	0	0	.2	.9	1	1	.1	0	
FJ20	x	0	250	500		0	25	50		0	50	75	100	12	15	18	24
	v	1	.1	0		1	.1	0		0	.1	.9	1	1	.9	.1	0

Table 47. Subjects' Elicited Simultaneous Equation Coefficients.

ID	FA	Ambush	Fuel	Time
FJ1	100	25	50	14
	250	10	75	14
	200	28	100	12
	300	8	5	24
FJ4	300	20	50	18
	300	35	100	12
	400	15	50	19
	200	12	20	24
FJ6	100	4	100	18
	300	2	60	20
	300	1	40	12
	125	10	60	19
FJ9	100	25	70	18
	400	18	50	18
	75	20	30	20
	200	10	90	20
FJ10	450	25	70	18
	200	22	20	20
	300	15	45	24
	50	28	30	18
FJ12	300	30	80	15
	100	0	40	21
	50	10	50	24
	300	5	50	18
FJ13	425	25	60	18
	350	50	30	15
	75	0	30	21
	200	15	50	24
FJ16	75	0	55	18
	75	15	75	18
	0	10	55	20
	0	5	30	20
FJ18	100	10	20	18
	200	10	80	12
	500	16	60	14
	250	15	100	18

Table 48. Subjects' Demographic Data.

ID	Ver	YOS	CA/NCA	Age	Gender	Grade
FJ1	B	6	CA	29	M	O3
FJ2	A	26	CA	47	M	E9
FJ3	A	2	NCA	24	M	O2
FJ4	B	14	NCA	33	M	O3
FJ5	B	14	CA	32	M	E6
FJ6	A	24	CA	49	M	O4
FJ7	B	2	NCA	21	M	E2
FJ8	A	10	NCA	44	M	O3
FJ9	B	16	CA	37	M	E7
FJ10	B	5	CA	29	M	O3
FJ11	B	11	CA	31	M	O3
FJ12	A	17	CA	36	M	E8
FJ13	B	6	NCA	27	F	O3
FJ14	B	3.5	NCA	25	F	O2
FJ15	B	8	CA	29	M	E6
FJ16	A	9	CA	28	M	E6
FJ17	A	7	CA	29	M	O3
FJ18	A	6	NCA	26	M	E4
FJ19	A	21	CA	42	M	O5
FJ20	A	20	CA	41	M	O5

Table 49. WRMSE Model Fit Data for $u\left(v\left(x\right)\right)$.

Subj		Linear	Exponential		Logarithmic		Power		Sigmoid	
		WRMSE	c	WRMSE	c	WRMSE	c	WRMSE	c	WRMSE
1	FA	0.075	-0.794	0.030	1.83E+13	0.070	1.334	0.024	-0.277	0.049
	Amb	0.098	0.223	0.097	3.19E+00	0.096	0.844	0.093	-0.251	0.037
	Fuel	0.182	2.307	0.125	4.59E+02	0.182	0.449	0.113	0.267	0.039
	Time	0.133	-1.463	0.211	4.49E+13	0.114	1.676	0.219	-0.306	0.050
2	FA	0.093	0.884	0.082	6.01E-01	0.080	0.699	0.076	-0.394	0.018
	Amb	0.588	112.967	0.342	1.14E+01	0.585	0.024	0.324	-0.537	0.029
	Fuel	0.275	-1.757	0.269	6.53E+04	0.275	2.012	0.269	0.215	0.022
	Time	0.228	-4.860	0.079	4.31E+11	0.228	4.641	0.088	-0.326	0.064
3	FA	0.067	-0.208	0.065	1.55E+06	0.067	1.014	0.067	-0.257	0.046
	Amb	0.155	2.245	0.135	7.40E-02	0.119	0.482	0.105	-0.307	0.035
	Fuel	0.193	2.227	0.155	5.47E+01	0.193	0.430	0.135	0.291	0.055
	Time	0.266	7.094	0.103	1.00E-02	0.061	0.307	0.062	-0.286	0.062
4	FA	0.091	-0.959	0.065	9.15E+09	0.091	1.308	0.075	-0.261	0.039
	Amb	0.227	5.796	0.146	3.79E+02	0.227	0.311	0.107	-0.256	0.034
	Fuel	0.111	1.435	0.073	2.72E-01	0.069	0.596	0.062	0.341	0.025
	Time	0.185	3.774	0.113	5.10E-02	0.096	0.459	0.099	-0.277	0.043
5	FA	0.153	-1.960	0.094	4.62E+07	0.153	1.816	0.110	-0.287	0.051
	Amb	0.235	5.963	0.159	3.63E+01	0.234	0.304	0.119	-0.243	0.036
	Fuel	0.092	0.485	0.088	1.35E+00	0.087	0.782	0.082	0.266	0.027
	Time	0.189	3.034	0.148	6.00E-02	0.128	0.479	0.122	-0.212	0.062
6	FA	0.188	2.279	0.147	7.00E-02	0.138	0.422	0.138	-0.251	0.043
	Amb	0.410	10.252	0.365	2.49E+01	0.409	0.130	0.363	-0.244	0.038
	Fuel	0.082	-0.459	0.075	1.03E+06	0.082	1.100	0.079	0.248	0.036
	Time	0.131	-1.744	0.064	2.91E+07	0.131	1.695	0.079	-0.326	0.049

Subj		Linear	Exponential		Logarithmic		Power		Sigmoid	
		WRMSE	c	WRMSE	c	WRMSE	c	WRMSE	c	WRMSE
7	FA	0.104	-0.880	0.089	2.95E+09	0.104	1.244	0.097	-0.268	0.041
	Amb	0.211	2.951	0.141	4.11E+02	0.211	0.370	0.117	-0.307	0.056
	Fuel	0.236	-4.194	0.153	1.01E+11	0.236	3.773	0.160	0.330	0.061
	Time	0.126	-0.554	0.121	1.02E+06	0.126	1.060	0.125	-0.254	0.044
8	FA	0.087	0.459	0.084	1.41E+00	0.083	0.794	0.079	-0.266	0.053
	Amb	0.068	-0.334	0.064	3.60E+09	0.068	1.048	0.068	-0.295	0.041
	Fuel	0.182	2.650	0.139	5.10E-02	0.122	0.417	0.111	0.274	0.054
	Time	0.151	-1.063	0.142	1.81E+05	0.151	1.311	0.147	-0.254	0.072
9	FA	0.098	-0.892	0.071	1.50E+06	0.098	1.404	0.070	-0.258	0.032
	Amb	0.273	4.346	0.205	3.40E+01	0.272	0.259	0.184	-0.326	0.062
	Fuel	0.122	-1.672	0.076	2.81E+07	0.122	1.712	0.086	0.386	0.058
	Time	0.110	-1.068	0.082	1.04E+11	0.110	1.347	0.093	-0.342	0.101
10	FA	0.146	0.064	0.147	6.92E+00	0.147	0.873	0.146	-0.277	0.032
	Amb	0.117	1.333	0.066	3.24E-01	0.063	0.627	0.056	-0.264	0.032
	Fuel	0.095	0.753	0.082	8.57E-01	0.081	0.788	0.080	0.276	0.037
	Time	0.113	1.271	0.090	8.42E+01	0.113	0.676	0.081	-0.266	0.029
11	FA	0.141	-1.359	0.114	8.01E+05	0.141	1.416	0.128	-0.235	0.067
	Amb	0.232	5.891	0.152	3.31E+01	0.231	0.307	0.113	-0.249	0.038
	Fuel	0.319	7.490	0.219	2.95E+01	0.318	0.183	0.195	0.447	0.098
	Time	0.185	4.231	0.074	4.30E-02	0.058	0.438	0.072	-0.328	0.028
12	FA	0.351	10.819	0.075	7.32E-03	0.113	0.292	0.150	-0.313	0.046
	Amb	0.183	2.980	0.075	5.80E-02	0.039	0.461	0.041	-0.280	0.031
	Fuel	0.193	1.100	0.190	2.10E-01	0.188	0.550	0.184	0.275	0.047
	Time	0.237	1.229	0.229	1.24E-01	0.218	0.527	0.202	-0.085	0.163
13	FA	0.131	-0.399	0.130	7.81E+03	0.131	0.976	0.131	-0.271	0.039
	Amb	0.106	-0.895	0.091	7.12E+09	0.106	1.272	0.098	-0.267	0.041
	Fuel	0.089	-1.121	0.061	6.54E+06	0.089	1.373	0.074	0.350	0.061
	Time	0.109	-0.534	0.104	5.03E+05	0.109	1.033	0.109	-0.242	0.049
14	FA	0.135	0.073	0.135	6.38E+00	0.135	0.862	0.134	-0.329	0.037
	Amb	0.323	5.069	0.271	3.13E+01	0.322	0.232	0.260	-0.256	0.041
	Fuel	0.092	0.952	0.068	5.78E-01	0.066	0.702	0.060	0.295	0.030
	Time	0.178	3.984	0.128	3.90E-02	0.102	0.449	0.099	-0.253	0.034
15	FA	0.367	6.729	0.016	1.50E+02	0.366	0.319	0.135	-0.254	0.037
	Amb	0.325	6.428	0.054	1.93E+02	0.325	0.327	0.106	-0.285	0.051
	Fuel	0.190	1.044	0.186	2.29E-01	0.184	0.562	0.180	0.364	0.064
	Time	0.293	6.713	0.065	1.10E-01	0.148	0.302	0.088	-0.460	0.065
16	FA	0.145	2.585	0.056	1.05E-01	0.040	0.544	0.053	-0.347	0.023
	Amb	0.320	5.224	0.088	1.85E+02	0.320	0.394	0.160	0.005	0.356
	Fuel	0.095	1.204	0.068	4.44E-01	0.065	0.721	0.063	0.326	0.059
	Time	0.135	1.143	0.116	3.19E-01	0.111	0.602	0.098	-0.251	0.082
17	FA	0.213	4.008	0.112	1.80E-02	0.055	0.376	0.054	-0.302	0.047
	Amb	0.169	2.536	0.079	4.48E+02	0.169	0.465	0.030	-0.314	0.050
	Fuel	0.366	19.519	0.080	1.63E+02	0.366	0.256	0.161	0.335	0.033
	Time	0.280	3.625	0.217	7.44E+01	0.280	0.287	0.186	-0.245	0.115
18	FA	0.152	-1.709	0.058	3.25E+07	0.152	1.851	0.049	-0.316	0.037
	Amb	0.326	-5.269	0.140	6.10E+10	0.326	4.925	0.150	-0.391	0.019
	Fuel	0.231	3.446	0.087	3.40E-02	0.037	0.381	0.037	0.303	0.081
	Time	0.115	-1.377	0.052	1.54E+07	0.115	1.603	0.068	-0.194	0.055
19	FA	0.082	-0.890	0.052	2.37E+13	0.073	1.309	0.060	-0.247	0.043
	Amb	0.354	7.302	0.317	3.47E+01	0.353	0.043	0.305	-0.296	0.051
	Fuel	0.322	7.665	0.042	1.97E+02	0.322	0.272	0.110	0.486	0.069
	Time	0.252	5.780	0.061	2.57E+02	0.252	0.366	0.079	-0.327	0.023
20	FA	0.186	3.446	0.075	4.80E-02	0.037	0.452	0.046	-0.282	0.040
	Amb	0.286	7.755	0.101	2.69E+02	0.286	0.316	0.085	-0.303	0.062
	Fuel	0.179	3.996	0.130	4.75E+01	0.178	0.451	0.101	0.272	0.048
	Time	0.220	6.813	0.111	3.22E+01	0.219	0.364	0.088	-0.294	0.042

Table 50. RMSE Model Fit Data for $u\left(v\left(x\right)\right)$.

Subj		Linear	Exponential		Logarithmic		Power		Sigmoid	
		RMSE	c	RMSE	c	RMSE	c	RMSE	c	RMSE
1	FA	0.082	-0.794	0.034	1.830E+13	0.078	1.334	0.027	-0.277	0.046
	Amb	0.101	0.223	0.100	3.189E+00	0.100	0.844	0.096	-0.251	0.034
	Fuel	0.206	2.307	0.139	4.589E+02	0.206	0.449	0.123	0.267	0.037
	Time	0.150	-1.463	0.239	4.493E+13	0.120	1.676	0.251	-0.306	0.052
2	FA	0.104	0.884	0.090	6.010E-01	0.088	0.699	0.082	-0.394	0.015
	Amb	0.612	112.967	0.390	1.137E+01	0.609	0.024	0.370	-0.537	0.031
	Fuel	0.315	-1.757	0.309	6.530E+04	0.315	2.012	0.309	0.215	0.020
	Time	0.258	-4.860	0.089	4.310E+11	0.258	4.641	0.100	-0.326	0.053
3	FA	0.072	-0.208	0.070	1.550E+06	0.072	1.014	0.072	-0.257	0.040
	Amb	0.173	2.245	0.143	7.400E-02	0.123	0.482	0.108	-0.307	0.034
	Fuel	0.214	2.227	0.164	5.466E+01	0.213	0.430	0.141	0.291	0.045
	Time	0.296	7.094	0.111	1.000E-02	0.067	0.307	0.066	-0.286	0.056
4	FA	0.100	-0.959	0.070	9.153E+09	0.100	1.308	0.081	-0.261	0.035
	Amb	0.256	5.796	0.156	3.792E+02	0.255	0.311	0.111	-0.256	0.032
	Fuel	0.125	1.435	0.078	2.720E-01	0.073	0.596	0.065	0.341	0.022
	Time	0.212	3.774	0.126	5.100E-02	0.105	0.459	0.105	-0.277	0.037
5	FA	0.167	-1.960	0.099	4.622E+07	0.167	1.816	0.116	-0.287	0.043
	Amb	0.264	5.963	0.169	3.629E+01	0.263	0.304	0.124	-0.243	0.034
	Fuel	0.100	0.485	0.095	1.353E+00	0.094	0.782	0.087	0.266	0.023
	Time	0.211	3.034	0.164	6.000E-02	0.141	0.479	0.132	-0.212	0.056
6	FA	0.217	2.279	0.167	7.000E-02	0.156	0.422	0.146	-0.251	0.040
	Amb	0.440	10.252	0.403	2.494E+01	0.440	0.130	0.402	-0.244	0.036
	Fuel	0.090	-0.459	0.084	1.029E+06	0.090	1.100	0.088	0.248	0.029
	Time	0.146	-1.744	0.069	2.910E+07	0.146	1.695	0.085	-0.326	0.042
7	FA	0.135	-0.554	0.131	1.022E+06	0.135	1.060	0.135	-0.254	0.035
	Amb	0.235	2.951	0.150	4.111E+02	0.235	0.370	0.122	-0.307	0.054
	Fuel	0.267	-4.194	0.165	1.005E+11	0.267	3.773	0.173	0.330	0.053
	Time	0.135	-0.554	0.131	1.022E+06	0.135	1.060	0.135	-0.254	0.037
8	FA	0.095	0.459	0.090	1.414E+00	0.089	0.794	0.083	-0.266	0.053
	Amb	0.070	-0.334	0.066	3.602E+09	0.070	1.048	0.070	-0.295	0.040
	Fuel	0.207	2.650	0.151	5.100E-02	0.131	0.417	0.117	0.274	0.048
	Time	0.163	-1.063	0.151	1.813E+05	0.163	1.311	0.157	-0.254	0.068
9	FA	0.107	-0.892	0.073	1.503E+06	0.107	1.404	0.071	-0.258	0.026
	Amb	0.308	4.346	0.229	3.399E+01	0.307	0.259	0.206	-0.326	0.061
	Fuel	0.137	-1.672	0.079	2.808E+07	0.137	1.712	0.091	0.386	0.059
	Time	0.117	-1.068	0.091	1.035E+11	0.117	1.347	0.102	-0.342	0.080
10	FA	0.166	0.064	0.166	6.923E+00	0.166	0.873	0.165	-0.277	0.025
	Amb	0.123	1.333	0.068	3.240E-01	0.063	0.627	0.054	-0.264	0.031
	Fuel	0.104	0.753	0.089	8.570E-01	0.087	0.788	0.086	0.276	0.031
	Time	0.125	1.271	0.098	8.418E+01	0.125	0.676	0.087	-0.266	0.024
11	FA	0.154	-1.359	0.123	8.014E+05	0.154	1.416	0.137	-0.235	0.057
	Amb	0.260	5.891	0.163	3.309E+01	0.259	0.307	0.118	-0.249	0.037
	Fuel	0.356	7.490	0.232	2.950E+01	0.354	0.183	0.441	0.447	0.080
	Time	0.214	4.231	0.084	4.300E-02	0.064	0.438	0.073	-0.328	0.025
12	FA	0.394	10.819	0.081	7.316E-03	0.122	0.292	0.155	-0.313	0.047
	Amb	0.204	2.980	0.082	5.800E-02	0.043	0.461	0.038	-0.280	0.029
	Fuel	0.222	1.100	0.217	2.100E-01	0.215	0.550	0.210	0.275	0.044
	Time	0.234	1.229	0.219	1.240E-01	0.205	0.527	0.186	-0.085	0.133
13	FA	0.144	-0.399	0.143	7.805E+03	0.144	0.976	0.144	-0.271	0.034
	Amb	0.113	-0.895	0.095	7.120E+09	0.113	1.272	0.104	-0.267	0.039
	Fuel	0.101	-1.121	0.067	6.540E+06	0.101	1.373	0.081	0.350	0.060
	Time	0.117	-0.534	0.112	5.025E+05	0.117	1.033	0.117	-0.242	0.042
14	FA	0.149	0.073	0.149	6.384E+00	0.149	0.862	0.147	-0.329	0.032
	Amb	0.360	5.069	0.306	3.130E+01	0.359	0.232	0.294	-0.256	0.039
	Fuel	0.102	0.952	0.073	5.780E-01	0.070	0.702	0.064	0.295	0.024
	Time	0.203	3.984	0.143	3.900E-02	0.112	0.449	0.106	-0.253	0.028
15	FA	0.392	6.729	0.015	1.502E+02	0.392	0.319	0.130	-0.254	0.033
	Amb	0.354	6.428	0.057	1.928E+02	0.354	0.327	0.100	-0.285	0.049
	Fuel	0.209	1.044	0.205	2.290E-01	0.203	0.562	0.199	0.364	0.056
	Time	0.333	6.713	0.074	1.100E-01	0.167	0.302	0.077	-0.460	0.055

Subj		Linear	Exponential		Logarithmic		Power		Sigmoid	
		RMSE	c	RMSE	c	RMSE	c	RMSE	c	RMSE
16	FA	0.168	2.585	0.060	1.050E-01	0.042	0.544	0.055	-0.347	0.020
	Amb	0.340	5.224	0.094	1.850E+02	0.340	0.394	0.162	0.005	0.328
	Fuel	0.107	1.204	0.071	4.440E-01	0.067	0.721	0.065	0.326	0.055
	Time	0.147	1.143	0.122	3.190E-01	0.116	0.602	0.102	-0.251	0.072
17	FA	0.244	4.008	0.126	1.800E-02	0.062	0.376	0.053	-0.302	0.043
	Amb	0.188	2.536	0.084	4.480E+02	0.188	0.465	0.028	-0.314	0.049
	Fuel	0.407	19.519	0.087	1.628E+02	0.406	0.256	0.167	0.335	0.033
	Time	0.311	3.625	0.230	7.437E+01	0.311	0.287	0.193	-0.245	0.091
18	FA	0.172	-1.709	0.064	3.254E+07	0.172	1.851	0.054	-0.316	0.037
	Amb	0.312	-5.269	0.110	6.103E+10	0.312	4.925	0.124	-0.391	0.070
	Fuel	0.252	3.446	0.090	3.400E-02	0.041	0.381	0.032	0.303	0.065
	Time	0.130	-1.377	0.048	1.543E+07	0.130	1.603	0.064	-0.194	0.048
19	FA	0.090	-0.890	0.056	2.368E+13	0.079	1.309	0.064	-0.247	0.038
	Amb	0.395	7.302	0.358	3.470E+01	0.395	0.043	0.345	-0.296	0.052
	Fuel	0.368	7.665	0.048	1.975E+02	0.368	0.272	0.099	0.486	0.057
	Time	0.288	5.780	0.069	2.572E+02	0.288	0.366	0.078	-0.327	0.018
20	FA	0.212	3.446	0.084	4.800E-02	0.042	0.452	0.045	-0.282	0.036
	Amb	0.319	7.755	0.109	2.688E+02	0.319	0.316	0.079	-0.303	0.057
	Fuel	0.207	3.996	0.145	4.746E+01	0.206	0.451	0.109	0.272	0.045
	Time	0.253	6.813	0.125	3.220E+01	0.252	0.364	0.090	-0.294	0.035

Table 51. Summary of Model Fits for $u(v(x))$ Employing RMSE. “Lin” stands for linear, “Log” for logarithmic, “Pow” for power functions.

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
1	FA	Power	0.90	Lin, Exp, Log,Sig	Near Linear
	Amb	Sigmoid	0.89	Lin, Exp, Log,Pow	S-Shape
	Fuel	Sigmoid	0.97	None	Concave
	Time	Sigmoid	0.88	None	Concave
2	FA	Sigmoid	0.98	Lin, Exp, Log,Pow	S-Shape
	Amb	Sigmoid	1.00	None	Concave
	Fuel	Sigmoid	1.00	None	S-Shape
	Time	Sigmoid	0.88	Exp,Power	S-Shape
3	FA	Sigmoid	0.69	Lin,Exp,Log,Pow	Near Linear
	Amb	Sigmoid	0.96	Pow	Concave
	Fuel	Sigmoid	0.96	None	Concave
	Time	Sigmoid	0.96	Exp, Log, Pow	S-Shape
4	FA	Sigmoid	0.50	Lin,Exp,Log,Pow	Convex
	Amb	Sigmoid	0.98	Power	Concave
	Fuel	Sigmoid	0.97	Exp,Log,Pow	Concave
	Time	Sigmoid	0.97	Log,Pow	S-Shape
5	FA	Sigmoid	0.94	Exp,Power	Convex
	Amb	Sigmoid	0.98	None	Concave

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
	Fuel	Sigmoid	0.95	Lin,Exp,Log,Pow	Concave
	Time	Sigmoid	0.61	None	S-Shape
6	FA	Sigmoid	0.97	None	Concave
	Amb	Sigmoid	0.99	None	Concave
	Fuel	Sigmoid	0.90	Lin,Exp,Log,Pow	Near Linear
	Time	Sigmoid	0.92	Exp,Power	Convex
7	FA	Sigmoid	0.90	Lin,Exp,Log,Pow	S-Shape
	Amb	Sigmoid	0.95	None	Concave
	Fuel	Sigmoid	0.96	None	S-Shape
	Time	Sigmoid	0.93	None	S-Shape
8	FA	Sigmoid	0.69	Lin,Exp,Log,Pow	Concave
	Amb	Sigmoid	0.68	Lin,Exp,Log,Pow	Near Linear
	Fuel	Sigmoid	0.95	Pow	Concave
	Time	Sigmoid	0.83	None	S-Shape
9	FA	Sigmoid	0.94	Lin,Exp,Log,Pow	S-Shape
	Amb	Sigmoid	0.96	None	Concave
	Fuel	Sigmoid	0.82	Exp,Pow	S-Shape
	Time	Sigmoid	0.53	Lin,Exp,Log,Pow	S-Shape
10	FA	Sigmoid	0.98	None	S-Shape
	Amb	Sigmoid	0.94	Exp, Log,Pow	Concave
	Fuel	Sigmoid	0.91	Lin, Exp, Log,Pow	S-Shape
	Time	Sigmoid	0.96	Exp, Pow	S-Shape
11	FA	Sigmoid	0.86	None	S-Shape
	Amb	Sigmoid	0.98	Power	Concave
	Fuel	Sigmoid	0.95	None	Concave
	Time	Sigmoid	0.99	Exp,Log,Pow	S-Shape
12	FA	Sigmoid	0.99	Exp	Concave
	Amb	Sigmoid	0.98	Exp,Log,Pow	Concave
	Fuel	Sigmoid	0.96	None	S-Shape
	Time	None	0.68	None	S-Shape
13	FA	Sigmoid	0.94	None	S-Shape
	Amb	Sigmoid	0.88	Lin,Exp,Log,Pow	S-Shape
	Fuel	Sigmoid	0.65	Lin,Exp,Log,Pow	Concave
	Time	Sigmoid	0.87	Lin,Exp,Log,Pow	S-Shape
14	FA	Sigmoid	0.95	None	S-Shape
	Amb	Sigmoid	0.99	None	Concave
	Fuel	Sigmoid	0.94	Lin, Exp, Log,Pow	Concave
	Time	Sigmoid	0.98	Log,Pow	S-Shape
15	FA	Expo	1.00	Sig	Concave

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
	Amb	Sigmoid	0.98	Exp,Power	Concave
	Fuel	Sigmoid	0.93	None	S-Shape
	Time	Sigmoid	0.97	Exp,Power	Concave
16	FA	Sigmoid	0.99	Exp,Log,Pow	Concave
	Amb	Expo	0.92	None	Concave
	Fuel	Sigmoid	0.73	Lin, Exp, Log,Pow	S-Shape
	Time	Sigmoid	0.76	Log,Pow	Concave
17	FA	Sigmoid	0.97	Log,Pow	Concave
	Amb	Power	0.98	Exp,Sig	Concave
	Fuel	Sigmoid	0.99	Exp	Concave
	Time	Sigmoid	0.91	None	Concave
18	FA	Sigmoid	0.95	Exp,Power	S-Shape
	Amb	Sigmoid	0.95	Exp	Convex
	Fuel	Power	0.98	Exp,Log,Sig	Concave
	Time	Sigmoid	0.87	Exp,Pow	S-Shape
19	FA	Sigmoid	0.82	Lin,Exp,Log,Pow	Near Linear
	Amb	Sigmoid	0.98	None	Concave
	Fuel	Expo	0.98	Power,Sig	Concave
	Time	Sigmoid	1.00	Exp,Power	Concave
20	FA	Sigmoid	0.97	Exp,Log,Pow	Concave
	Amb	Sigmoid	0.97	Exp,Power	Concave
	Fuel	Sigmoid	0.95	Power	S-Shape
	Time	Sigmoid	0.98	Exp,Power	S-Shape

Table 52. Summary of Model Fits for $u_{ij}^{\text{model}}\left(v_{ij}\left(x\right)\right)$ Employing WRMSE. "Lin" stands for linear, "Log" for logarithmic, "Pow" for power functions.

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
1	FA	Power	0.9007	Lin,Exp,Log,Sig	Near Linear
	Amb	Sigmoid	0.8601	Lin,Exp,Log,Pow	S-Shape
	Fuel	Sigmoid	0.9538	None	Concave
	Time	Sigmoid	0.8588	None	Concave
2	FA	Sigmoid	0.9607	Lin,Exp,Log,Pow	S-Shape

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
	Amb	Sigmoid	0.9975	None	Concave
	Fuel	Sigmoid	0.9934	None	S-Shape
	Time	Sigmoid	0.9216	Exp,Pow	S-Shape
3	FA	Sigmoid	0.5345	Lin,Exp,Log,Pow	Near Linear
	Amb	Sigmoid	0.9483	Power	Concave
	Fuel	Sigmoid	0.9198	None	Concave
	Time	Log	0.9470	Exp,Pow,Sig	S-Shape
4	FA	Sigmoid	0.8111	Lin,Exp,Log,Pow	Convex
	Amb	Sigmoid	0.9775	Pow	Concave
	Fuel	Sigmoid	0.9510	Exp,Log,Pow	Concave
	Time	Sigmoid	0.9456	Log,Pow	S-Shape
5	FA	Sigmoid	0.8901	Exp	Convex
	Amb	Sigmoid	0.9763	None	Concave
	Fuel	Sigmoid	0.9117	Lin,Exp,Log,Pow	Concave
	Time	Sigmoid	0.8903	None	S-Shape
6	FA	Sigmoid	0.9473	None	Concave
	Amb	Sigmoid	0.9915	None	Concave
	Fuel	Sigmoid	0.8095	Lin,Exp,Log,Pow	Near Linear
	Time	Sigmoid	0.8584	Exp,Pow	Convex
7	FA	Sigmoid	0.8444	Lin,Exp,Log,Pow	S-Shape
	Amb	Sigmoid	0.9295	None	Concave
	Fuel	Sigmoid	0.9334	None	S-Shape
	Time	Sigmoid	0.8756	None	S-Shape
8	FA	Sigmoid	0.6219	Lin,Exp,Log,Pow	Concave
	Amb	Sigmoid	0.6345	Lin,Exp,Log,Pow	Near Linear
	Fuel	Sigmoid	0.9107	None	Concave
	Time	Sigmoid	0.7692	None	S-Shape
9	FA	Sigmoid	0.8938	Lin,Exp,Log,Pow	S-Shape
	Amb	Sigmoid	0.9494	None	Concave
	Fuel	Sigmoid	0.7740	Exp,Pow	S-Shape
	Time	Expo	0.4439	Pow,Sig	S-Shape
10	FA	Sigmoid	0.9530	None	S-Shape
	Amb	Sigmoid	0.9246	Exp,Log,Pow	Concave
	Fuel	Sigmoid	0.8460	Lin,Exp,Log,Pow	S-Shape
	Time	Sigmoid	0.9356	Exp,Pow	S-Shape
11	FA	Sigmoid	0.7766	None	S-Shape
	Amb	Sigmoid	0.9726	None	Concave
	Fuel	Sigmoid	0.9051	None	Concave

Subject		Best Acceptable Fit	G	Other Acceptable	Curve Shape
	Time	Sigmoid	0.9770	Exp,Log,Pow	S-Shape
12	FA	Sigmoid	0.9830	Exp	Concave
	Amb	Sigmoid	0.9711	Exp,Log,Pow	Concave
	Fuel	Sigmoid	0.9393	None	S-Shape
	Time	None	0.5240	None	S-Shape
13	FA	Sigmoid	0.9109	None	S-Shape
	Amb	Sigmoid	0.8477	Lin,Exp,Log,Pow	S-Shape
	Fuel	Expo	0.5359	Lin,Log,Pow,Sig	Concave
	Time	Sigmoid	0.7950	Exp	S-Shape
14	FA	Sigmoid	0.9233	None	S-Shape
	Amb	Sigmoid	0.9841	None	Concave
	Fuel	Sigmoid	0.8953	Lin,Exp,Log,Pow	Concave
	Time	Sigmoid	0.9635	Log,Pow	S-Shape
15	FA	Expo	0.9981	Sig	Concave
	Amb	Sigmoid	0.9751	Exp,Pow	Concave
	Fuel	Sigmoid	0.8877	None	S-Shape
	Time	Sigmoid	0.9512	Exp,Pow	Concave
16	FA	Sigmoid	0.9747	Exp,Log,Pow	Concave
	Amb	Expo	0.9248	None	Concave
	Fuel	Sigmoid	0.6097	Lin,Exp,Log,Pow	S-Shape
	Time	Sigmoid	0.6289	Pow	Concave
17	FA	Sigmoid	0.9516	Log,Pow	Concave
	Amb	Power	0.9680	Exp,Sig	Concave
	Fuel	Sigmoid	0.9918	Exp	Concave
	Time	None	0.8318	None	Concave
18	FA	Sigmoid	0.9394	Exp,Pow	S-Shape
	Amb	Sigmoid	0.9968	None	Convex
	Fuel	Power	0.9748	Exp,Log,Sig	Concave
	Time	Expo	0.7959	Pow,Sig	S-Shape
19	FA	Sigmoid	0.7212	Lin,Exp,Log,Pow	Near Linear
	Amb	Sigmoid	0.9796	None	Concave
	Fuel	Expo	0.9830	Sig	Concave
	Time	Sigmoid	0.9919	Exp,Pow	Concave
20	FA	Log	0.9604	Exp,Pow,Sig	Concave
	Amb	Sigmoid	0.9532	Exp,Power	Concave
	Fuel	Sigmoid	0.9285	Power	S-Shape
	Time	Sigmoid	0.9637	Power	S-Shape

The results of the multidimensional analysis of the tactical problem are tabulated in Table 53. The results are listed by subject by route for the various preference functions: value function, $v(x)$; utility function determined by the certainty-equivalent method, $u_{CE}(x)$; utility function determined by the probability-equivalent method, $u_{PE}(x)$; utility function determined by averaging the certainty-equivalent and probability-equivalent methods, $\bar{u}(x)$; and Kirkwood's (1997) multiattribute utility method, $u_m(x)$. The last function has subscripts of one and two. Two functions are present for those subjects who were never ambivalent between the alternatives presented. The two functions bound the ambivalence of the subject.

Table 53. Route Alternative Mathematical Expectation Scores for Preference Functions.

Subject	Route	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\bar{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
FJ1	A	0.404	0.487	0.523	0.505	0.501	0.414
	B	0.254	0.341	0.372	0.356	0.335	0.262
	C	0.180	0.263	0.303	0.280	0.244	0.185
	D	0.218	0.291	0.344	0.317	0.290	0.225
	E	0.679	0.673	0.703	0.688	0.762	0.688
	F	0.610	0.613	0.660	0.637	0.702	0.620
FJ2	A	0.182	0.462	0.461	0.439	0.021	-
	B	0.144	0.313	0.230	0.271	0.016	-
	C	0.044	0.250	0.171	0.207	0.004	-
	D	0.073	0.308	0.271	0.290	0.007	-
	E	0.487	0.671	0.773	0.722	0.123	-
	F	0.294	0.616	0.642	0.629	0.052	-
FJ3	A	0.470	0.588	0.621	0.605	0.307	-
	B	0.398	0.411	0.491	0.451	0.248	-
	C	0.220	0.268	0.407	0.336	0.122	-
	D	0.150	0.249	0.361	0.305	0.080	-
	E	0.539	0.516	0.556	0.536	0.373	-
	F	0.486	0.496	0.562	0.529	0.326	-
FJ4	A	0.282	0.497	0.419	0.458	0.274	0.544

Subject	Route	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\overline{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	B	0.234	0.347	0.289	0.318	0.227	0.474
	C	0.140	0.278	0.226	0.248	0.136	0.313
	D	0.135	0.345	0.262	0.304	0.131	0.298
	E	0.709	0.715	0.700	0.708	0.701	0.900
	F	0.617	0.651	0.617	0.634	0.607	0.847
FJ5	A	0.101	0.287	0.448	0.367	0.120	-
	B	0.072	0.181	0.301	0.241	0.085	-
	C	0.056	0.167	0.301	0.221	0.068	-
	D	0.108	0.268	0.413	0.340	0.127	-
	E	0.889	0.822	0.881	0.852	0.908	-
	F	0.829	0.771	0.824	0.798	0.856	-
FJ6	A	0.289	0.495	0.398	0.447	0.600	0.077
	B	0.203	0.344	0.256	0.300	0.462	0.048
	C	0.106	0.271	0.181	0.222	0.277	0.021
	D	0.129	0.322	0.226	0.274	0.325	0.027
	E	0.539	0.727	0.676	0.702	0.829	0.023
	F	0.555	0.712	0.656	0.684	0.837	0.245
FJ7	A	0.251	0.406	0.430	0.418	0.251	-
	B	0.186	0.278	0.276	0.277	0.186	-
	C	0.116	0.218	0.228	0.215	0.116	-
	D	0.197	0.308	0.304	0.306	0.197	-
	E	0.775	0.745	0.724	0.735	0.775	-
	F	0.740	0.687	0.674	0.681	0.740	-
FJ8	A	0.664	0.673	0.528	0.701	0.767	-
	B	0.513	0.502	0.402	0.532	0.629	-
	C	0.211	0.331	0.280	0.347	0.294	-
	D	0.121	0.230	0.250	0.265	0.176	-
	E	0.347	0.371	0.378	0.386	0.463	-
	F	0.267	0.353	0.361	0.375	0.365	-
FJ9	A	0.442	0.570	0.535	0.553	0.384	0.394
	B	0.362	0.436	0.384	0.410	0.308	0.317
	C	0.272	0.306	0.231	0.264	0.226	0.234
	D	0.225	0.305	0.229	0.267	0.187	0.193
	E	0.530	0.543	0.526	0.534	0.471	0.481
	F	0.567	0.509	0.494	0.502	0.510	0.520
FJ10	A	0.390	0.461	0.498	0.479	0.234	0.390
	B	0.247	0.282	0.340	0.311	0.132	0.247
	C	0.082	0.202	0.256	0.227	0.039	0.082
	D	0.131	0.255	0.298	0.277	0.065	0.131
	E	0.642	0.693	0.696	0.694	0.468	0.642
	F	0.580	0.667	0.669	0.668	0.403	0.580
FJ11	A	0.551	0.715	0.670	0.692	0.073	-

Subject	Route	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\bar{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	B	0.471	0.512	0.417	0.465	0.048	-
	C	0.257	0.369	0.318	0.341	0.012	-
	D	0.168	0.353	0.307	0.330	0.006	-
	E	0.482	0.504	0.496	0.500	0.050	-
	F	0.472	0.526	0.512	0.519	0.053	-
FJ12	A	0.507	0.620	0.702	0.661	0.119	0.490
	B	0.179	0.503	0.535	0.519	0.052	0.345
	C	0.041	0.325	0.389	0.355	0.018	0.208
	D	0.039	0.203	0.340	0.271	0.010	0.150
	E	0.394	0.388	0.480	0.434	0.029	0.268
	F	0.198	0.310	0.426	0.368	0.023	0.220
FJ13	A	0.663	0.664	0.682	0.673	0.249	0.356
	B	0.581	0.510	0.567	0.539	0.182	0.277
	C	0.281	0.313	0.396	0.351	0.051	0.093
	D	0.163	0.247	0.321	0.284	0.025	0.048
	E	0.374	0.406	0.436	0.422	0.066	0.123
	F	0.344	0.362	0.425	0.394	0.068	0.120
FJ14	A	0.460	0.664	0.600	0.632	0.421	-
	B	0.376	0.476	0.396	0.436	0.341	-
	C	0.076	0.268	0.237	0.251	0.066	-
	D	0.039	0.286	0.231	0.259	0.033	-
	E	0.430	0.536	0.508	0.522	0.392	-
	F	0.431	0.535	0.511	0.523	0.393	-
FJ15	A	0.369	0.694	0.629	0.662	0.001	-
	B	0.135	0.494	0.448	0.471	0.000	-
	C	0.071	0.349	0.316	0.329	0.000	-
	D	0.092	0.405	0.315	0.360	0.000	-
	E	0.442	0.610	0.587	0.598	0.002	-
	F	0.461	0.623	0.584	0.604	0.003	-
FJ16	A	0.188	0.393	0.434	0.382	0.006	0.021
	B	0.106	0.214	0.194	0.181	0.003	0.010
	C	0.057	0.125	0.150	0.132	0.001	0.005
	D	0.082	0.192	0.248	0.221	0.002	0.008
	E	0.521	0.752	0.808	0.807	0.077	0.150
	F	0.571	0.780	0.812	0.805	0.110	0.195
FJ17	A	0.300	0.516	0.518	0.517	0.437	-
	B	0.154	0.315	0.345	0.330	0.246	-
	C	0.060	0.254	0.283	0.264	0.100	-
	D	0.085	0.300	0.359	0.329	0.139	-
	E	0.635	0.748	0.780	0.764	0.768	-
	F	0.599	0.714	0.729	0.721	0.741	-
FJ18	A	0.420	0.416	0.382	0.399	0.812	0.219

Subject	Route	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\bar{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	B	0.341	0.366	0.309	0.338	0.714	0.171
	C	0.371	0.351	0.337	0.323	0.762	0.186
	D	0.463	0.404	0.428	0.416	0.847	0.253
	E	0.856	0.745	0.833	0.789	0.985	0.719
	F	0.780	0.693	0.770	0.732	0.972	0.602
FJ19	A	0.387	0.618	0.616	0.617	0.670	-
	B	0.341	0.387	0.462	0.425	0.616	-
	C	0.215	0.264	0.385	0.322	0.442	-
	D	0.150	0.294	0.389	0.342	0.329	-
	E	0.579	0.613	0.642	0.627	0.824	-
	F	0.543	0.606	0.654	0.630	0.796	-
FJ20	A	0.215	0.447	0.459	0.453	0.453	-
	B	0.128	0.305	0.326	0.315	0.315	-
	C	0.055	0.251	0.295	0.267	0.267	-
	D	0.076	0.346	0.374	0.360	0.360	-
	E	0.730	0.760	0.756	0.758	0.758	-
	F	0.541	0.658	0.684	0.671	0.671	-

The route alternatives ordered in descending preference appear in Table 54.

Table 54. Mathematical Expectation Ordered Route Alternatives for Preference Functions.

Subject	Order	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\bar{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
FJ1	1	E	E	E	E	E	E
	2	F	F	F	F	F	F
	3	A	A	A	A	A	A
	4	B	B	B	B	B	B
	5	D	D	D	D	D	D
	6	C	C	C	C	C	C
FJ2	1	E	E	E	E	E	
	2	F	F	F	F	F	
	3	A	A	A	A	A	
	4	B	B	D	D	B	
	5	D	D	B	B	D	
	6	C	C	C	C	C	
FJ3	1	E	A	A	A	E	
	2	F	E	F	E	F	

Subject	Order	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\bar{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	3	A	F	E	F	A	
	4	B	B	B	B	B	
	5	C	C	C	C	C	
	6	D	D	D	D	D	
FJ4	1	E	E	E	E	E	E
	2	F	F	F	F	F	F
	3	A	A	A	A	A	A
	4	B	B	B	B	B	B
	5	C	D	D	D	C	C
	6	D	C	C	C	D	D
FJ5	1	E	E	E	E	E	
	2	F	F	F	F	F	
	3	D	A	A	A	D	
	4	A	D	D	D	A	
	5	B	B	C	B	B	
	6	C	C	B	C	C	
FJ6	1	F	E	E	E	F	F
	2	E	F	F	F	E	A
	3	A	A	A	A	A	B
	4	B	B	B	B	B	D
	5	D	D	D	D	D	E
	6	C	C	C	C	C	C
FJ7	1	E	E	E	E	E	
	2	F	F	F	F	F	
	3	A	A	A	A	A	
	4	D	D	D	D	D	
	5	B	B	B	B	B	
	6	C	C	C	C	C	
FJ8	1	A	A	A	A	A	
	2	B	B	B	B	B	
	3	E	E	E	E	E	
	4	F	F	F	F	F	
	5	C	C	C	C	C	
	6	D	D	D	D	D	
FJ9	1	F	A	A	A	F	F
	2	E	E	E	E	E	E
	3	A	F	F	F	A	A
	4	B	B	B	B	B	B
	5	C	C	C	D	C	C
	6	D	D	D	C	D	D
FJ10	1	E	E	E	E	E	E
	2	F	F	F	F	F	F

Subject	Order	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\overline{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	3	A	A	A	A	A	A
	4	B	B	B	B	B	B
	5	D	D	D	D	D	D
	6	C	C	C	C	C	C
FJ11	1	A	A	A	A	A	
	2	E	F	F	F	F	
	3	F	B	E	E	E	
	4	B	E	B	B	B	
	5	C	C	C	C	C	
	6	D	D	D	D	D	
FJ12	1	A	A	A	A	A	A
	2	E	B	B	B	B	B
	3	F	E	E	E	E	E
	4	B	C	F	F	F	F
	5	C	F	C	C	C	C
	6	D	D	D	D	D	D
FJ13	1	A	A	A	A	A	A
	2	B	B	B	B	B	B
	3	E	E	E	E	F	E
	4	F	F	F	F	E	F
	5	C	C	C	C	C	C
	6	D	D	D	D	D	D
FJ14	1	A	A	A	A	A	
	2	F	E	F	F	F	
	3	E	F	E	E	E	
	4	B	B	B	B	B	
	5	C	D	C	D	C	
	6	D	C	D	C	D	
FJ15	1	F	A	A	A	A	
	2	E	F	E	F	F	
	3	A	E	F	E	E	
	4	B	B	B	B	B	
	5	D	D	C	D	D	
	6	C	C	D	C	C	
FJ16	1	F	F	F	E	F	F
	2	E	E	E	F	E	E
	3	A	A	A	A	A	A
	4	B	B	D	D	B	B
	5	D	D	B	B	D	D
	6	C	C	C	C	C	C
FJ17	1	E	E	E	E	E	
	2	F	F	F	F	F	

Subject	Order	$v(x)$	$u_{CE}(x)$	$u_{PE}(x)$	$\overline{u}(x)$	$u_{m_1}(x)$	$u_{m_2}(x)$
	3	A	A	A	A	A	
	4	B	B	D	B	B	
	5	D	D	B	D	D	
	6	C	C	C	C	C	
FJ18	1	E	E	E	E	E	E
	2	F	F	F	F	F	F
	3	D	A	D	D	D	D
	4	A	D	A	A	A	A
	5	C	B	C	B	C	C
	6	B	C	B	C	B	B
FJ19	1	E	A	F	F	E	
	2	F	E	E	E	F	
	3	A	F	A	A	A	
	4	B	B	B	B	B	
	5	C	D	D	D	C	
	6	D	C	C	C	D	
FJ20	1	E	E	E	E	E	
	2	F	F	F	F	F	
	3	A	A	A	A	A	
	4	B	D	D	D	D	
	5	D	B	B	B	B	
	6	C	C	C	C	C	

Appendix C. Information Operations Detailed Results

Information Operations Value and Utility Functions.

Attack Information Realm Hierarchy. Attack Information.

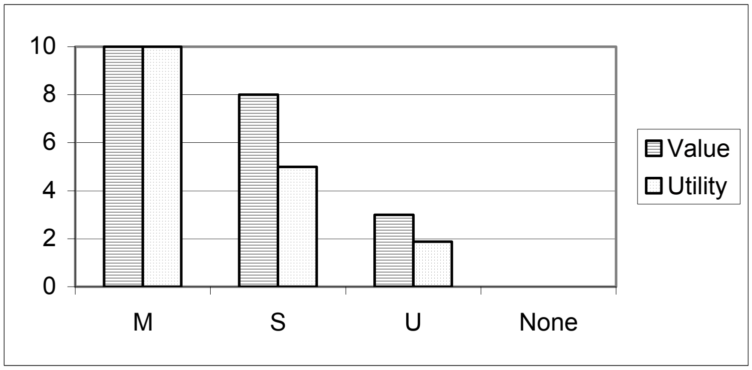


Figure 113. Increase Minimum Update Goal.

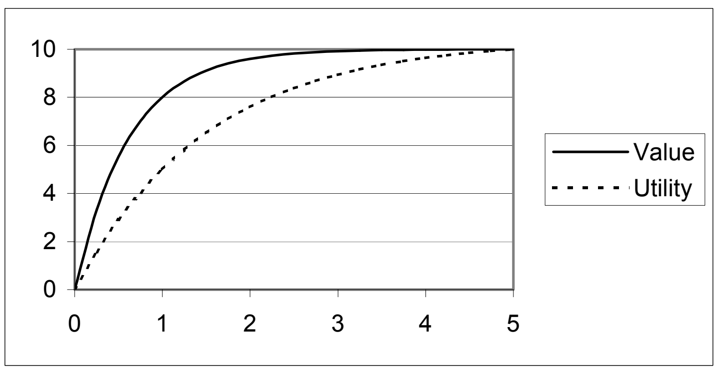


Figure 114. Age Devaluation Goal.

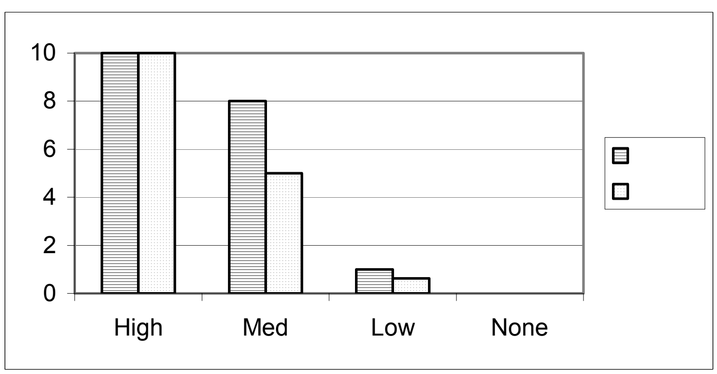


Figure 115. Likely Accept False Goal.

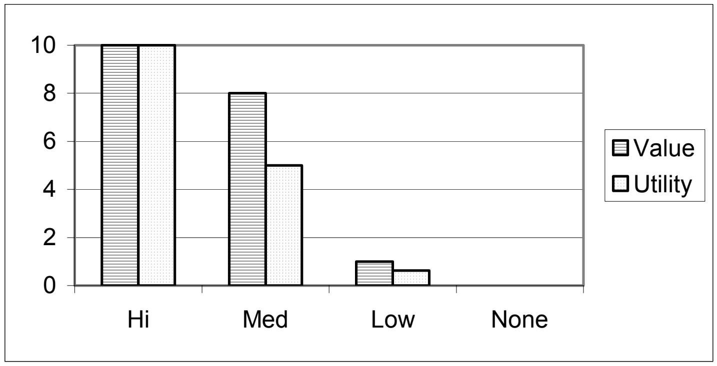


Figure 116. Likely Accept True Reject False Goal.

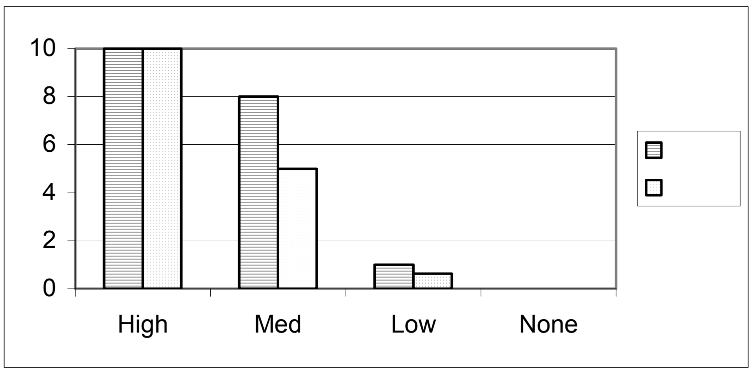


Figure 117. Likely Reject Truth Goal.

Attack Information Systems.

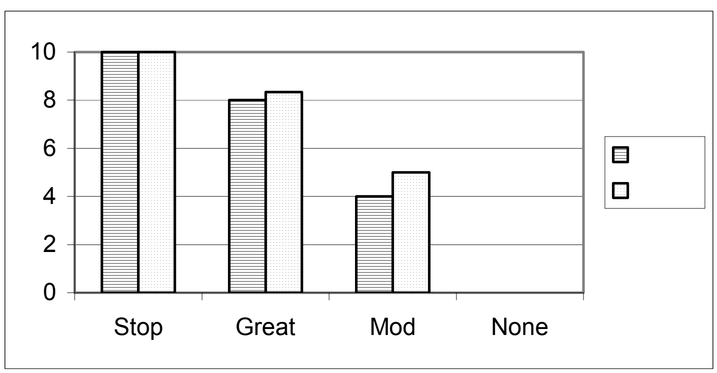


Figure 118. Reduce Bandwidth Goal.

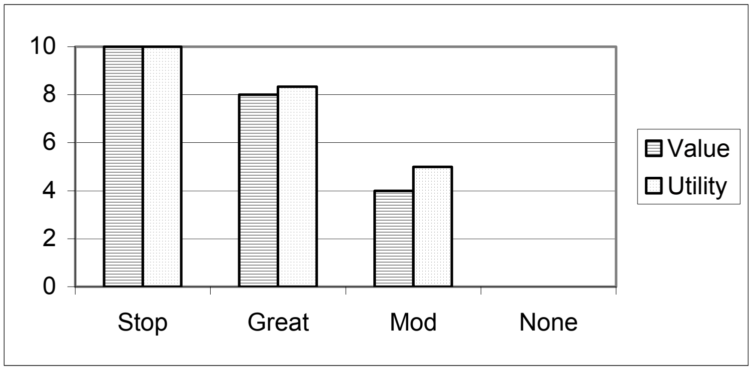


Figure 119. Reduce Throughput Goal.

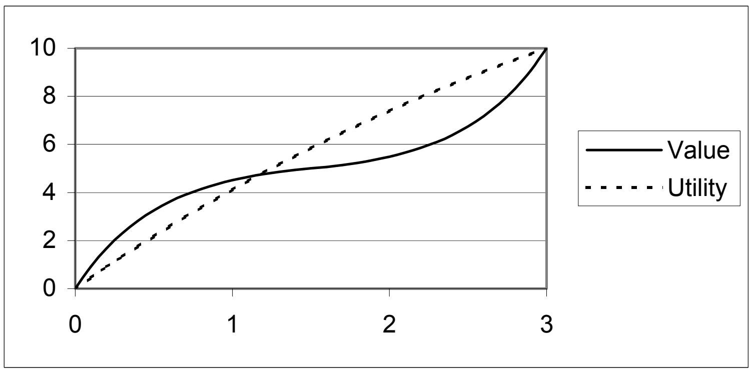


Figure 120. Degrade Update Goal.

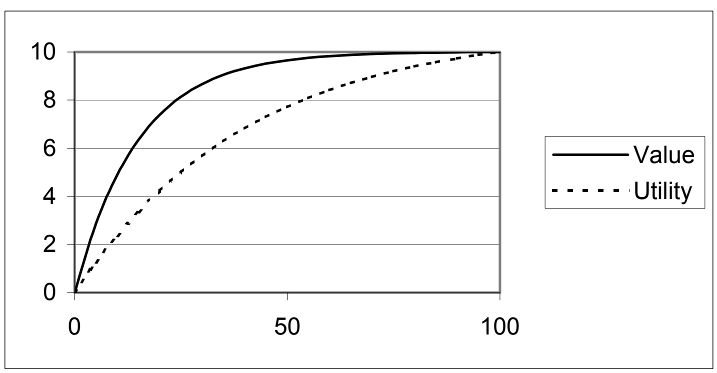


Figure 121. Increase Error Content Goal.

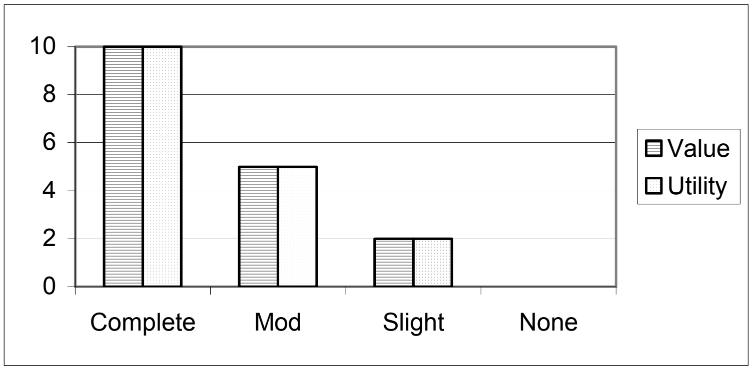


Figure 122. Penetrate System Goal.

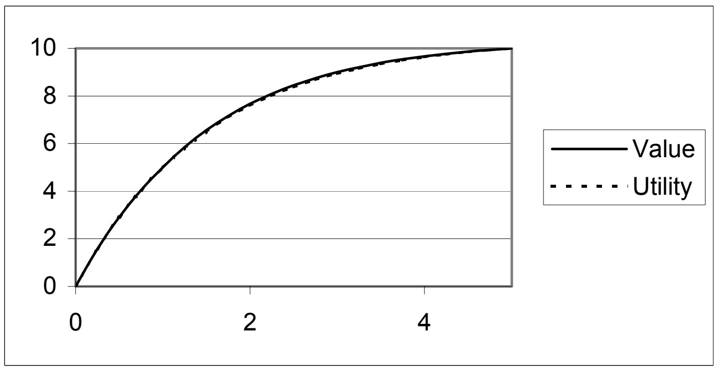


Figure 123. Increase Recovery Time Goal.

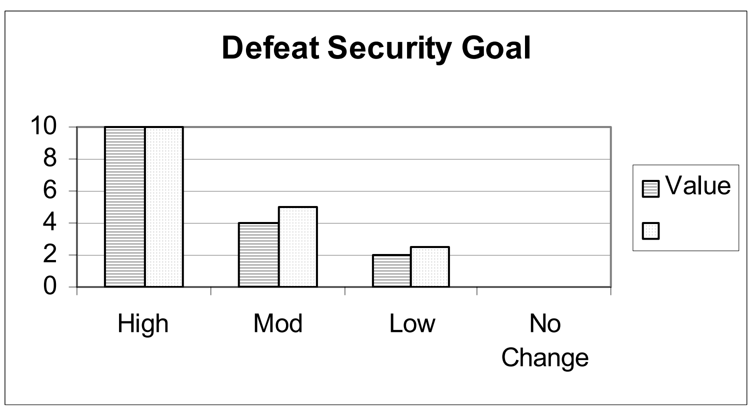


Figure 124. Defeat Security Goal.

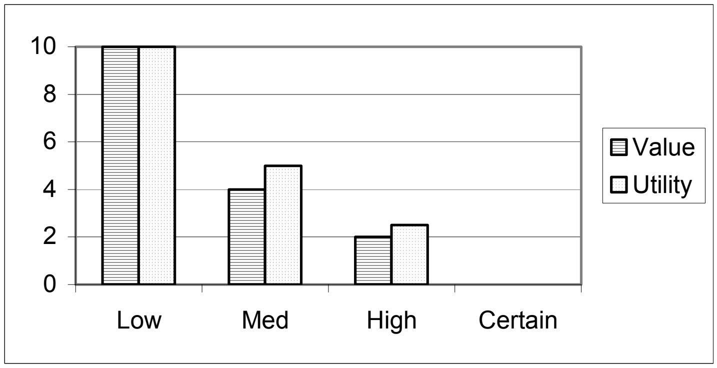


Figure 125. Defeat Detection Goal.

Attack Information-Based Processes.

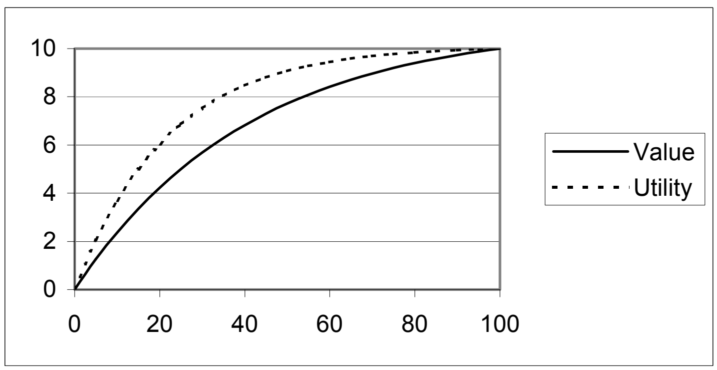


Figure 126. Consume Essential Resources Goal.

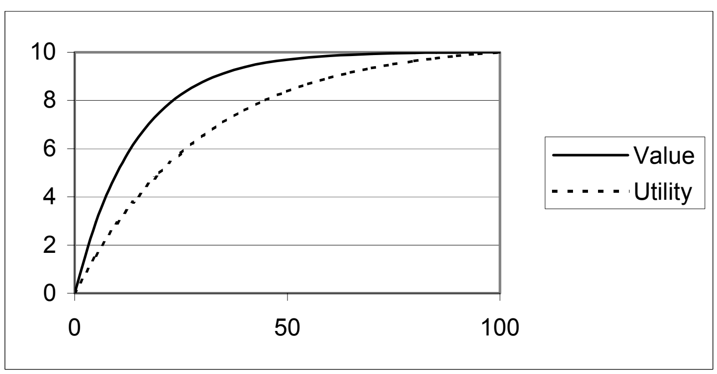


Figure 127. Consume Essential Time Goal.

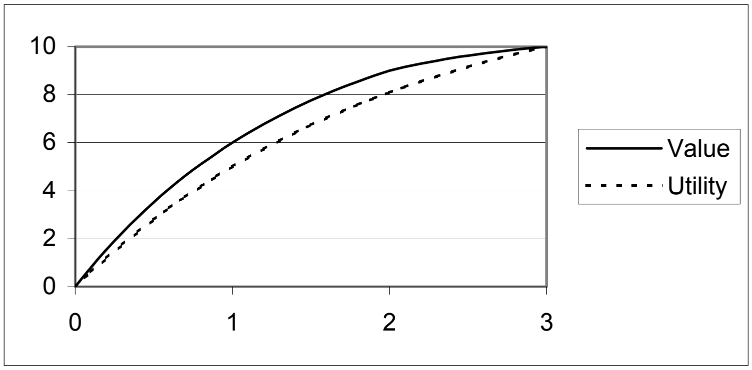


Figure 128. Reduce Timeliness Goal.

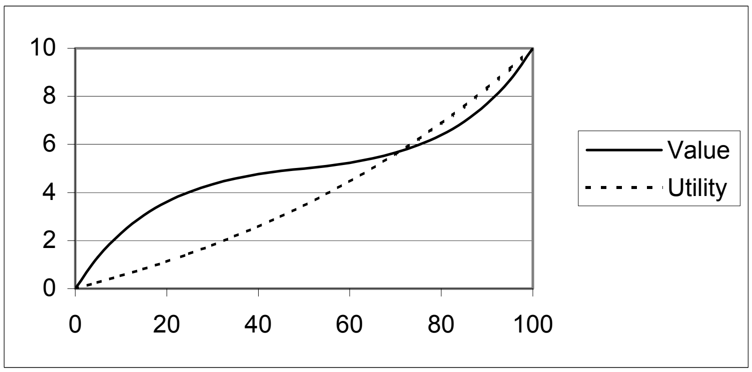


Figure 129. Decrease Accuracy Goal.

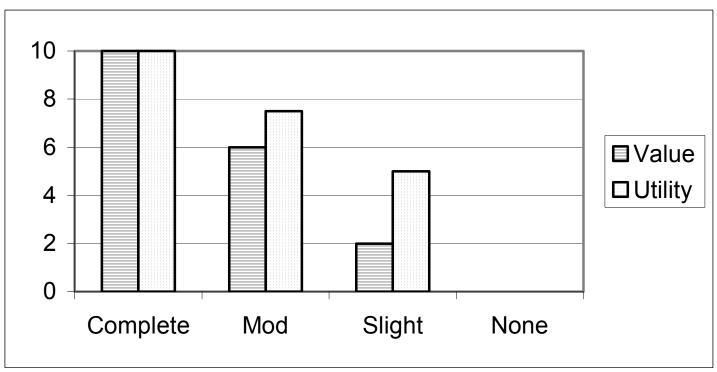


Figure 130. Reduce Focus Goal.

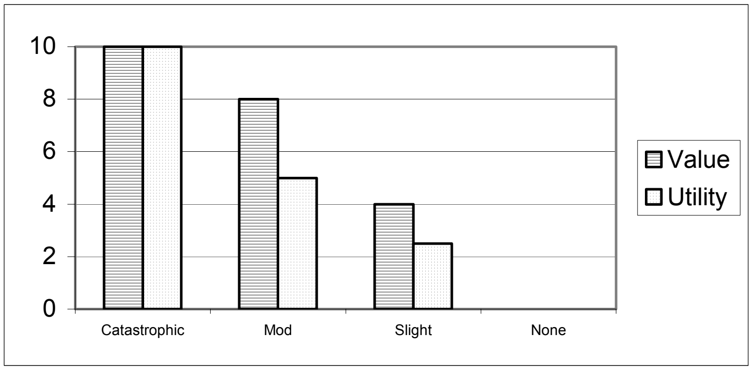


Figure 131. Reduce Resilience Goal.

Minimize Cost Hierarchy.

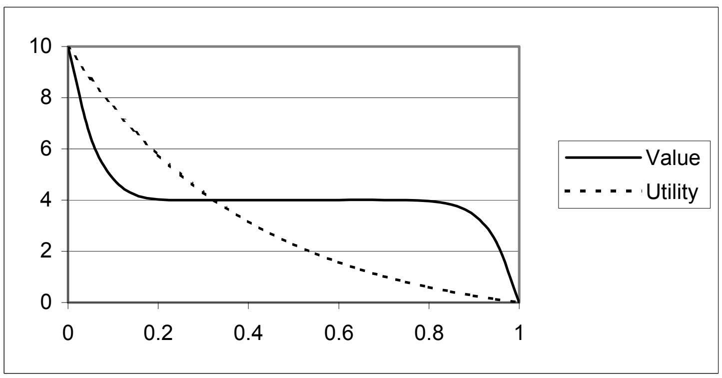


Figure 132. Friendly at Risk Cost.

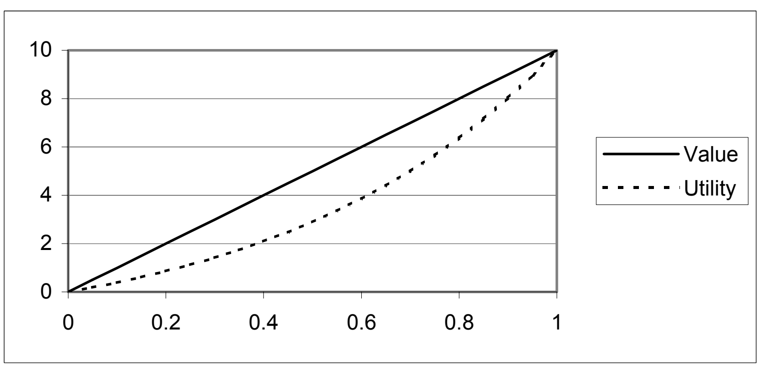


Figure 133. Maximum System Survivability Cost.

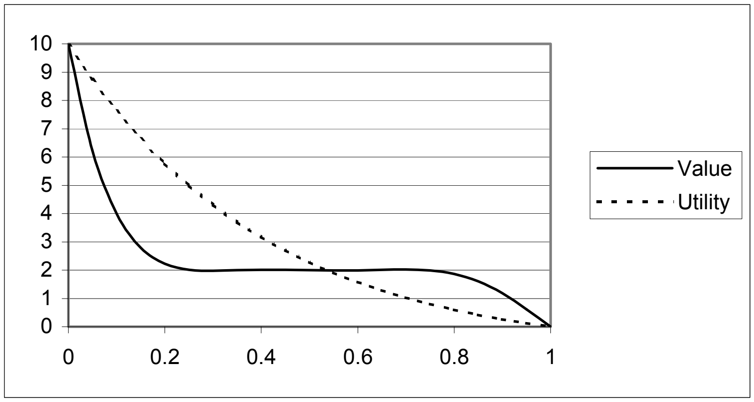


Figure 134. Minimize Collateral Damage.

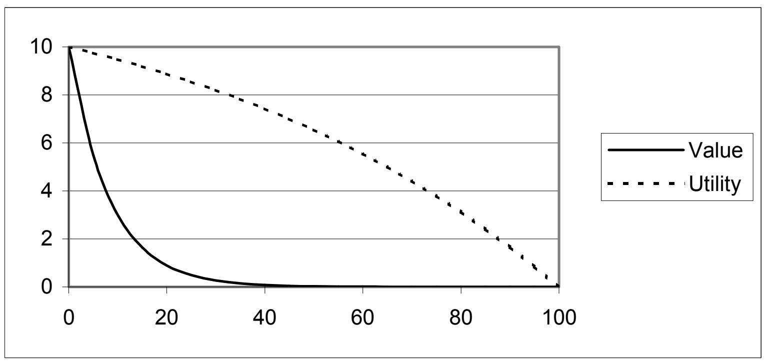


Figure 135. Sensitive Information Security Cost.

Table 55. WRMSE Model Fit Data for $u(v(x))$.

	Linear	Exponential		Logarithmic		Power		Sigmoid	
	WRMSE	C	WRMSE	C	WRMSE	c	WRMSE	c	WRMSE
Increase Recovery Time	0.00406	-0.045	0.00310	2.58E+08	0.00406	1.014	0.00353	0.528	0.0612
Degrade Update	0.122	0.516	0.111	1.628	0.112	0.871	0.117	0.292	0.0402
Increase Error Content	0.182	-3.745	0.0682	1.31E+11	0.182	3.396	0.0817	0.437	0.0588
Age Devaluation	0.166	-3.556	0.0661	8.08E+10	0.166	3.21	0.0786	0.529	0.0611
Decrease Accuracy	0.143	-0.962	0.109	2661000	0.143	1.421	0.0986	0.336	0.0478
Reduce Timeliness	0.0683	-0.934	0.0232	1.06E+13	0.0774	1.37	0.0331	0.349	0.0499
Consume Essential Resources	0.111	1.589	0.0140	0.263	0.0246	0.595	0.0434	0.701	0.0589

	Linear	Exponential		Logarithmic		Power		Sigmoid	
	WRMSE	C	WRMSE	C	WRMSE	c	WRMSE	c	WRMSE
Consume Essential Time	0.140	-2.709	0.0510	9E+09	0.140	2.508	0.0648	0.528	0.0612
Friendly at Risk Cost	0.224	-0.808	0.214	310700	0.224	1.317	0.210	-0.437	0.0588
System Survivability Cost	0.138	-1.802	1.38E-05	1.93E+08	0.138	1.821	0.0211	-0.378	0.684
Collateral Damage Cost	0.191	1.11	0.158458	0.412	0.162	0.779	0.170	-0.437	0.0588
Sensitive Information Security Cost	0.524	1255	0.15913	13.293	0.522	0.098	0.131	-0.336	0.0478

Table 56. RMSE Model Fit Data for $u\left(v\left(x\right)\right)$.

	Linear	Exponential		Logarithmic		Power		Sigmoid	
	RMSE	c	RMSE	c	RMSE	c	RMSE	c	RMSE
Increase Recovery Time	0.00470	-0.045	0.00332	2.58E+08	0.00470	1.014	0.00380	0.528	0.0494
Degrade Update	0.128	0.516	0.114	1.628	0.115	0.871	0.120	0.292	0.0337
Increase Error Content	0.196	-3.745	0.0728	1.31E+11	0.196	3.396	0.0853	0.437	0.0480
Age Devaluation	0.174	-3.556	0.0701	8.08E+10	0.174	3.21	0.0812	0.529	0.0494
Decrease Accuracy	0.155	-0.962	0.112	2661000	0.155	1.421	0.102	0.336	0.0399
Reduce Timeliness	0.0770	-0.934	0.0250	1.06E+13	0.0866	1.37	0.0349	0.349	0.0415
Consume Essential Resources	0.121	1.589	0.0143	0.263	0.0250	0.595	0.0402	0.701	0.0477
Consume Essential Time	0.148	-2.709	0.0540	9E+09	0.148	2.508	0.0663	0.528	0.0494
Friendly at Risk Cost	0.226	-0.808	0.206	310700	0.226	1.317	0.203	-0.437	0.0480
System Survivability Cost	0.156	-1.802	1.55E-05	1.93E+08	0.156	1.821	0.0220	-0.378	0.633
Collateral Damage Cost	0.191	1.11	0.163	0.412	0.168	0.779	0.171	-0.437	0.0480
Sensitive Information Security Cost	0.571	1255	0.178	13.293	0.569	0.098	0.113	-0.336	0.0399

Table 57. Number of Best Acceptable Fits By Model Type.

	RMSE	WRMSE
Linear	0	0
Exponential	5	5
Logarithmic	0	0
Power	0	0
Sigmoid	7	7
None	0	0

Table 58. Total Fits by Model Type.

	RMSE		WRMSE	
	Number	Percent	Number	Percent
Linear	2	16.7	4	33.3
Exponential	9	75.0	9	75.0
Logarithmic	4	33.3	4	33.3
Power	10	83.3	9	75.0
Sigmoid	11	91.7	11	91.7

Appendix D. Application of Decision Analysis to Automatic Target Recognition Programmatic Decisions

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TECHNICAL REPORT
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Application of Decision Analysis to Automatic Target Recognition Programmatic Decisions

William K. Klimack, Col, USA
Christopher B. Bassham, Capt, USAF
Dr. Kenneth W. Bauer, Jr.

**GRADUATE SCHOOL OF ENGINEERING AND MANAGEMENT
AIR FORCE INSTITUTE OF TECHNOLOGY
WRIGHT-PATTERSON AIR FORCE BASE, OHIO**

Abstract

The purpose of this research is to demonstrate the application of decision analysis (DA) techniques to the decisions made throughout the lifecycle of Automatic Target Recognition (ATR) technology development. This work is accomplished in the hopes of improving the means by which ATR technologies are evaluated. The first step in this research was to create a flexible decision analysis framework that could be applied to a variety of decisions across several different ATR programs evaluated by the Comprehensive ATR Scientific Evaluation (COMPASE) Center of the Air Force Research Laboratory (AFRL). For the purposes of this research, a single COMPASE Center representative provided the value, utility, and preference functions for the DA framework through elicitation meetings with the authors. The DA framework employs performance measures collected during ATR classification system (CS) testing to calculate value and utility scores. The authors gathered data from the Moving and Stationary Target Acquisition and Recognition (MSTAR) program to demonstrate how the decision framework could be used to evaluate three different ATR CSs. A decision-maker may use the resultant scores to gain insight into any of the decisions that occur throughout the lifecycle of ATR technologies. Additionally, a means of evaluating ATR CS self-assessment ability is presented. This represents a new criterion that emerged from this study, and no present evaluation metric is known.

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Application of Decision Analysis to Automatic Target Recognition Programmatic Decisions

1.0 Summary. The purpose of this research is to demonstrate the implementation of a decision analysis (DA) approach towards programmatic decision-making within the Automatic Target Recognition (ATR) field of research. Several decisions within the lifecycle of ATR research are based upon evaluations using many performance measures and program characteristics. Often, the evaluation of these raw performance measures and characteristics leads to non-dominating solutions among the various ATR programs and technologies competing for further development. Therefore, the decisions are finalized using the preferences and values of the decision-maker. Thus, it is clear that ATR research decision-makers should employ a formalized decision analysis framework to aid in selecting the best ATR research direction or ATR product available based upon the given performance characteristics of ATR classification systems (CSs) and to ensure evaluation fairness among competing technologies. The DA framework presented quantifies and rank-orders the preferences of the decision-maker, i.e., the desirable traits of a good ATR CS are weighted more heavily than less desirable traits. The numerous performance measures and characteristics of a given set of ATR CSs may then be synthesized through the DA model. The result is a single utility, or value, measure associated with each ATR CS under consideration. This methodology provides the decision-maker with a defensible approach that includes his own value structure of the problem for use in making programmatic decisions. This research uses past ATR data to illustrate that this technique is feasible and potentially very useful to the decision-makers involved in ATR research.

1.1 Introduction. Automatic Target Recognition is a processing problem where an image is examined in order to detect and classify objects of interest, or *targets*. The image is provided by one or more sensors, which typically are forward-looking infrared, millimeter wave, synthetic aperture radar, or laser radar systems. An ATR CS, in the form of a pattern recognition and classification algorithm, is then applied to the image in order to identify particular regions of interest (ROIs) and to then classify whether the ROI is a target or not. It is necessary to point out that both enemy and friendly objects of interest are referred to as targets. Typically, the targets are difficult to separate both from normal environmental objects, generally referred to as *clutter*, and objects with target-like signatures found within the image, often referred to in ATR research as *confusers*. Currently, human analysis far exceeds the capabilities of automated ATR systems. It is highly desirable to improve automated ATR capabilities, which would increase analytic capacity in military intelligence systems as well as permit ATR systems to be employed on unmanned platforms. Automatic Target Recognition is widely acknowledged as a critical military capability [4].

ATR CSs generally fall into three classes: statistical pattern recognition, neural networks, or model-based recognition. All ATRs have a number of quantifiable evaluation measures, such as ATR CS performance, robustness, estimate accuracy, employment doctrine, and cost, which may be used to compare multiple ATR CSs. In general, these measures are not assessed in total, but specific measures are selected when considering decisions for a specific program. No examples of employment of DA techniques with respect to ATR selection have been found in the literature.

2.0 Methods, Assumptions, and Procedures. The following section details the steps taken to construct a DA framework. The first step is defining the decision situation with a decision-maker. Next, the DA model is generated using quantitative representations of the preferences held by the decision-maker. Finally, the performance measures of multiple ATR CSs may then be introduced to the model for analysis.

2.1 The Decision Situation. The objective of ATR evaluation is to determine which system performs best via performance measure assessment and comparison. During development of past ATR CSs, decisions that compared competing ATR CSs were not performed in a manner that collectively considered all pertinent aspects of the decision. Typically a subset of criteria was examined. Employment of decision analytic techniques permits all criteria to be considered and trade-offs performed when comparing systems. As an example of how these techniques can be applied, the Moving and Stationary Target Acquisition and Recognition (MSTAR) program, which was an investigative effort into synthetic aperture radar (SAR), model-based ATR development, provides an ideal scenario [5]. At the time, three ATR systems were tested as candidates for furthering the development of an advanced ATR system [5]. These ATR CSs are referred to as ATR 33, ATR 55, and ATR 89.

There are two basic ATR employment profiles under consideration. The Combat Identification (CID) employment profile is implemented when the primary objective of the ATR CS is to select targets for weapon systems. In this scenario, the system is allowed to sacrifice detection performance in order to gain classification accuracy. Thus, the selection of a target must be associated with a high degree of confidence as to

minimize the number of false alarms. The Intelligence/Surveillance/Reconnaissance (ISR) employment profile is used when the primary objective of the ATR CS is to collect information for many potential targets. In this scenario, the CS is allowed to sacrifice classification accuracy for improved detection performance. Thus, the goal of the ATR CS is to detect as many potential targets as possible as to minimize the number of targets that evade detection.

In order to assist the Comprehensive ATR Scientific Evaluation (COMPASE) Center, an Air Force Research Laboratory (AFRL) organization within the Sensors Directorate, with its assessment of which future ATR systems should be retained for further development, a feasibility study of value-focused decision analysis is performed upon the MSTAR data. Dr. Timothy Ross, COMPASE Center Director, served as the decision-maker. Dr. Ross participated in two elicitation sessions. The initial session developed the subject's value hierarchy and elicited single dimensional value functions for the evaluation measures. Single dimensional utility functions were elicited during the second session as well as follow up clarification on several minor issues. Additionally, the authors discussed minor points with Dr. Ross via e-mail. Throughout the elicitation process, the subject was comfortable expressing his preferences in a quantified manner. Discussions with Dr. Ross confirmed and refined the evaluation measures for the decision situation.

2.2 Encoding the Value Hierarchy. In a test environment, ATR CS evaluation is accomplished using performance measure assessment and comparison. Assessment refers to the collection of several quantitative performance measures. Understanding the design of the testing environment is key to the understanding of performance measure

assessment. To begin, the test image is broken into two separate areas: the *target truth area* and the *clutter scene area*. Testing personnel place targets and confusers within the target truth area. The ATR CS examines this section of the scene for potential targets. Any target declaration made against a confuser, or *non-targets*, is considered a *false alarm*. Next, the ATR CS scans the clutter scene area, which is known to be devoid of targets. Any target declarations made by the ATR CS in this section are considered to be false alarms *in clutter*, since no confusers exist in this area. Figure 2.1 illustrates the manner in which an ATR test area is divided. When an ATR CS has scanned both areas, ATR performance may then be assessed.

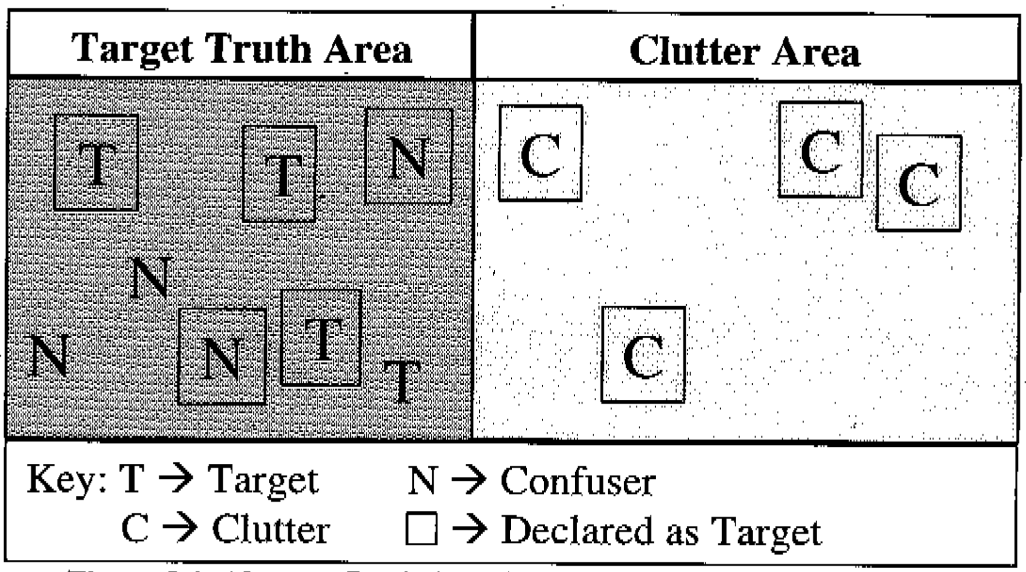

Figure 2.1 Abstract Depiction of Target Truth and Clutter Areas.

The primary performance measure is the probability of detection, P_D. This measure is defined as:

$$P_D = N_{DET} / N_{TGT} \qquad (1)$$

where N_{DET} is the number of targets detected by the ATR CS and N_{TGT} is the total number of targets to be detected within the image. The probability of detection provides insight into how well the ATR CS is detecting the target it is designed to find. Beyond detecting targets, it is desirable that the ATR CS provides a conjecture of the taxon, or label, of the object. The ATR may or may not further refine the description of the target. Typically, a refined identification is bifurcated into class and type, but a target may be labeled at any level desirable to the decision-maker. *Class* describes a broad category of materiel. Example class taxa are main battle tanks (MBTs) and surface-to-air systems. The *type* of the object is the specific nomenclature. The M-1A1 and T-72 main battle tanks are 'type' examples of the MBT class. Given the targets detected by the ATR CS, additional performance measures conditional to P_D may then be calculated concerning how well the ATR algorithm classifies the ROIs that it considers targets. The first performance measure, the probability of correct classification (P_{CC}), is the ratio of the number of targets correctly classified by class (N_{CC}) to the total number of detected targets (N_{DET}). Thus, P_{CC} is defined as:

$$P_{CC} = N_{CC}/N_{DET}. \qquad (2)$$

In other words, P_{CC} measures the proportion of correctly labeled targets to the number of correctly detected targets, e.g., classifying a detected target as a MBT when it indeed is a MBT, and serves as an estimate of the probability of correctly classifying a target. A similar, but slightly more specific, performance measure is the probability of correct identification, or P_{ID}. This measure is the ratio of the number of targets correctly classified by type (N_{ID}) to the total number of detected targets (N_{DET}). Thus, P_{ID} is defined as:

$$P_{ID} = N_{ID}/N_{DET}. \tag{3}$$

In other words, P_{ID} measures the proportion of correctly labeling, for example, a ROI as a T-72 MBT when it indeed is a T-72 MBT.

While the previous performance measure indicate how well an ATR CS performs in detecting targets, the next set of measures provides insight into how often an ATR CS mistakes non-targets as targets. The probability of false alarm measure, denoted P_{FA}, is the ratio of the number of detected confusers, or non-targets, (N_C) to the total number of known confusers in the image (N_{FA}), and is defined as:

$$P_{FA} = N_C / N_{FA}. \tag{4}$$

A similar performance measure is the false alarm rate (FAR), which is the ratio of the number of false alarms in clutter (N_{CL}) to the clutter scene area (A), defined as:

$$FAR = N_{CL} / A. \tag{5}$$

This performance measure indicates how likely an ATR is to mistake terrain or natural objects as a potential target.

Typically, P_D is changed for an ATR algorithm by adjusting a detection threshold internal to the CS. P_D may be increased to any desired level as the detection threshold is adjusted, but there is a corresponding degradation in the ATR performance as more clutter and confusers are incorrectly declared as targets. That is, the false alarm rate (FAR) and probability of false alarm, P_{FA}, never decrease, and should increase, as the probability of detection increases. For a given ATR CS, a receiver operating characteristic (ROC) curve illustrates the trade-off between the detection performance and the false alarm rate. Figure 2.2 depicts sample ROC curves along with an area under the curve measure, which is typically used in comparing multiple ROC curves. When

evaluating ATRs, either the P_D is fixed for a particular mission profile, or the ROC curve is employed. In both cases, P_D is not considered an evaluation measure as the ATR performs at that level by definition. The false alarm performance is then the concern.

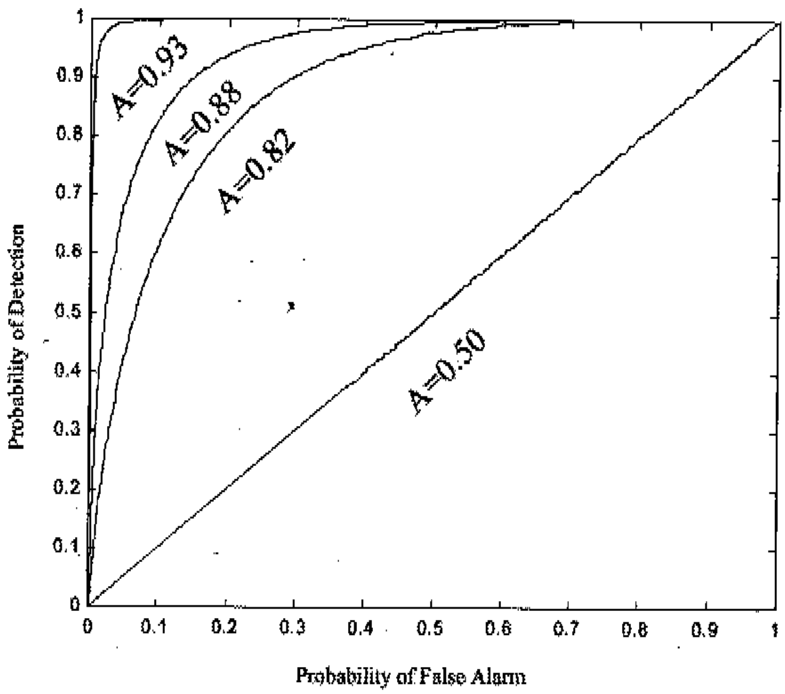

Figure 2.2 Sample Receiver Operating Characteristic (ROC) Curve [2]. The ROC curve represents the performance of a given ATR CS as an internal detection threshold is varied. The plot above illustrates the ROC curve for four different ATR CSs and provides a sample area under the curve (A) performance measure for each (higher is better).

Another desirable characteristic of an ideal ATR CS is its frequency of making declarations of targets and non-targets to be detected. Thus, a superior ATR system will declare a larger set of the known target population in an image than an inferior system. This measure hints at the confidence an ATR has in its detection ability. The probability of declaration, or P_{DEC}, is defined as:

[301]

$$P_{DEC} = N_{DEC} / N_{TOTAL} \tag{6}$$

where N_{DEC} represents the number of correct declarations (declaring a ROI to either a target or a non-target) made by the ATR CS and N_{TOTAL} is the total number of test objects, both targets and confusers. Thus, N_{DEC} consists of all ROIs declared as targets that are targets and all ROIs declared to be non-targets by the ATR CS that are confusers.

It is also desirable that ATR performance is robust to target, environmental, and sensor differences; provide an assessment of its confidence in its target estimates; and have a well-developed employment concept. ATR CSs are typically trained against a given set of baseline, or *nominal*, target images. An example of ATR performance robustness occurs when an ATR CS is able to detect and possibly correctly classify an MBT even though it has not been presented with the particular configuration that the vehicle exhibits. Thus, the ATR CS detects and/or correctly classifies the MBT when a variety of external differences from the nominal image training set, such as open hatches, turret articulation, or external stores, appear. Performance measures based upon the nominal training set, which are generally near perfect, are then compared to the performance measures where *at least one change* is made in the target configuration. It is desirable that the probabilities of detection, typification, and classification (P_D, P_{ID}, P_{CC}) remain as close to their nominal values as possible. Degradation is measured in percent change from the nominal values for some specific target set where the targets are perturbed in some fashion. The percent change values for probability of detection are calculated in the following manner, and P_{CC} and P_{ID} are assessed similarly. A nominal probability of detection measure ($P_{D\text{-}NOM}$) is assessed against targets at the baseline configuration. Next, a probability of detection measure is assessed against all other target

configuration deviations of interest ($P_{D\text{-}DEV}$), which may include sensor, target, environmental, or some combination of changes to the configuration. The difference between the two measures is assessed:

$$\Delta P_D = P_{D\text{-}NOM} - P_{D\text{-}DEV} \tag{7}$$

and the resultant difference estimate is between 0 and 1. For a large number of observations on a target array, the Central Limit Theorem permits the assumption of normality, and the formula:

$$\Phi(x) = \Delta P_D \pm Z_\alpha \sqrt{\frac{\Delta P_D (1 - \Delta P_D)}{n}} \tag{8}$$

can be used to create a probability density function, $\Phi(x)$, around the estimate for use in the decision analysis model [3]. To complete the procedure and create a percent change in the detection difference measure ($\%\Delta P_D$), the upper, lower, and mean values of the probability of detection difference estimate must be multiplied by 100. Though these estimates could be expressed as a normal distribution within the decision analysis model, they may be approximated by a triangular distribution. The triangular distribution forces the realizations to be bounded within the domain of elicitation of the DA information.

The ATR CS self-assessment is a confidence, expressed as a probability of accuracy, for the detection, typification, and classification estimates, C^D, C^{ID}, C^{CC}, respectively, as determined by the CS. For example, an ATR CS may have difficulty with the correct identification of a target. Perhaps the target exhibits characteristics that indicate to some degree that it is a T-72 MBT, but other characteristics indicate that it is a T-80 MBT. The ATR CS provides an identification, declaring that it is either a T-72 or a T-80, and an associated confidence, C_{ID}, for its declaration. A CS may be designed to

return a single target type with an associated confidence (e.g., 90 percent confidence that the target is a T-80), or a list of several possible target types each with an associated confidence (e.g., 60 percent confidence that the target is a T-80 and 40 percent confidence that the target is a T-72). Detection and classification confidences perform similarly.

This self-confidence score is compared to the true target identity to assess the accuracy of the confidence estimate. The accuracy of these confidence estimates for detection, typification, and classification estimates ($E_{S\text{-}PD}$, $E_{S\text{-}PCC}$, $E_{S\text{-}PID}$) are scored on the unit interval. The inclusion of the self-assessment measure accuracy in the DA model is for use on future ATR systems with this capability. The ATR CSs of the MSTAR program did not have the capability to provide or use these measures.

One possible method of assessing $E_{S\text{-}PD}$, $E_{S\text{-}PCC}$, $E_{S\text{-}PID}$ would be to compare C^{D}, C^{ID}, C^{CC} to the true nature of the target. Continuing with the identification case, each target is assigned C_{ij}^{ID} by the ATR CS, where the variable i indicates the target and the variable j indicates various identifications of the same target i by the CS. The self-assessment accuracy, $E_{S\text{-}PID}$, may be determined from

$$E_{S-PID} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{J}\left[\left(1-\frac{\sum_{i=1}^{J}\left(1-T_{ij}\right)\left(C_{ij}^{ID}\right)}{\sum_{i=1}^{J}C_{ij}^{ID}}\right)T_{ij}\left(C_{ij}^{ID}\right)\right] \tag{9}$$

where J is the total number of possible target identifications for the ith target, N is the total number of targets, M is the total number of target identifications made by the CS, and T_{ij} is an indicator variable defined as:

$$T_{ij} = \begin{cases} 1, & \text{ID correct} \\ 0, & \text{ID incorrect} \end{cases} \tag{10}$$

Similarly, $E_{S\text{-}PD}$, and $E_{S\text{-}PCC}$ are defined by

$$E_{S-PD} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{J}\left[\left(1-\frac{\sum_{i=1}^{J}(1-T_{ij})(C_{ij}^{D})}{\sum_{i=1}^{J}C_{ij}^{D}}\right)T_{ij}\left(C_{ij}^{D}\right)\right] \tag{11}$$

and

$$E_{S-PCC} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{J}\left[\left(1-\frac{\sum_{i=1}^{J}(1-T_{ij})(C_{ij}^{CC})}{\sum_{i=1}^{J}C_{ij}^{CC}}\right)T_{ij}\left(C_{ij}^{CC}\right)\right] \tag{12}$$

respectively.

Finally, cost is a consideration. Costs may be grouped by developmental costs, redeployment costs, and operational, or use, costs. Developmental costs are those incurred when bringing the ATR to an operational status. Redeployment costs are those incurred when moving the ATR from its base to an operational area. Operational costs are those incurred when employing the system. Each of these categorical costs may be further decomposed into the sub-costs of funding, time, expertise, and risk, except that there are no operational risks (with respect to the system performance).

The value hierarchy that emerged from the elicitation is depicted in Figure 2.3. The parenthetical numbers are the weights representing the relative importance, within the parent value, of the values. The value hierarchy was decomposed until the bottom tier members were measurable.

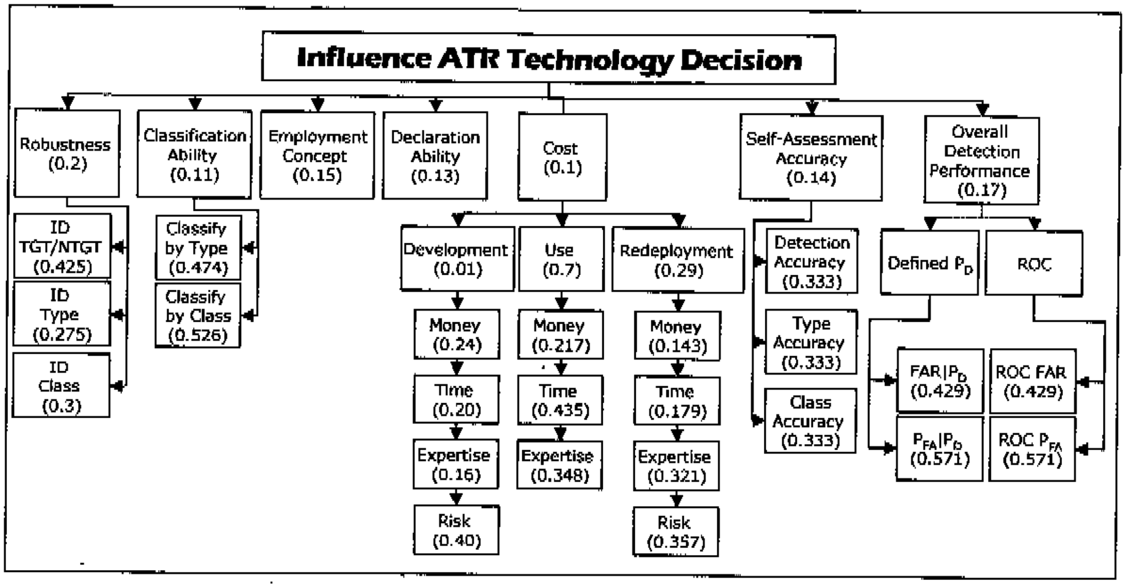

Figure 2.3 . The Value Hierarchy for ATR Decisions. The dashed line under Overall Detection Performance indicates that either the Defined P_D or the ROC measures are employed. The parenthetical numbers indicate the relative weights for a value, within the parent value.

In order to assess the categorical data, scales were constructed. For the adequacy of employment concept measure, the created scale categorized the employment concept as well defined, strongly defined, moderately defined, poorly defined, and no definition. Cost risks were assessed on a scale with low, medium, and high risk. Because these scales were developed in cooperation with the subject, no precise definitions were specified for the terms.

2.3 Single Dimensional Value and Utility Function Elicitation. The single dimensional value functions were elicited for each of the evaluation measures (the bottom tier of the value hierarchy). It was explained that most and least preferred levels of an evaluation measure mapped to one and zero respectively, and the function captures intervening preference levels. Initial discussion introduced the idea of comparing relative

differences in preference between pairs of evaluation measure observations. However, the subject was very comfortable with expressing preferences in a quantitative manner, usually commenting on a functional form and providing a value to fix the curve. For example, he often observed that his preference would decrease exponentially on a measure and pass through a point he specified. Direct assessment of the value functions was employed.

Single dimensional utility functions were elicited for continuous evaluation measures employing the certainty equivalent lottery technique. In this process the subject chooses between uncertain alternatives with two equally likely outcomes, each at the extremes of the evaluation measure domain, or a certain alternative with some specified level of the evaluation measure. The certain alternative is varied until the subject is indifferent between the alternatives. As is considered standard practice, the certainty equivalent was approached by alternately providing values from the margins of the evaluation measure domain to avoid anchoring. The initial lottery considered the entire domain for the evaluation measure under consideration at that point. This provides the datum point (known as a mid-value point) for which the subject assigns a utility of 0.5. This process was repeated for the sub-domains created about the mid-value point (the fractile approach). Thus each utility function is established with five data points, three of which are elicited and two defined. The subject again was comfortable with the elicitation methodology and often quickly determined his indifference points. As no functional form was evident from these elicitations, linear interpolation was employed for utilities between elicited points.

For the evaluation measures with categorical scales, the probability equivalent lottery method was used. The subject is provided a choice between uncertain and certain alternatives. The uncertain alternative provides the most preferred level of the alternative with some probability p and the least preferred with probability $(1-p)$. The certain alternative provides the evaluation measure level for which the corresponding utility will be determined. The p is varied until the subject is indifferent. The utility of the certain alternative is then equal to p. By considering each category between the least and most preferred levels, the utility for each category is determined. Again the subject rapidly provided indifference points for p.

2.4 ATR CS Alternatives. Data for most of the evaluation measures of the three alternatives, ATR 33, ATR 55, and ATR 89, were available from past MSTAR test data. As the MSTAR data did not include all evaluation measures, assumptions were made to permit analysis. These assumptions were:

- Developmental Costs are sunk. Value, $v(x)$, and utility, $u(x)$ are set equal to one in the model.
- Robustness for classify by class (% ΔP_{CC}) data was not collected. Data have $v(x) = u(x) = 1$.
- ATR CSs evaluated in the MSTAR program did not have the self-assessment capability: $v(x) = u(x) = 0$.
- No Classify by Class data was collected, so it was assumed that $P_{CC} = P_{ID}$. This provides a conservative estimate of P_{CC}.
- Assumed that costs for use time greater than 300 CPU seconds have $v(x)=u(x)=0$ and were truncated to stay within elicited domain for this evaluation measure.
- The preference functions for the cost of redeployment of the system were elicited on a domain normalized for the Global Hawk system. The Global Hawk costs were assumed to be $60k for this study.
- Operational costs were based on a $10k price for a workstation with a three-year lifecycle.
- Distributions were assumed to be triangular.

- P_{DEC} for all ATR CSs is assumed to be 1 since MSTAR CSs must make a declaration decision.

Some data were fictionalized from their test values to mask competition sensitive information. Fictional data are representative and the results may be used to validate the methodological approach, but the results are not valid for ATR selection. The analysis would have to be repeated with true data for an ATR selection decision. The data for the alternatives is available through the COMPASE Center.

3.0 Results and Discussion. The expected value results for the alternatives examined with value functions (under conditions of uncertainty and certainty where the mean was treated as deterministic) and utility functions (under uncertainty) are provided in Table 3.1. The results for the value functions under certainty differ slightly from those employing the value function under uncertainty (note the CID profile for ATR 55). This illustrates that using the mean of a distribution and treating it deterministically and using a value function does not provide identical results as considering the stochastic nature of the problem within the analysis.

Table 3.1 ATR CS Expected Value and Expected Utility Results.

		ATR 33	**ATR 55**	**ATR 89**
Value Functions (Certainty)	**CID**	0.509	0.537	0.525
	ISR	0.497	0.531	0.497
Value Functions (Uncertainty)	**CID**	0.509	0.556	0.525
	ISR	0.497	0.531	0.497
Utility Functions (Uncertainty)	**CID**	0.572	0.507	0.518
	ISR	0.414	0.455	0.439

Comparing, by rank ordering the alternatives, the results of the value functions under uncertainty and the utility functions in Table 3.2, we see that the recommended alternative differs for the CID case. This indicates that the constructs of value and utility provide differing answers. As the problem involves uncertainty, the utility results are the appropriate choice. The two employment profiles provide differing recommendations. For CID missions, ATR 33 is the best choice. For ISR missions, ATR 55 is the preferred alternative. However ATR 89 was the second choice under both profiles. Should one ATR be required to perform both missions, ATR 89 may be the most appropriate choice. These results are depicted graphically in Figure 3.1. The risk profiles (as cumulative distribution functions [CDFs] are referred to in DA terminology) for the utility distributions for the CID and ISR profiles are provided in Figures 3.2 and 3.3. Figures 3.4 and 3.5 present tornado diagrams for the two profiles where the weights were varied by ten percent. The lack of color change (shading) in the bars indicates that the ATR choice is not sensitive to the weights elicited from the decision maker.

Table 3.2 Recommended ATR CS Alternative by Expected Value and Utility.

		ATR 33	**ATR 55**	**ATR 89**
Value Functions (Certainty)	**CID**	3	1	2
	ISR	2	1	2
Value Functions (Uncertainty)	**CID**	3	1	2
	ISR	2	1	2
Utility Functions (Uncertainty)	**CID**	1	3	2
	ISR	3	1	2

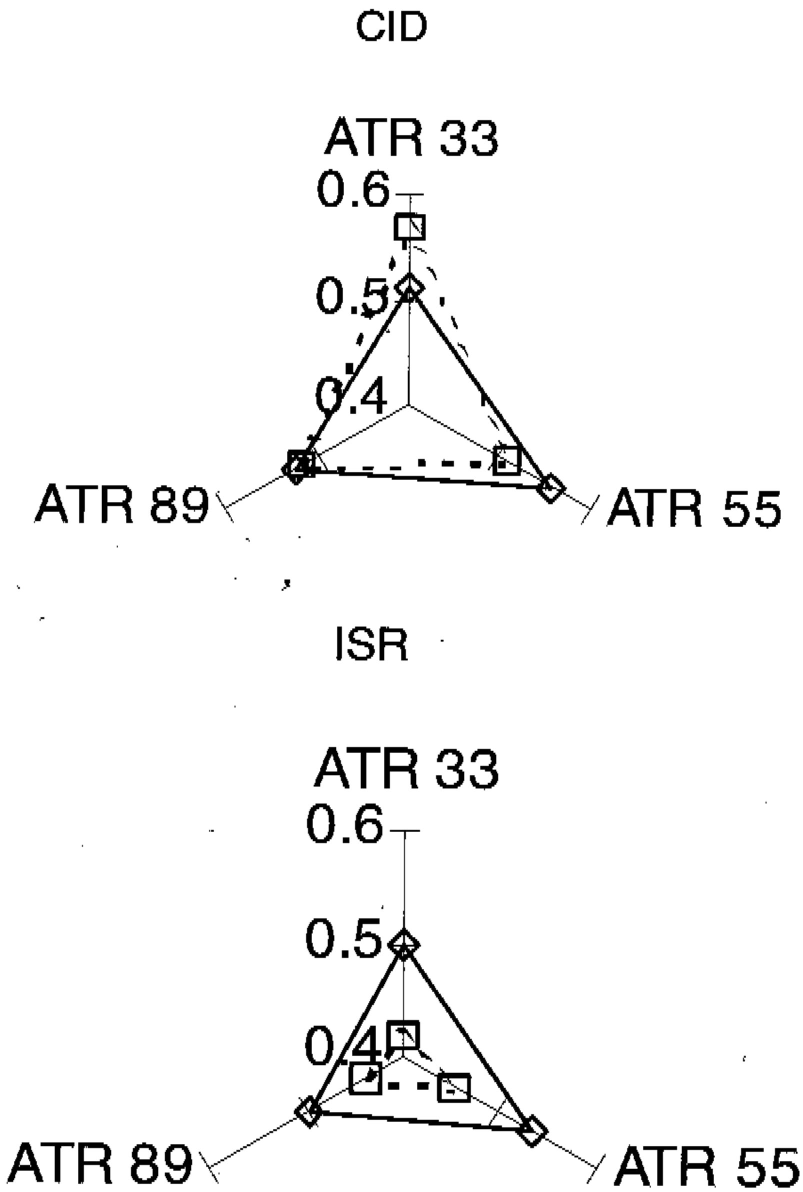

Figure 3.1 Radar Plots of ATR Value and Utility. Value results are represented by solid lines, utility by dashed. The Combat Identification (CID) results are above, the Intelligence/Surveillance/Reconnaissance (ISR) are below.

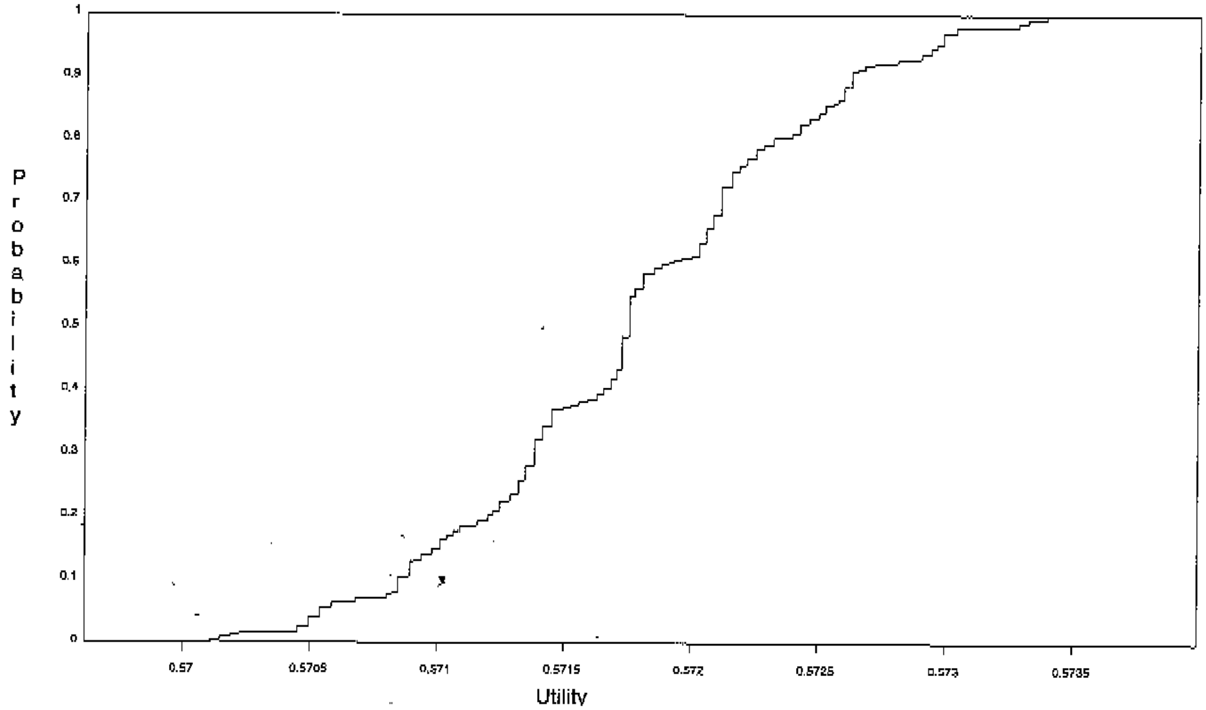

Figure 3.2 Risk Profile for CID.

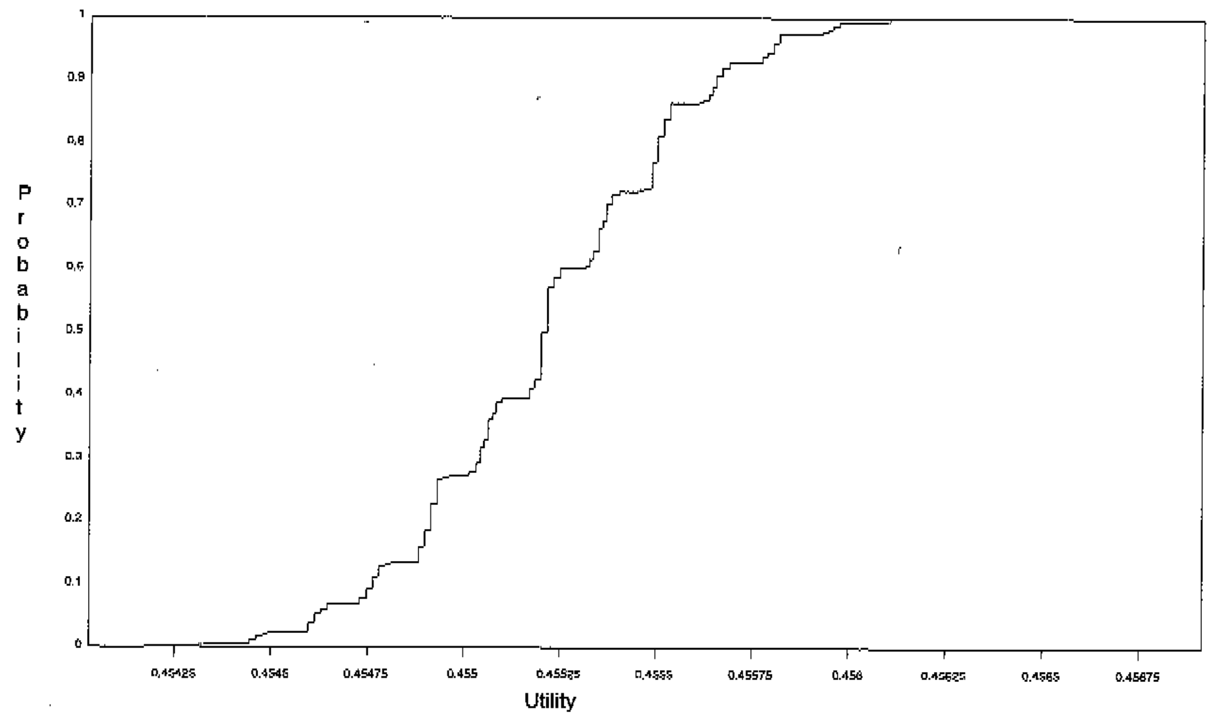

Figure 3.3 Risk Profile for ISR.

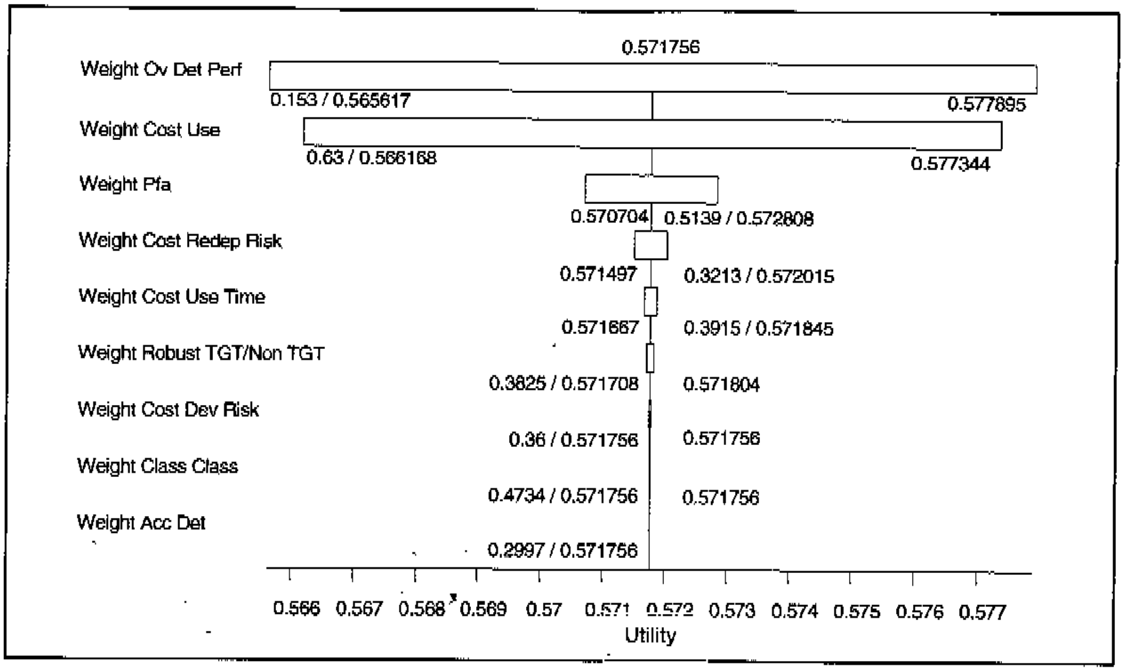

Figure 3.4 Sensitivity Analysis for CID Scenarios.

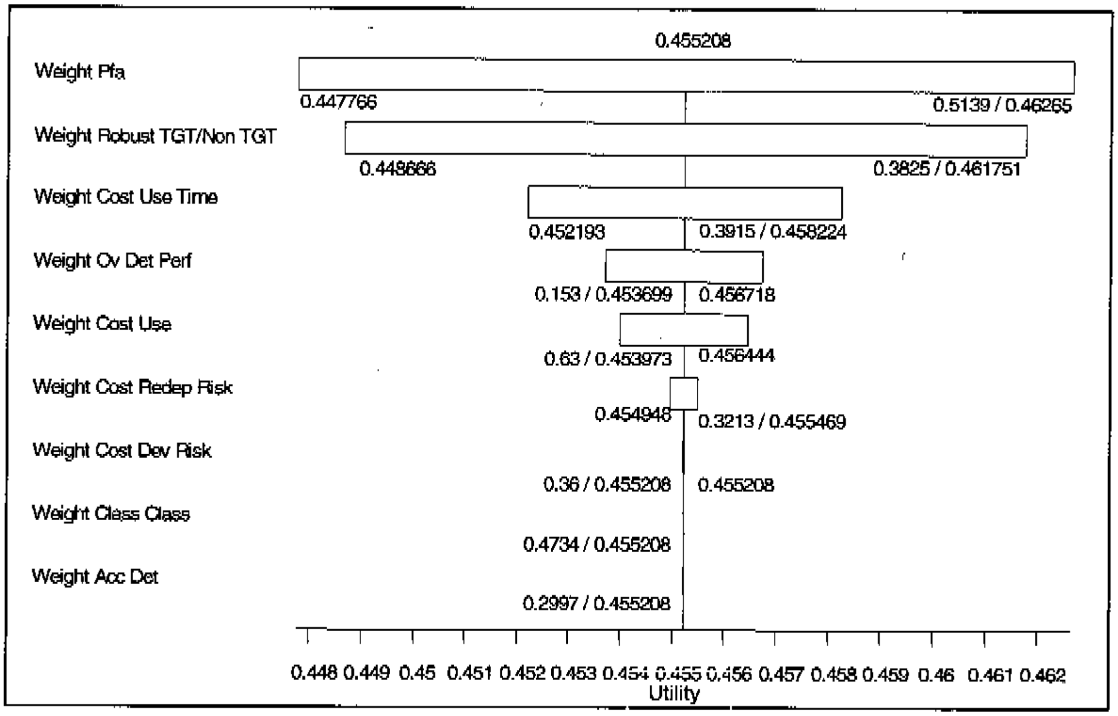

Figure 3.5 Sensitivity Analysis for ISR Scenarios.

The following figures illustrate that differing approaches, deterministic value, uncertain value and uncertain utility, provide differing answers. For the certain value approach, Figure 3.6 shows that for the CID employment profile, increasing cost of the ATR CS provides increasing value (benefit). For the ISR profile, ATR 33 dominates ATR 89. ATR 55 does provide improved value for an increased cost. These results hold for the uncertain value case, as illustrated in Figure 3.7. When utility is employed for the CID profile (Figure 3.8), ATR 33 dominates ATR 89, which in turn dominates ATR 55. For the ISR profile, increasing cost provides increasing utility (benefit). These interpretations agree with those of Table 3.2. Clearly both the mission profile and the DA methodology significantly affect the recommendation (answer).

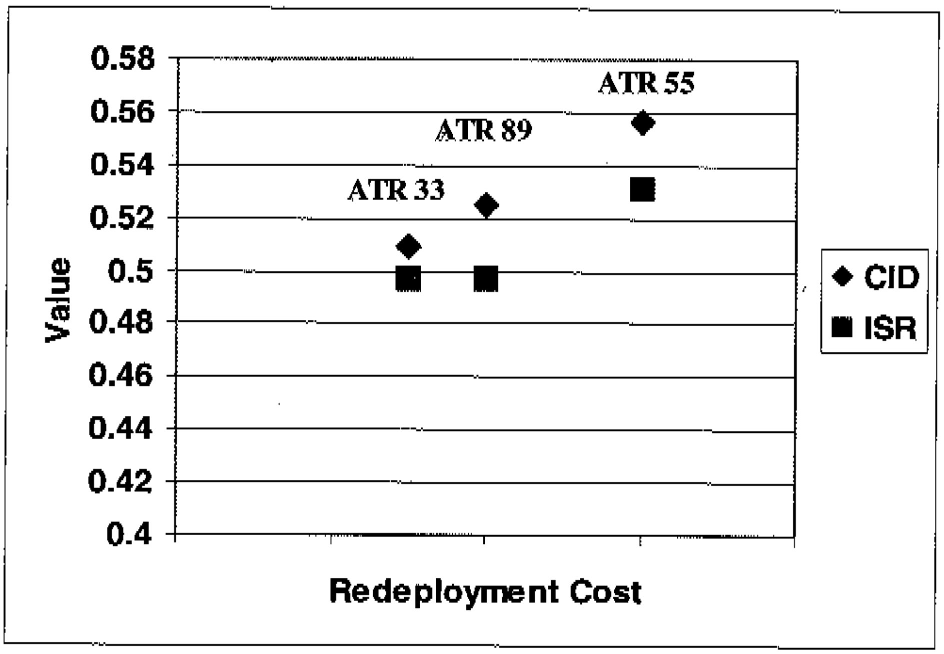

Figure 3.6 ATR Value (With Uncertainty) Versus Redeployment Cost.

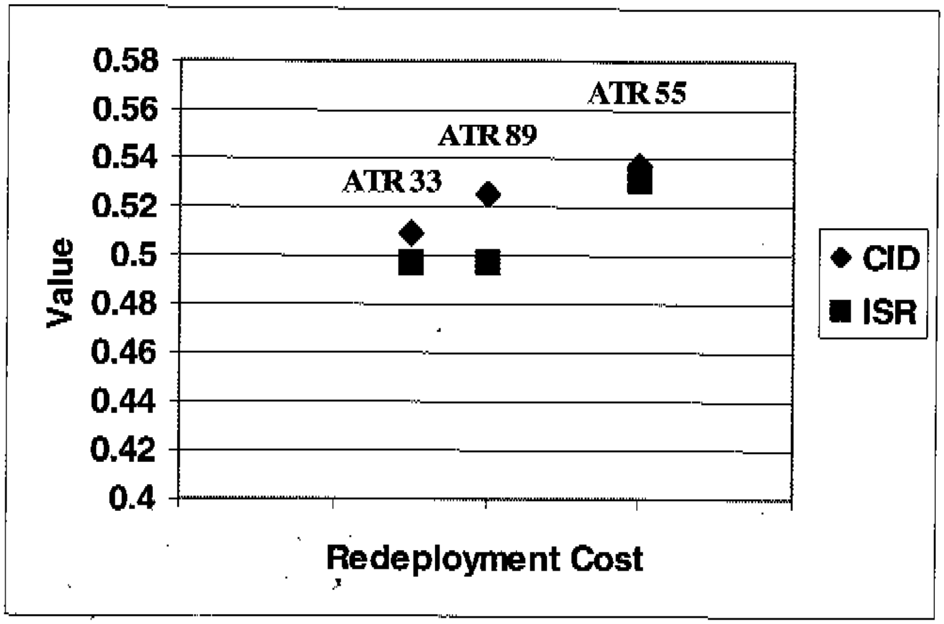

Figure 3.7 ATR Value (With Uncertainty) Versus Redeployment Cost.

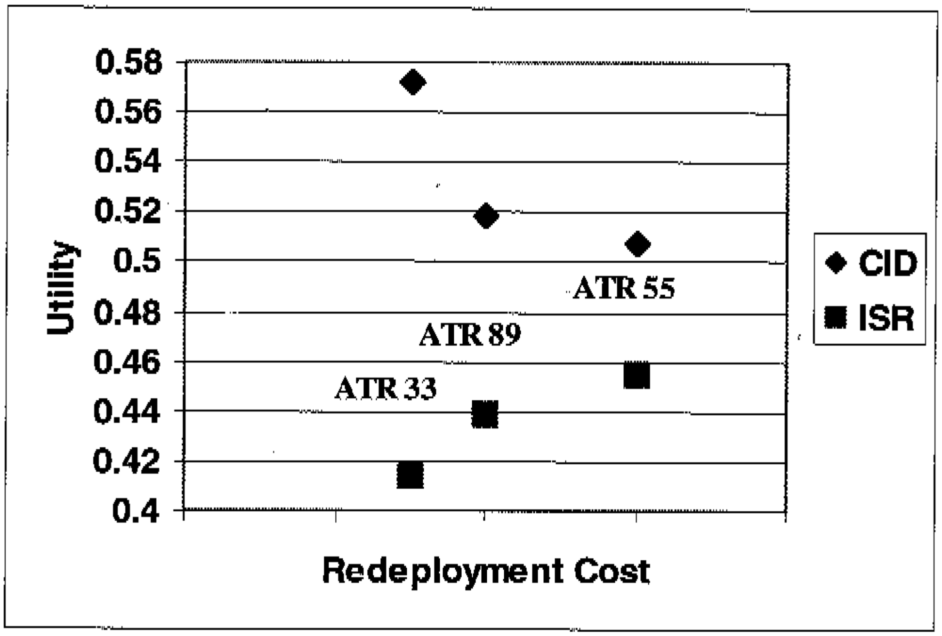

Figure 3.8 ATR Utility (With Uncertainty) Versus Redeployment Cost.

4.0 Conclusions. The conclusion can be drawn that the DA approach does indeed work. The decision-maker's preferences are encapsulated in the framework, and the resultant

utility measure provides a defensible argument towards selecting a given ATR CS over another.

5.0 Recommendations. The authors recommend that these results should be applied towards a warfighter's perspective of the same problem. In a method parallel to the one detailed in this report, a DA framework should be constructed that provides a utility function measure for combat model results generated using similar ATR CS performance measures. The utility of the warfighter perspective may then be used in conjunction with the ATR technology developer's utility results in order to positively affect the manner in which ATR products are evaluated and judged.

It is recommended that the decision analysis approach should be incorporated into ATR research and development programs. Design of experiments and data collection efforts should serve to provide data to score alternatives in accordance with the value hierarchy. Assessment of ATR programs should include mission profile considerations. Decision analysis evaluations should be utility based unless all criteria are considered to be under conditions of certainty, which is unlikely.

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Appendix A – Elicitation Results

The results of the elicitation meetings between the authors and the COMPASE Center representative are summarized below. Elicitation sessions also included hand-drawn plots of the value functions, which are not presented here.

1. Robustness.
    a. Top level weight: 0.2
    b. Subvalues:
        i. Detection Robustness
            1. Units: percent change in P_d.
            2. Domain: $[0,1]$.
            3. Weight: 0.425.
            4. Value Function: exponential fitting $\{(0,1),(25,0.2),(50,0)\}$.
            5. Utility Function: $\{(0,1),(3,0.75),(10,0.5),(10,0.25),(50,0)\}$.
        ii. Identification Robustness
            1. Units: percent change in P_{id}.
            2. Domain: $[0,1]$.
            3. Weight: 0.275.
            4. Value Function: exponential fitting $\{(0,1),(25,0.2),(50,0)\}$.
            5. Utility Function: $\{(0,1),(3,0.75),(10,0.5),(10,0.25),(50,0)\}$.
        iii. Classification Robustness
            1. Units: percent change in P_{cc}.
            2. Domain: $[0,1]$.
            3. Weight: 0.3.
            4. Value Function: exponential fitting $\{(0,1),(25,0.2),(50,0)\}$.
            5. Utility Function: $\{(0,1),(3,0.75),(10,0.5),(10,0.25),(50,0)\}$.
2. Overall Detection Performance. Note that this value decomposes into either the "defined P_d" subvalue or the "ROC" subvalue set, depending on which metric is used.
    a. Top level weight: 0.17.
    b. Subvalues:
        i. Defined P_d.
            1. $FAR \mid P_d$. Note, $FAR \mid P_d = 0.9$ or $FAR \mid P_d = 0.5$, depending on mission profile.
                a. Units: $occurences / km^2$.
                b. Domain: $[0,1]$.

c. Weight: 0.429.
d. Value Function: exponential fitting $\{(0,1),(1,0.3),(1000,0)\}$.
e. Utility Function: $\{(0,1),(3,0.75),(10,0.5),(50,0.25),(1000,0)\}$.

2. $P_{FA} \mid P_d$. Note, $P_{FA} \mid P_d = 0.9$ or $P_{FA} \mid P_d = 0.5$, depending on mission profile.
   a. Units: probability.
   b. Domain: $[0,1]$.
   c. Weight: 0.571.
   d. Value Function: linear fitting $\{(0,1),(1,0)\}$.
   e. Utility Function: $\{(0,1),(0.1,0.75),(0.3,0.5),(0.4,0.25),(1,0)\}$.

ii. ROC.

1. *ROC FAR.*
   a. Units: area under ROC curve.
   b. Domain: $[0,1000]$.
   c. Weight: 0.429.
   d. Value Function: exponential fitting $\{(0,0),(500,0.1),(1000,1)\}$.
   e. Utility Function: $\{(0,0),(600,0.25),(750,0.5),(875,0.75),(1000,1)\}$.
2. *ROC* P_{FA}.
   a. Units: normalized area under ROC curve.
   b. Domain: $[0,1]$.
   c. Weight: 0.571.
   d. Value Function: exponential fitting $\{(0.5,0),(0.75,0.3),(1,1)\}$.
   e. Utility Function: $\{(0.5,0),(0.8,0.25),(0.875,0.5),(0.938,0.75),(1,1)\}$.

3. Employment Concept.
   a. Units: categorical.
   b. Domain: {None, Poorly Defined, Moderately Defined, Strongly Defined, Well Defined}.
   c. Weight: 0.15.
   d. Value Function: {(None, 0), (Poorly Defined, 0.4), (Moderately Defined, 0.6), (Strongly Defined, 0.9), (Well Defined, 1)}.
   e. Utility Function: {(None, 0), (Poorly Defined, 0.1), (Moderately Defined, 0.5), (Strongly Defined, 0.9), (Well Defined, 1)}.
4. Declaration Ability.
   a. Units: probability.
   b. Domain: $[0.5,1]$.
   c. Weight: 0.13.

d. Value Function: exponential fitting $\{(0,0),(0.5,0.7),(1,1)\}$.
e. Utility Function: $\{(0,0),(0.25,0.25),(0.5,0.5),(0.75,0.75),(1,1)\}$.

5. Classification Ability.
    a. Top level weight: 0.11.
    b. Subvalues:
        i. Correctly Classify by Type (P_{ID}).
            1. Units: probability.
            2. Domain: $[0.5,1]$.
            3. Weight: 0.474.
            4. Value Function: exponential fitting $\{(0.5,0),(0.75,0.3),(1,1)\}$.
            5. Utility Function: $\{(0.5,0),(0.6,0.25),(0.7,0.5),(0.9,0.75),(1,1)\}$.
        ii. Correctly Classify by Class (P_{CC}).
            1. Units: probability.
            2. Domain: $[0,1]$.
            3. Weight: 0.526.
            4. Value Function: exponential fitting $\{(0.5,0),(0.75,0.2),(1,1)\}$.
            5. Utility Function: $\{(0.5,0),(0.6,0.25),(0.7,0.5),(0.9,0.75),(1,1)\}$.
6. Cost
    a. Top level weight: 0.1.
    b. Subvalues:
        i. Development
        ii. Weight: 0.01.
            1. Money.
                a. Units: \$M.
                b. Domain: $[0,10]$.
                c. Weight: 0.24.
                d. Value Function: exponential fitting $\{(0,1),(5,0.3),(10,0)\}$.
                e. Utility Function: $\{(0,1),(2.5,0.75),(5,0.5),(7.5,0.25),(10,0)\}$.
            2. Time.
                a. Units: months.
                b. Domain: $[0,18]$.
                c. Weight: 0.2.
                d. Value Function: exponential fitting $\{(0,1),(9,0.3),(18,0)\}$.
                e. Utility Function: $\{(0,1),(4.5,0.75),(9,0.5),(13.5,0.25),(18,0)\}$.
            3. Expertise
                a. Units: categorical.

b. Domain: {Technical Training, BS in Engineering, Graduate-level Engineer, Multi-location subject matter experts (SME), Single Site SME}.
c. Weight: 0.16.
d. Value Function: {(Technical Training, 1), (BS in Engineering, 0.8), (Graduate-level Engineer, 0.4), (Multi-location subject matter experts (SME), 0.2), (Single Site SME, 0)}.
e. Utility Function: {(Technical Training, 1), (BS in Engineering, 0.9), (Graduate-level Engineer, 0.8), (Multi-location subject matter experts (SME), 0.7), (Single Site SME, 0)}.

4. Risk
   a. Units: categorical.
   b. Domain: Low, Medium, and High.
   c. Weight: 0.4.
   d. Value Function: exponential fitting: {(Low, 1), (Medium, 0.5), (High, 0)}.
   e. Utility Function: {(Low, 1), (Medium, 0.8), (High, 0)}.

iii. Redeployment.
iv. Weight: 0.29.

1. Money.
   a. Units: normalized on Global Hawk.
   b. Domain: $[0,1]$.
   c. Weight: 0.143.
   d. Value Function: linear, $\{(0,1),(1/3,0),(1,0)\}$.
   e. Utility Function: $\{(0,1),(0.15,0.75),(0.333,0.5),(0.9999,0.25),(1,0)\}$.
2. Time.
   a. Units: days.
   b. Domain: $[0,90]$.
   c. Weight: 0.179.
   d. Value Function: exponential fitting $\{(0,1),(45,0.2),(90,0)\}$.
   e. Utility Function: $\{(0,1),(15,0.75),(30,0.5),(50,0.25),(90,0)\}$.
3. Expertise
   a. Units: categorical.
   b. Domain: {Technical Training, BS in Engineering, Graduate-level Engineer, Multi-location subject matter experts (SME), Single Site SME}.
   c. Weight: 0.321.
   d. Value Function: {(Technical Training, 1), (BS in Engineering, 0.95), (Graduate-level Engineer, 0.9),

(Multi-location subject matter experts (SME), 0.5), (Single Site SME, 0)}.

e. Utility Function: {(Technical Training, 1), (BS in Engineering, 0.95), (Graduate-level Engineer, 0.9), (Multi-location subject matter experts (SME), 0.8), (Single Site SME, 0)}.

4. Risk
   a. Units: categorical.
   b. Domain: Low, Medium, and High.
   c. Weight: 0.357.
   d. Value Function: exponential fitting: {(Low, 1), (Medium, 0.5), (High, 0)}.
   e. Utility Function: {(Low, 1), (Medium, 0.2), (High, 0)}.

v. Use.

vi. Weight: 0.7.

1. Money.
   a. Units: normalized on Global Hawk.
   b. Domain: $[0,2]$.
   c. Weight: 0.217.
   d. Value Function: exponential fitting $\{(0,1),(1,0.2),(2,0)\}$.
   e. Utility Function: $\{(0,1),(0.5,0.75),(1,0.5),(1.5,0.25),(2,0)\}$.
2. Time.
   a. Units: minutes.
   b. Domain: $[0,5]$.
   c. Weight: 0.435.
   d. Value Function: exponential fitting $\{(0,1),(2:30,0.3),(5,0)\}$.
   e. Utility Function: $\{(0,1),(1,0.75),(2,0.5),(3,0.25),(5,0)\}$.
3. Expertise
   a. Units: categorical.
   b. Domain: {Technical Training, BS in Engineering, Graduate-level Engineer, Multi-location subject matter experts (SME), Single Site SME}.
   c. Weight: 0.348.
   d. Value Function: {(Technical Training, 1), (BS in Engineering, 0.4), (Graduate-level Engineer, 0.3), (Multi-location subject matter experts (SME), 0.1), (Single Site SME, 0)}.
   e. Utility Function: {(Technical Training, 1), (BS in Engineering, 0.2), (Graduate-level Engineer, 0.1),

(Multi-location subject matter experts (SME), 0.05), (Single Site SME, 0)}.

7. Self-Assessed Accuracy.
   a. Es-Pd
      i. Units:
      ii. Domain: [0,1].
      iii. Weight: 0.333.
      iv. Value Function: exponential fitting $\{(0,1),(0.5,0.7),(1,0)\}$.
      v. Utility Function: $\{(0,1),(0.25,0.75),(0.5,0.5),(0.75,0.25),(1,0)\}$.
   b. Es-Pid
      i. Units:
      ii. Domain: [0,1].
      iii. Weight: 0.333.
      iv. Value Function: exponential fitting $\{(0,1),(0.5,0.7),(1,0)\}$.
      v. Utility Function: $\{(0,1),(0.25,0.75),(0.5,0.5),(0.75,0.25),(1,0)\}$.
   c. Es-Pcc
      i. Units:
      ii. Domain: [0,1].
      iii. Weight: 0.333.
      iv. Value Function: exponential fitting $\{(0,1),(0.5,0.7),(1,0)\}$.
      v. Utility Function: $\{(0,1),(0.25,0.75),(0.5,0.5),(0.75,0.25),(1,0)\}$.

List of Acronyms

AFRL	Air Force Research Laboratory
AGRI	Air-to-Ground Radar Imaging
ATR	Automatic Target Recognition
CID	Combat Identification
COMPASE Center	Comprehensive ATR Scientific Evaluation Center
CS	Classification System
DA	Decision Analysis
ISR	Intelligence/Surveillance/Reconnaissance
MBT	Main Battle Tank
MSTAR	Moving and Stationary Target Acquisition and Recognition
ROC	Receiver Operating Characteristic
SAR	synthetic aperture radar
TGT	target

Appendix E. ATR CS Hybrid Utility Detailed Results

Background. An application of decision analysis to the problem of selection of an Automatic Target Recognition (ATR) classification system (CS) is provided in Chapter V. Detailed information is presented below. Simply stated, these results are a series of group screening steps investigating the significance of the evaluation measures.

First Group Screening. The ATR CS hybrid utility information presented in chapter V included the algorithm results involving the significant variables. The detailed experimental setup is shown in Table 59. The subscripts for the exponential constant variables of the initial group screening, ρ_i, are:

1. Robustness,
2. Overall Detection Performance,
3. Employment Concept,
4. Declaration Ability,
5. Classification Ability,
6. Cost, and
7. Self-Assessed Accuracy.

Explanation of the variables and the values corresponding to the settings for each row are presented in Chapter V.

Table 59. Detailed Experimental Design and Results for ATR Problem.

Row	Pattern	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	$\tilde{\mathbf{w}}$	Profile	Alternative	$\hat{U}$
1	----------	-.1	-.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.278
2	---------0	-.1	-.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 55	0.302
3	---------+	-.1	-.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.292
4	+-+---+++-	.1	-.1	.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.535
5	+-+---+++0	.1	-.1	.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 55	0.505

Row	Pattern	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	$\tilde{\mathbf{w}}$	Profile	Alternative	$\hat{U}$
6	+-+---++++	.1	-.1	.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.506
7	++-+---++-	.1	.1	-.1	.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.480
8	++-+---++0	.1	.1	-.1	.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 55	0.544
9	++-+---+++	.1	.1	-.1	.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.546
10	-++-+---+-	-.1	.1	.1	-.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.601
11	-++-+---+0	-.1	.1	.1	-.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.645
12	-++-+---++	-.1	.1	.1	-.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 89	0.647
13	+-++-+----	.1	-.1	.1	.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.574
14	+-++-+---0	.1	-.1	.1	.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 55	0.622
15	+-++-+---+	.1	-.1	.1	.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.627
16	++-++-+---	.1	.1	-.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.610
17	++-++-+--0	.1	.1	-.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 55	0.623
18	++-++-+--+	.1	.1	-.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.602
19	+++-++-+--	.1	.1	.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.772
20	+++-++-+-0	.1	.1	.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.806
21	+++-++-+-+	.1	.1	.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 89	0.795
22	-+++-++-+-	-.1	.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.512
23	-+++-++-+0	-.1	.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.565
24	-+++-++-++	-.1	.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 89	0.583
25	--+++-++--	-.1	-.1	.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.500
26	--+++-++-0	-.1	-.1	.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.545
27	--+++-++-+	-.1	-.1	.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 89	0.541
28	---+++-++-	-.1	-.1	-.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.381
29	---+++-++0	-.1	-.1	-.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 55	0.360
30	---+++-+++	-.1	-.1	-.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.376
31	+---+++-+-	.1	-.1	-.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.524
32	+---+++-+0	.1	-.1	-.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.515
33	+---+++-++	.1	-.1	-.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 89	0.531
34	-+---+++--	-.1	.1	-.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.414
35	-+---+++-0	-.1	.1	-.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.433
36	-+---+++-+	-.1	.1	-.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 89	0.414
37	00000000--	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 33	0.509
38	00000000-0	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 55	0.556
39	00000000-+	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 89	0.525
40	00000000+-	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 33	0.497
41	00000000+0	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 55	0.531

Row	Pattern	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	$\tilde{\mathbf{w}}$	Profile	Alternative	$\hat{U}$
42	00000000++	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 89	0.497

Second Group Screening. The group screening experimental design for the robustness group is presented in Table 60.

Table 60. Robustness Screening Experimental Design and Response for ATR Problem.

Row	**Pattern**	ρ_1	ρ_2	ρ_3	$\tilde{\mathbf{w}}$	**Profile**	**Alternative**	$\hat{U}$
1	---++-	-.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.0289
2	+++---	.1	.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.949
3	-++---	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.577
4	+--++-	.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.745
5	-+--+-	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.577
6	+-++--	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.745
7	--++-0	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.286
8	++--+0	.1	.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.996
9	--+-+0	-.1	-.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.331
10	++-+-0	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.995
11	-+-+-0	-.1	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.534
12	+-+-+0	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.708
13	-+++++	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.526
14	+----+	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.701
15	-----+	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.324
16	++++++	.1	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.987
17	-----+	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.324
18	++++++	.1	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.987
19	000011	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 33	0.618
20	000012	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 55	0.671
21	000013	∞	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 89	0.584
22	000021	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 33	0.618
23	000022	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 55	0.671
24	000023	∞	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 89	0.584

Third Group Screening. The group screening experimental design for the cost group is presented in Table 61.

Table 61. Cost Screening Design and Response for ATR Problem.

Row	Pattern	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	$\tilde{\mathbf{w}}$	ATR	$\hat{U}$
1	--++--++-+---	-.1	-.1	.1	.1	-.1	-.1	.1	.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.642
2	---+++++++-+0	-.1	-.1	-.1	.1	.1	.1	.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	55	0.617
3	+-+--++-+-+--	.1	-.1	.1	-.1	-.1	.1	.1	-.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	33	0.612
4	-------+----+	-.1	-.1	-.1	-.1	-.1	-.1	-.1	.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	89	0.370
5	+----++--+---	.1	-.1	-.1	-.1	-.1	.1	.1	-.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.439
6	+++-+--++---0	.1	.1	.1	-.1	.1	-.1	-.1	.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	55	0.496
7	++++-++++--+-	.1	.1	.1	.1	-.1	.1	.1	.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	33	0.703
8	--+-+++++--+0	-.1	-.1	.1	-.1	.1	.1	.1	.1	.1	-.1	.-1	$\tilde{\mathbf{w}}^+$	55	0.617
9	++-----+++++-	.1	.1	-.1	-.1	-.1	-.1	-.1	.1	.1	.1	.1	$\tilde{\mathbf{w}}^+$	33	0.778
10	++-++-+--+-+0	.1	.1	-.1	.1	.1	-.1	.1	-.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^+$	55	0.443
11	+---++++-+++0	.1	-.1	-.1	-.1	.1	.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	55	0.546
12	++++++-+++--+	.1	.1	.1	.1	.1	.1	-.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^-$	89	0.694
13	-++-++-+--++-	-.1	1	.1	-.1	.1	.1	-.1	.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	33	0.703
14	-++---+--+-+0	-.1	.1	.1	-.1	-.1	-.1	.1	-.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^+$	55	0.443
15	++-+-+---+-++	.1	.1	-.1	.1	-.1	.1	-.1	-.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^+$	89	0.804
16	---+---++-++-	-.1	-.1	-.1	.1	-.1	-.1	-.1	.1	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	33	0.664
17	--++-+----+-0	-.1	-.1	.1	.1	-.1	.1	-.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^-$	55	0.351
18	--+-+-+-+++-+	-.1	-.1	.1	-.1	.1	-.1	.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^-$	89	0.756
19	-+-+++--++---	-.1	.1	-.1	.1	.1	.1	-.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.705
20	+---+--+-+---	.1	-.1	-.1	-.1	.1	-.1	-.1	.1	-.1	.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.642
21	+-+--+--++-++	.1	-.1	.1	-.1	-.1	.1	-.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	89	0.815
22	+++---+++--++	.1	.1	.1	-.1	-.1	-.1	.1	.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	89	0.526
23	+-+++---++++0	.1	-.1	.1	.1	.1	-.1	-.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^+$	55	0.444
24	+---+++-+--++	.1	-.1	-.1	-.1	.1	.1	.1	-.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	89	0.456
25	+++++++---+--	.1	.1	.1	.1	.1	.1	.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^-$	33	0.612
26	++----+-+-+-0	.1	.1	-.1	-.1	-.1	-.1	.1	-.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	55	0.483
27	+-++--++-++-+	.1	-.1	.1	.1	-.1	-.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	89	0.747
28	-++--------++	-.1	.1	.1	-.1	-.1	-.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^+$	89	0.445
29	+++-++-+-++-0	.1	.1	.1	-.1	.1	.1	-.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	55	0.477
30	--+++-------+	-.1	-.1	.1	.1	.1	-.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	89	0.370
31	-++++-+++----	-.1	.1	.1	.1	.1	-.1	.1	.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.549
32	-----++-+---0	-.1	-.1	-.1	-.1	-.1	.1	.1	-.1	.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	55	0.522
33	++-+-+++--+++	.1	.1	-.1	.1	-.1	.1	.1	.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	89	0.552
34	+-++----++++-	.1	-.1	.1	.1	-.1	-.1	-.1	-.1	.1	.1	.1	$\tilde{\mathbf{w}}^+$	33	0.778
35	-+-------++-0	-.1	.1	-.1	-.1	-.1	-.1	-.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	55	0.312

Row	Pattern	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	$\tilde{\mathbf{w}}$	ATR	$\hat{U}$
36	-+--++-++++-+	-.1	.1	-.1	-.1	.1	.1	-.1	.1	.1	.1	.1	$\tilde{\mathbf{w}}^-$	89	0.703
37	-++-+++--+++-	-.1	.1	.1	-.1	.1	.1	.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	33	0.817
38	++-++-+-----0	.1	.1	-.1	.1	.1	-.1	.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	55	0.425
39	+--++---+-+++	.1	-.1	-.1	.1	.1	-.1	-.1	-.1	.1	-.1	.1	$\tilde{\mathbf{w}}^+$	89	0.456
40	-+--+-----++-	-.1	.1	-.1	-.1	.1	-.1	-.1	-.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	33	0.664
41	-+++-+--++-+0	-.1	.1	.1	.1	-.1	.1	-.1	-.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	55	0.468
42	-+-+-+++-++-+	-.1	.1	-.1	.1	-.1	.1	.1	.1	-.1	.1	.1	$\tilde{\mathbf{w}}^-$	89	0.807
43	+---++--------	.1	-.1	-.1	-.1	.1	.1	-.1	-.1	-.1	-.1	-.1	$\tilde{\mathbf{w}}^-$	33	0.612
44	---+-+-+--++0	-.1	-.1	-.1	.1	-.1	.1	-.1	.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^+$	55	0.475
45	--+-+-+--++++	-.1	-.1	.1	-.1	.1	-.1	.1	-.1	-.1	.1	.1	$\tilde{\mathbf{w}}^+$	89	0.837
46	---++-++++-+-	-.1	-.1	-.1	.1	.1	-.1	.1	.1	.1	.1	-.1	$\tilde{\mathbf{w}}^+$	33	0.778
47	+-+++--+--+-0	.1	-.1	.1	.1	.1	-.1	-.1	.1	-.1	-.1	.1	$\tilde{\mathbf{w}}^-$	55	0.438
48	-+-++++-+-+-+	-.1	.1	-.1	.1	.1	.1	.1	-.1	.1	-.1	.1	$\tilde{\mathbf{w}}^-$	89	0.553
49	00000000000-	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	33	0.719
50	000000000000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	55	0.500
51	00000000000+	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	$\tilde{\mathbf{w}}^0$	89	0.670

Fourth Group Screening. The group screening experimental design for the detection performance group is presented in Table 62.

Table 62. Detection Performance Screening Experimental Design and Response for ATR Problem.

Row	Pattern	ρ_1	ρ_2	$\tilde{\mathbf{w}}$	Profile	ATR	$\hat{U}$
1	---+-	-.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.0850
2	+++--	.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.680
3	-++--	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.317
4	+--+-	.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.0850
5	-+---	-.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.260
6	+-++-	.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.104
7	--++0	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 55	0.0133
8	++--0	.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 55	0.919
9	--+-0	-.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.331
10	++-+0	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.996
11	-+-+0	-.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.522
12	+-+-0	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.373
13	-++++	-.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.626
14	+---+	.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.486
15	----+	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.383

Row	Pattern	ρ_1	ρ_2	$\tilde{\mathbf{w}}$	Profile	ATR	$\hat{U}$
16	+++++	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.991
17	----+	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.383
18	+++++	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.991
19	000--	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 33	0.204
20	000-0	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 55	0.570
21	000-+	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 89	0.464
22	000+-	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 33	0.468
23	000+0	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 55	0.562
24	000++	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 89	0.480

Fifth Group Screening. The group screening experimental design for the classification ability group is presented in Table 63.

Table 63. Classification Ability Screening Design and Response for ATR Problem.

Row	Pattern	ρ_1	ρ_2	$\tilde{\mathbf{w}}$	Profile	ATR	$\hat{U}$
1	---+-	-.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.00106
2	+++--	.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	1.000
3	-++--	-.1	.1	$\tilde{\mathbf{w}}^+$	CID	ATR 33	0.684
4	+--+-	.1	-.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 33	0.508
5	-+---	-.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 33	0.603
6	+-++-	.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 33	0.410
7	--++0	-.1	-.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 55	0.0150
8	++--0	.1	.1	$\tilde{\mathbf{w}}^-$	CID	ATR 55	0.995
9	--+-0	-.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.110
10	++-+0	.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.990
11	-+-+0	-.1	.1	$\tilde{\mathbf{w}}^-$	ISR	ATR 55	0.484
12	+-+-0	.1	-.1	$\tilde{\mathbf{w}}^+$	CID	ATR 55	0.455
13	-++++	-.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.552
14	+---+	.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.529
15	----+	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.0436
16	+++++	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.967
17	----+	-.1	-.1	$\tilde{\mathbf{w}}^-$	CID	ATR 89	0.0436
18	+++++	.1	.1	$\tilde{\mathbf{w}}^+$	ISR	ATR 89	0.967
19	00011	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 33	0.811
20	00012	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 55	0.769
21	00013	∞	∞	$\tilde{\mathbf{w}}^0$	CID	ATR 89	0.655
22	00021	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 33	0.293

23	00022	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 55	0.557
24	00023	∞	∞	$\tilde{\mathbf{w}}^0$	ISR	ATR 89	0.374

Hybrid Value-Utility Algorithm. The results from applying the algorithm to all evaluation measures for each mission profile are presented here.

Table 64. CID Profile Value, Utility, and Hybrid Utility Results, All Evaluation Measures.

	Notes	ATR 33	ATR 55	ATR 89
Goal	Utility Model	0.572	0.507	0.518
Iteration				
0	Value Model	0.509	0.556	0.525
1	Employment Concept	0.464	0.511	0.480
2	Detection Robustness	0.455	0.471	0.442
3	Probability of False Alarm	0.529	0.500	0.512
4	Classification by Class	0.536	0.509	0.524
5	Classification by Type	0.538	0.511	0.528
6	Identification Robustness	0.529	0.505	0.509
7	False Alarm Rate	0.569	0.506	0.510
8	Declaration Ability	0.569	0.506	0.510
9	Cost Use Time	0.569	0.506	0.513
10	Classification Robustness	0.569	0.506	0.513
11	Cost Redeployment Time	0.570	0.507	0.513
12	Cost Use Expertise	0.570	0.507	0.513
13	Cost Redeployment Risk	0.570	0.504	0.513
14	Cost Redeployment Expertise	0.570	0.506	0.516
15	Cost Use Money	0.570	0.507	0.516
16	Cost Development Expertise	0.570	0.507	0.516
17	Redeployment Money	0.572	0.507	0.518
18	Cost Development Money	0.572	0.507	0.518
19	Cost Development Risk	0.572	0.507	0.518
20	Cost Development Time	0.572	0.507	0.518
21	Es Pd	0.572	0.507	0.518
22	Es Pid	0.572	0.507	0.518
23	Es Pcc	0.572	0.507	0.518

Table 65. ISR Profile Value, Utility, and Hybrid Utility Results, All Evaluation Measures.

	Notes	ATR 33	ATR 55	ATR 89
Goal	Utility Model	0.415	0.455	0.439
Iteration				
0	Value Model	0.497	0.531	0.500
1	Employment Concept	0.452	0.486	0.452
2	Detection Robustness	0.443	0.446	0.414
3	Probability of False Alarm	0.371	0.415	0.383
4	Classification by Class	0.392	0.430	0.403
5	Classification by Type	0.405	0.435	0.413
6	Identification Robustness	0.395	0.429	0.394
7	False Alarm Rate	0.412	0.455	0.432
8	Declaration Ability	0.412	0.455	0.432
9	Cost Use Time	0.412	0.455	0.434
10	Classification Robustness	0.412	0.455	0.434
11	Cost Redeployment Time	0.413	0.455	0.434
12	Cost Use Expertise	0.413	0.455	0.434
13	Cost Redeployment Risk	0.413	0.452	0.434
14	Cost Redeployment Expertise	0.413	0.455	0.437
15	Cost Use Money	0.413	0.455	0.437
16	Cost Development Expertise	0.413	0.455	0.437
17	Redeployment Money	0.415	0.455	0.439
18	Cost Development Money	0.415	0.455	0.439
19	Cost Development Risk	0.415	0.455	0.439
20	Cost Development Time	0.415	0.455	0.439
21	Es Pd	0.415	0.455	0.439
22	Es Pid	0.415	0.455	0.439
23	Es Pcc	0.415	0.455	0.439

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Vita

Colonel William K. Klimack graduated from Lehigh University, Bethlehem, Pennsylvania, with a Bachelor of Science degree in Chemical Engineering in May 1979. A Distinguished Military Graduate of the Reserve Officer Training Corps, he received a Regular Army commission as a Second Lieutenant, Infantry. Subsequently he earned a Master of Science in Applied Mathematics from the Johns Hopkins University, Baltimore, Maryland, and a Master of Military Arts and Science from the United States Army Command and General Staff College, Fort Leavenworth, Kansas. His thesis topic was entitled An Analysis of Direct-Fire, Time-Fused, Bursting Munitions. He has been selected to attend the United States Army War College.

He has served at every level from platoon to army and in five divisions. Key assignments included commanding Company A, 1st Battalion, 325th Infantry (Airborne), 82nd Airborne Division and 2nd Battalion, 28th Infantry (Basic Combat Training). He also served as the Chief of Operations, Assistant Chief of Staff G3, 101st Airborne Division (Air Assault) and as Chief of Joint/Combined Staff, United States Forces Haiti. He is a combat veteran of Operation Urgent Fury. He is a graduate of Basic and Advanced Airborne School, Air Assault School (Distinguished Graduate), the Operations Research Systems Analysis Military Applications Course I (Distinguished Honor Graduate), and Ranger School.

Awards include the Humanitarian Service Medal with Oak Leaf Cluster, the Armed Forces Expeditionary Medal, the Southwest Asia Service Medal, the Meritorious Service Medal with four Oak Leaf Clusters, the Defense Meritorious Service medal, and

the Expert Infantryman Badge. The West German and Turkish Armies also have awarded him jump wings. He is a member of the Tau Beta Pi and Omega Rho national honorary fraternities. Additionally, he is a Fellow of the United Kingdom's Royal Geographic Society and the Explorers Club. He serves on the latter's Science Advisory Board and also serves on the Board of Directors of the National Speleological Society.

He currently serves in the Department of Systems Engineering, United States Military Academy, West Point, New York.

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14. ABSTRACT

This research examined the relationship between value and utility functions in multiobjective decision analysis in a military decision making context. The impact of these differences was examined to improve implementation efficiency. The robustness of the decision model was examined with respect to the preference functions to reduce the time burden imposed on the decision maker.

Data for decision making in a military context supports the distinction between value and utility functions. Relationships between value and utility functions and risk attitudes were found to be complex. Elicitation error was significantly smaller than the difference between value and utility functions. Risk attitudes were generally neither constant across the domain of the evaluation measure nor consistent between evaluation measures. An improved measure of differences between preference functions, the weighted root means square, is introduced and a goodness of fit criterion established. An improved measure of risk attitudes employing utility functions is developed.

Response Surface Methodology was applied to improve the efficiency of decision analysis utility model applications through establishing the robustness of decision models to the preference functions. An algorithm was developed and employs this information to provide a hybrid value-utility model that offers increased elicitation efficiency.

15. SUBJECT TERMS

Decision Analysis, Utility Theory, Elicitation Error, Operations Research, Decision Making, Response Surface Methodology, Decision Theory, Risk Attitudes, Systems Engineering.

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U	U	U	UU	423	(937) 255-2549, e-mail: Kenneth.Bauer@afit.edu

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